KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2024 Mathematics 2025

Unit 12 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents Page

1 Expanding Double Brackets

Worked Example	Your Turn
Expand and simplify: a) $(x+3)(x-4)$ b) $(2x+3)(3x-4)$	Expand and simplify: a) $(x+3)(x-7)$ b) $(2x+3)(3x-7)$

Dr Frost 299b and 299e

Worked Example	Your Turn
Expand and simplify: a) $(x-3)^2$ b) $(2x-3)^2$	Expand and simplify: a) $(x-7)^2$ b) $(3x-7)^2$

Worked Example	Your Turn
Expand and simplify: a) $(8x^2 - 7x)(4x^2 + 9x - 5)$ b) $(8x^2 - 9x + 4)(6x^2 + 5x)$	Expand and simplify: a) $(9y^2 - 7y)(5y^2 + 4y + 5)$ b) $(5y^2 - 6y + 7)(5y^2 + 8y)$

Worked Example	Your Turn
Expand and simplify: $(3x - 6)(3x + 1) - 4x(x + 1)$	Expand and simplify: $(2x - 2)(x - 4) - 3(x + 2)$

Worked Example	Your Turn
Expand and simplify: $(x + 5)(2x - 1) - (2x + 3)^2$	Expand and simplify: $(2x + 5)^2 - (3x - 4)(3x + 6)$

Worked Example	Your Turn
Expand and simplify: $(2x + 3)^2 - (2x - 1)^2$	Expand and simplify: $(3x - 1)^2 - (2x - 3)^2$

Worked Example	Your Turn
Expand and simplify: $\left(3-\frac{5}{z}\right)^2$	Expand and simplify: $\left(2-\frac{4}{y}\right)^2$

Dr Frost 299p

Extra Notes

2 Factorising by Grouping

Worked Example	Your Turn
Factorise: a) $3x(x+1)-5(x+1)$ b) $3x(x+1)^2-5(x+1)$	Factorise: a) $5x(x+1)-3(x+1)$ b) $5x(x+1)^2-3(x+1)$

Worked Example	Your Turn
Factorise: a) $2(x+8)^2 - 6(x+8)$ b) $2(x+8) - 6(x+8)^2$	Factorise: a) $8(x+2) - 4(x+2)^2$ b) $8(x+2)^2 - 4(x+2)$

Worked Example	Your Turn
Factorise: a) $2x^2 + 2x - 3x - 3$ b) $2x^2 + 2x + 3x + 3$	Factorise: a) $2x^2 - 2x - 3x + 3$ b) $2x^2 - 2x + 3x - 3$

Worked Example	Your Turn
Factorise $35xa - 20x + 21a - 12$	Factorise $10px - 35p + 16x - 56$

Extra Notes

3 Factorising Quadratics

Sum and Product

	Sum is Positive	**Sum is Negative**
Product is Positive	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = 9$	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = -9$
Product is Negative	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = 5$	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = -5$

	Sum is Positive	**Sum is Negative**
Product is Positive	Positive and Positive	Negative and Negative
Product is Negative	Positive and Negative where the size of the positive is greater than the size of the negative	Positive and Negative where the size of the negative is greater than the size of the positive

Worked Example	Your Turn
$\_\_ \times \_\_ = 140$ $\_\_ + \_\_ = -33$	$\_\_ \times \_\_ = 280$ $\_\_ + \_\_ = -43$

Fill in the Gaps

Q	*Integer 1*	*Integer 2*	*Sum*	*Product*
1	5	7		
2	-5	7		
3			-2	-35
4	-5		-12	
5			8	15
6	-3			-15
7	3			-15
8	-3	-5		
9			10	24
10	-4	6		
11			-2	-24
12		-6	-10	24

Quadratics

quadratum *v.* a square (Latin)

quadrat *n.*
1. A square frame, used to mark out an area of land to study its plants, animals, soil or other natural processes.

quadratic *n.*
1. A polynomial where the highest power of the variable is the second power (i.e. square).
Examples:
$x^2 + 3x - 5$ is a quadratic expression in x.
$t^2 - 9$ is a quadratic expression in t.
$5n^2 + n$ is a quadratic expression in n.

The general form of a quadratic expression is: $ax^2 + bx + c$
where a, b and c are numbers, $a \neq 0$ and x is the variable.

Quadratic expressions in x	Not quadratic expressions
x^2+4x+3	x^3+3x^2+6x+7
$2x^2+7x-\frac{13}{2}$	$3x^2+6x^{-1}+7$
$4x^2-3.5$	$x^2-6x^{\frac{1}{2}}+9$
$-x^2+3x$	
$-5x^2-x$	
$7-x^2$	
$7+\frac{x}{2}+5x^2$	

Monic means you have a single x^2, i.e. $a = 1$ in the general form $ax^2 + bx + c$

- The **coefficient** of an algebraic term is the number/constant in front of it. So, the coefficient of $3x^2$ is 3 and the coefficient of $5x^3$ is 5.
- A **constant term** is one without any variables in it. So, in $3x^2 + x + 5$, the constant term is 5.

Worked Example	Your Turn
Factorise: $3x^2 + 10x + 8$	Factorise: $3x^2 - 10x + 8$

Dr Frost 364a

Worked Example	Your Turn
Factorise: $3x^2 + 2x - 8$	Factorise: $3x^2 - 2x - 8$

Dr Frost 364b, 364c and 364d

Fill in the Gaps

Quadratic	$a \times c$	$\times$ to give ac $+$ to give b	Split the middle term	Group and Factorise	Factorised Quadratic
$2x^2 + 7x + 6$	12	$+4, +3$	$2x^2 + 4x + 3x + 6$	$2x(x + 2) + 3(x + 2)$	$(2x + 3)(x + 2)$
$3x^2 + 19x + 6$	18	$+18, +1$	$3x^2 + 18x + x + 6$	$3x(x + 6) + 1(x + 6)$	
$8x^2 + 6x - 9$	-72	$+12, -6$			
$5x^2 + 12x - 9$					
$9x^2 - 9x - 10$					
$6x^2 + x - 5$					
$8x^2 - 18x + 7$				$2x(4x - 7) - 1(4x - 7)$	
$4x^2 - 12x + 5$					
		$+15, +2$	$6x^2 + 15x + 2x + 5$		
				$4x(3x - 2) + 5(3x - 2)$	

Worked Example	Your Turn
Finish factorising: a) $(x + 2)(10x + 50)$ b) $(4x + 2)(10x + 50)$	Finish factorising: a) $(x + 2)(5x + 15)$ b) $(4x + 2)(5x + 15)$

Worked Example	Your Turn
Factorise: $6x^2 + 20x + 16$	Factorise: $6x^2 - 4x - 16$

Worked Example	Your Turn
Factorise: $x^2 + 20x + 96$	Factorise: $x^2 - 4x - 96$

Dr Frost 362a, 362b, 362c and 362d

Fill in the Gaps

Quadratic Expression	Sum	Product	Pair of Values		Factorised Expression
$x^2 + 8x + 15$	$+8$	$+15$	$+5$	$+3$	$(x + 5)(x + 3)$
$x^2 + 5x + 6$	$+5$	$+6$	$+3$		
$x^2 + 6x + 5$	$+6$	$+5$	$+5$		
$x^2 + 10x + 21$		$+21$			
$x^2 + 14x + 24$	$+14$				
$x^2 - 7x + 10$			-5		
$x^2 - 11x + 18$					
$x^2 + 3x - 10$			$+5$		
$x^2 + 3x - 18$					
$x^2 + 11x + 18$					
$x^2 - 4x - 21$			-7		
$x^2 - 8x - 9$					
$x^2 - 6x + 9$					
$x^2 + x - 20$	$+1$				
$x^2 - x - 6$					
$x^2 - 19x - 42$					
			-8	-3	
	$+4$		-1		

Worked Example	Your Turn
Factorise: $-9x^2 + 30x + 24$	Factorise: $-9x^2 + 42x - 24$

Worked Example	Your Turn
Factorise: $6x^2 - 23xy + 20y^2$	Factorise: $5x^2 - 17xy + 6y^2$

Extra Notes

4 Difference of Two Squares

Worked Example	Your Turn
Factorise: a) $x^2 - 9$ b) $9 - x^2$ c) $x^2 - 9y^6$ d) $16x^2 - 9y^6$	Factorise: a) $x^2 - 25$ b) $25 - x^2$ c) $x^2 - 25y^4$ d) $16x^2 - 25y^4$

Worked Example	Your Turn
Factorise: a) $2x^2 - 8$ b) $2x^2 - 8y^6$	Factorise: a) $2x^2 - 50$ b) $2x^2 - 50y^4$

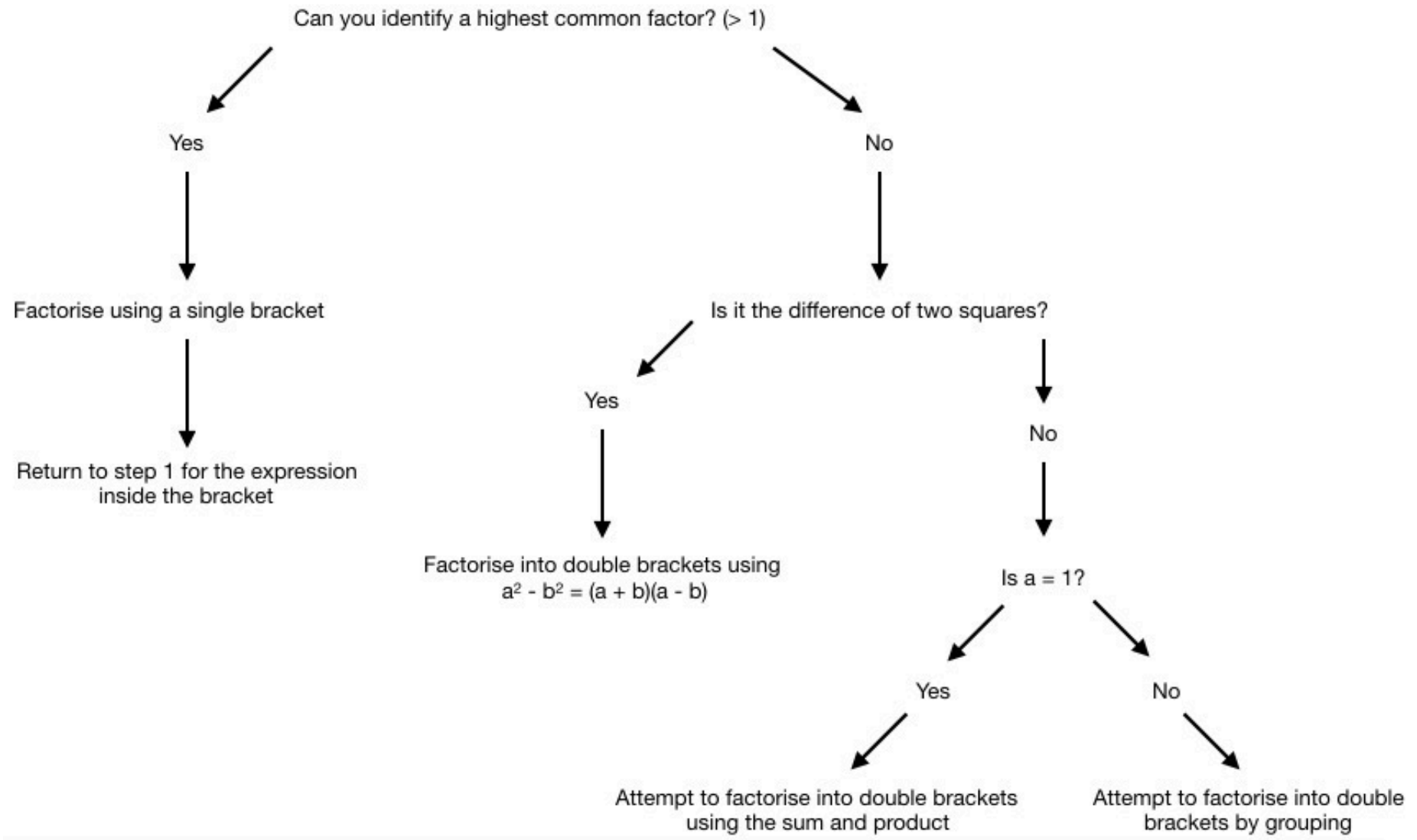
Worked Example	Your Turn
Factorise: a) $16x^2y - 9yz^2$ b) $2x^2y^3 - 8y^3z^2$	Factorise: a) $16xy^2 - 25xz^2$ b) $2x^4y^2 - 50x^4z^2$

Dr Frost 363e and 363f

Worked Example	Your Turn
Work out $51^2 - 49^2$	Work out $53^2 - 47^2$

Dr Frost 363g

Flowchart


Can you identify a highest common factor? (> 1)
Yes
No
Factorise using a single bracket
Return to step 1 for the expression inside the bracket
Is it the difference of two squares?
Yes
Factorise into double brackets using $a^2 - b^2 = (a + b)(a - b)$
No
Is a = 1?
Yes
Attempt to factorise into double brackets using the sum and product
No
Attempt to factorise into double brackets by grouping

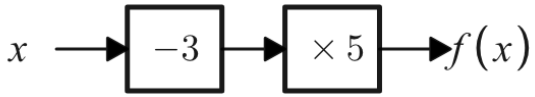
Worked Example	Your Turn
Factorise: a) $8x^3 - 18x$ b) $x^3 + 3x^2 + 2x$	Factorise: a) $8x^2 - 2$ b) $x^4 - x^3 - 6x^2$

Extra Notes

5 Basic Functions

Worked Example

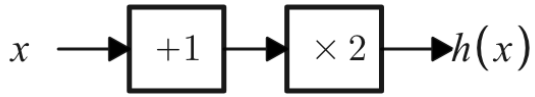
Below is a function machine.



Find an expression for $f(x)$.

Your Turn

Below is a function machine.



Find an expression for $h(x)$.

Fill in the Gaps

Input	Function Machine		Output	Function
x →	$\times 3$ →	$+8$ →	$f(x)$	$f(x) = 3x + 8$
x →	$\times 5$ →	-1 →	$f(x)$	
x →	$\times 2$ →	→	$g(x)$	$g(x) = 2x - 7$
x →	-1 →	→	$f(x)$	$f(x) = 4(x - 1)$
x →	→	$+2$ →	$h(x)$	$h(x) = \frac{x}{3} + 2$
x →	$\div 2$ →	-5 →	$f(x)$	
x →	$+7$ →	$\div 4$ →	$f(x)$	
x →	*square* →	$+3$ →	$g(x)$	
x →	$+2$ →	*square root* →	$f(x)$	
x →	→	→	$f(x)$	$f(x) = 10x^2$
x →	→	→		$g(x) = \sqrt{x} + 8$
x →	→	→		$h(x) = \frac{x^3}{2}$
x →	*reciprocal* →	$+8$ →	$f(x)$	

Fill in the Gaps

Input	Function Machine			Output	Function
x →	$\times 3$ →	-1 →	$\div 4$ →	$f(x)$	$f(x) = \dfrac{3x-1}{4}$
x →	$+2$ →	$\div 3$ →	*square root* →	$f(x)$	
x →	$+3$ →	*square* →	-5 →	$h(x)$	
x →	*square root* →	→	$+1$ →	$f(x)$	$f(x) = 4\sqrt{x} + 1$
x →	*reciprocal* →	→	→	$g(x)$	$g(x) = 2\left(\dfrac{1}{x} - 3\right)$
x →	→	→	→	$f(x)$	$f(x) = \dfrac{1}{3x} - 1$
x →	→	→	→		$f(x) = \left(\dfrac{x+2}{3}\right)^2$
x →	→	→	→		$g(x) = \dfrac{1}{4x-3}$

Worked Example	Your Turn
If $f(x) = 3x + 4$, evaluate: a) $f(2)$ b) $f(-4)$	If $g(x) = -3x + 7$, evaluate: a) $g(5)$ b) $g(-2)$

Worked Example	Your Turn
If $f(x) = 2x^2 + 3x$, evaluate: a) $f(4)$ b) $f(-2)$	If $g(x) = 3x^2 - 4x$, evaluate: a) $g(5)$ b) $g(-2)$

Dr Frost 432a

Find the values of:

$f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$ for the following functions:

	$f(3)$	$f(-5)$	$f(\frac{1}{2})$
$f(x) = x + 5$			
$f(x) = 2x + 5$			
$f(x) = \frac{x}{2} + 5$			
$f(x) = \frac{x}{2} + 5x$			
$f(x) = x^2 + 5x$			
$f(x) = 5x^2$			
$f(x) = \frac{5}{x^2}$			
$f(x) = \frac{1}{5x^2}$			
$f(x) =$	16	-24	$\frac{7}{2}$
	7		
		$\frac{25}{2}$	
			-11

For the last three questions, find a possible function for $f(x)$, and therefore, find $f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$.

Worked Example	Your Turn
If $f(x) = 3x + 4$, find x when $f(x) = 19$	If $g(x) = -3x + 7$, find x when $g(x) = 1$

Worked Example	Your Turn
If $f(x) = x^2 + 3$, find x when $f(x) = 19$	If $g(x) = x^2 - 4$, find x when $g(x) = 21$

Worked Example	Your Turn
Given $f(x) = 2x^2 + 2x - 2$ $g(x) = 2x^2 + x - 4$ Solve $f(x) = g(x)$	Given $g(x) = 3x^2 + x + 19$ $h(x) = 3x^2 - 3x + 3$ Solve $g(x) = h(x)$

Worked Example	Your Turn
If $f(x) = x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 5x - 2$, evaluate $f(x - 2)$	If $g(x) = 5x^2 - 2x + 3$, evaluate $g(x - 3)$

Worked Example	Your Turn
Given $f(t) = 4t^2 + 3t$ Find $f(t) - f(t-2)$, giving your answer in the form $at + b$.	Given $f(t) = 3t^2 - 4t$ Find $f(t-4) - f(t)$, giving your answer in the form $at + b$.

Dr Frost 432g

Extra Notes

6 Changing the Subject

Frayer Model – Formula

Definition	**Characteristics**
Examples	**Non-Examples**

Frayer Model – Subject of a Formula

Definition	**Characteristics**
Examples	**Non-Examples**

Is a the subject?

$a = 3x + 1$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 1 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4a = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4b + 2 = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = 5a - 7b + 3$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a^2 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{1}{2}b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{7b + 55c}{2}$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{b} = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{a} = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 0 = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject

Fluency Practice

Formula	Is a the subject?
$a = b + 3$	
$b + 3 = a$	
$a + 3 = b$	
$a + c = b$	
$ac = b$	
$a = bc$	
$a = bc - 6$	
$a = bc - x$	
$a = bc - a$	
$a = bc - a^2$	
$-a = b + 3$	
$\frac{1}{a} = b + 3$	

Formula	Is a the subject?
$a^2 = b + 3$	
$a = b^2 + 3$	
$2a = b^2 + 3$	
$\sqrt{a} = b^2 + 3$	
$a = \sqrt{\frac{b^2 + 3}{2}}$	
$\sqrt{\frac{b^2 + 3}{2}} = a$	
$\sqrt{\frac{b^2 + 3}{2a}} = a$	

Worked Example	Your Turn
Make x the subject of the following formulae: a) $y = mx + c$ b) $y = -efx + c^2$	Make x the subject of the following formulae: a) $y = abx - c$ b) $y = cdx - e^2$

Worked Example	Your Turn
Make x the subject of the following formulae: a) $y = \frac{x}{m} + c$ b) $y = -\frac{x}{ef} + c^2$	Make x the subject of the following formulae: a) $y = \frac{x}{ab} - c$ b) $y = -\frac{x}{cd} - e^2$

Worked Example	Your Turn
Make x the subject of the following formulae: a) $y = p(x + q)$ b) $y = 2p(3x + 4q)$	Make x the subject of the following formulae: a) $y = p(x - q)$ b) $4y = 3p(2x + q)$

Worked Example	Your Turn
Make a the subject of the following formulae: a) $2(a+b)^2 = 4c$ b) $2\sqrt{a-b} = 4c$	Make a the subject of the following formulae: a) $3(a-b)^2 = 6c$ b) $3\sqrt{a+b} = 6c$

Fill in the Gaps

Q	$\mathbf{a} =$	$\mathbf{b} =$	$\mathbf{c} =$
1	$a = b + c$		
2	$a = b - c$		
3		$b = ac$	
4			$c = \frac{2b}{a}$
5	$a = 2b + c$		
6		$b = \frac{a + c}{2}$	
7		$b = \frac{a}{2} + c$	
8			$c = b^2 - \frac{a}{2}$
9		$b = \frac{a}{2} + \sqrt{c}$	
10	$a = \frac{2b - 2\sqrt{c}}{3}$		

Worked Example	Your Turn
Make a the subject of the following formula: $ax + ay = 3$	Make a the subject of the following formula: $ak + am = 5$

Dr Frost 391a

Worked Example	Your Turn
Make a the subject of the following formula: $ax + 2y = 5y + am$	Make a the subject of the following formula: $ab + 3y = 7y + ak$

Worked Example	Your Turn
Make x the subject of the following formula: $ax + ay = cx + by$	Make x the subject of the following formula: $yx + wz = 3xz + 3yz$

Dr Frost 391c

Worked Example	Your Turn
Make x the subject of the following formula: $w = \frac{x + a}{x - a}$	Make x the subject of the following formula: $w = \frac{x + 2y}{x - y}$

Worked Example	Your Turn
Make a the subject of the formula: $b = \sqrt{\frac{3a+1}{4a-3}}$	Make x the subject of the formula: $y = \sqrt{\frac{5x-6}{2x-1}}$

Worked Example	Your Turn
Given that $a > 0$, make a the subject of the formula: $b = \frac{4 - 3a^2}{5a^2 - 1}$	Given that $x > 0$, make x the subject of the formula: $y = \frac{6x^2 + 4}{3x^2 - 5}$

Extra Notes

7 Inverse Functions

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x + 3}{4}$	Find the inverse function: $g(x) = \frac{4x - 3}{2}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{3}{2-5x}$	Find the inverse function: $g(x) = \frac{4}{5-3x}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x - 3}{x + 2}$	Find the inverse function: $g(x) = \frac{4x - 5}{x - 3}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \sqrt{\frac{3x-2}{x-4}}$	Find the inverse function: $g(x) = \sqrt{\frac{5x-4}{x+3}}$

Worked Example	Your Turn
Given that $f(x) = \left(\frac{5x-3}{9x-10}\right)^2$, find $f^{-1}(x)$	Given that $g(x) = \left(\frac{2x-9}{10x-1}\right)^2$, find $g^{-1}(x)$

Fill in the Gaps

$f(x)$	Write as $y = \cdots$	Swap x and y	Make y the subject	Write as $f^{-1}(x) = \cdots$
$f(x) = 3x - 1$	$y = 3x - 1$	$x = 3y - 1$	$x + 1 = 3y$ $\quad \frac{x+1}{3} = y$	$f^{-1}(x) = \frac{x+1}{3}$
$f(x) = 2x + 5$				
$f(x) = x^2 + 8$				
$f(x) = \sqrt{x-3}$	$y = \sqrt{x-3}$	$x = \sqrt{y-3}$	$x^2 = y - 3$	
$f(x) = \frac{x+2}{7}$				
$f(x) = \frac{x}{3} - 5$				
$f(x) = \frac{9}{x}$				
$f(x) = \frac{4}{x+3}$				

Extra Notes