



Year 8 2024 Mathematics 2025 Unit 7 Booklet

HGS Maths







Dr Frost Course



Name:

Class:

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1 Prime Factorisation

1.1 Prime Factors

3 is a prime factor of 36 (True / False)

9 is a prime factor of 36 (True / False)

1 is a prime factor of 36 (True / False)

2 is a prime factor of 36 (True / False)

7 is a prime factor of 36 (True / False)

intelligent i lactice
7 is a prime factor of 12 (True / False)
6 is a prime factor of 12 (True / False)
5 is a prime factor of 12 (True / False)
4 is a prime factor of 12 (True / False)
3 is a prime factor of 12 (True / False)
2 is a prime factor of 12 (True / False)
1 is a prime factor of 12 (True / False)
1 is a prime factor of 27 (True / False)
2 is a prime factor of 27 (True / False)
3 is a prime factor of 27 (True / False)
7 is a prime factor of 27 (True / False)
9 is a prime factor of 27 (True / False)
13 is a prime factor of 27 (True / False)
13 is a prime factor of 26 (True / False)
3 is a prime factor of 26 (True / False)
2 is a prime factor of 26 (True / False)
2 is a prime factor of 25 (True / False)
5 is a prime factor of 25 (True / False)
12.5 is a prime factor of 25 (True / False)

1.2 Product of Prime Factors

Product of Prime Factors	Yes / No ?
9 × 11	
19 × 11	
19×11^2	
$2 \times 19 \times 11^2$	
$2 \times 19 \times 101^2$	

Product of Prime Factors	Yes / No ?
5 + 7	
5 × 7	
4×7	
3 × 7	
2 × 7	
1×7	
$1 \times 7 \times 9$	
$2 \times 7 \times 9$	
$2 \times 7 \times 11$	
$2 \times 7 + 11$	
$2 \times 7 \times 11 \times 21$	
$2 \times 7 \times 11 \times 31$	
$1 \times 2 \times 7 \times 11 \times 31$	
$2 \times 7 \times 7 \times 11 \times 31$	
$2 \times 7^2 \times 11 \times 31$	
$2^2 \times 7^2 \times 11 \times 31$	
$2^3 \times 7^2 \times 11 \times 31$	
$2^3 \times 7^2 \times 11^5 \times 31^4$	
$1^3 \times 7^2 \times 11^5 \times 31^4$	
$2^3 \times 7^2 \times 11^5 \times 41^4$	

Worked Example	е	Your Turn								
Express 24 as a product of prime factors		Express 48 as a product of prime factors								

Worked Example	Your Turn							
Express 924 as a product of prime factors	Express 588 as a product of prime factors							

Worked Example	Your Turn							
Express $2^3 \times 3$ as an ordinary number	Express $3^2 \times 5$ as an ordinary number							

Fill in the Gaps

Number	Prime Factor Decomposition	Index Form
6		
	$2 \times 2 \times 3$	
48		
240		
		$2^4 \times 3^2 \times 5$
	$2 \times 2 \times 2 \times 3 \times 3$	
216		
		$2^2 \times 3^2$
	$2 \times 2 \times 3 \times 3 \times 5 \times 5$	
		$2 \times 3 \times 5$
420		
12 600		



Fill in the Gaps

Number	Factor Tree	Product of Prime Factors	Number	Factor Tree	Product of Prime Factors
10		2 × 3 × 3	20	20	
18		2×3^{2}	20	5 2	
4.2			55	55	
72			55	5	
10			45	45	
12			45		
27	27		26	36	
27			36		
	60		106		
60			126		
				·	



Fill in the Gaps

Factor Tree	Product of Prime Factors	Factor Tree	Product of Prime Factors
135	135	330	330
	3 × 3 × 3 × 5		
	$3^{3} \times 5$		
220	220	525	525
	$2 \times 2 \times 5 \times 5$		
		9	
0000	□ × □ ³	0000	$x^{2} \times 13$

1.3 Using Product of Prime Factors

Worked Example	Your Turn $84 = 2^2 \times 3 \times 7$ How is 504 written as its product of prime factors?							
$84 = 2^2 \times 3 \times 7$ How is 840 written as its product of prime factors?								

Worked Example							Your Turn											
$X = 378 \times 12^4$ Write X as a product of its prime factors.								$N = 242 \times 15^2$ Write N as a product of its prime factors.										
						,												

Worked Example	Your Turn									
$C = 3^a \times 5^b$	$D = 3^e \times 7^f$									
a) 3 <i>C</i> b) 5 <i>C</i> c) 25 <i>C</i>	a) 3 <i>D</i> b) 7 <i>D</i> c) 27 <i>D</i>									

			F	ill in	the	Gaps
$a \div b$	2					32
a imes b	2 ⁹ ×3 ¹⁶ ×5 ⁶				2 ⁹ ×3 ¹⁷ ×5 ⁶	
a - b	$ (2-1) \times 2^4 \times 3^8 \times 5^3 $ = $2^4 \times 3^8 \times 5^3 $			$2^{4} \times 3^{9} \times 5^{3}$		
a + b	$(2+1) \times 2^4 \times 3^8 \times 5^3$ = $2^4 \times 3^9 \times 5^3$		2 ⁵ ×3 ⁹ ×5 ³			
q	2 ⁴ ×3 ⁸ ×5 ³	$2^{4} \times 3^{8} \times 5^{3}$	$2^{4} \times 3^{8} \times 5^{3}$	$2^{4} \times 3^{8} \times 5^{3}$	$2^{4} \times 3^{8} \times 5^{3}$	$2^{4} \times 3^{8} \times 5^{3}$
ø	2 ⁵ ×3 ⁸ ×5 ³	$2^{4} \times 3^{9} \times 5^{3}$				

1.4 Factors from Prin	ne Factors
10 is a factor of $2 \times 5 \times 7 \times 11 \times 17$	(True / False)
10 is a factor of $2 \times 5^3 \times 7 \times 11 \times 17$	(True / False)
15 is a factor of $2 \times 5^3 \times 7 \times 11 \times 17$	(True / False)
25 is a factor of $2 \times 5^3 \times 7 \times 11 \times 17$	(True / False)
22 is a factor of $2 \times 5^3 \times 7 \times 11 \times 17$	(True / False)

2 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
3 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
5 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
7 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
4 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
6 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
14 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
21 is a factor of $2 \times 3 \times 7 \times 13$	(True / False)
15 is a factor of 2 \times 3 \times 7 \times 13	(True / False)
15 is a factor of $2 \times 3 \times 5 \times 7 \times 13$	(True / False)
30 is a factor of 2 \times 3 \times 5 \times 7 \times 13	(True / False)

6	
9 is a factor of $2 \times 3 \times 5 \times 7 \times 13$	(True / False)
9 is a factor of $2 \times 3^2 \times 5 \times 7 \times 13$	(True / False)
9 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
4 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
4 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
8 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
16 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
2 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
28 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
28 is a factor of $2^2 \times 3^2 \times 5 \times 7 \times 23$	(True / False)
28 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$	(True / False)

Fluency Practice

Number	Prime Factor Decomposition	Factor	Yes/No
2520	$2^3 \times 3^2 \times 5 \times 7$	$15 = 3 \times 5$	Yes
2520		8	
2520		25	
2520		45	
1320		22	
1320		45	
1320		88	
20250		12	
20250		27	
20250		15	
20250		75	
15120		16	
15120		21	
15120		70	
15120		18	

1.5 Types of Numbers from Prime Factors

- Square numbers have even powers in their prime factorisation.
- Cube numbers have powers which are multiples of 3.

Product of Prime Factors	Square Number	Cube Number	Neither
$5^2 \times 11$			
$5^2 \times 11^8$			
$5^{6} \times 11^{8}$			
5 ⁶ × 11 ⁹			
$5^6 \times 11^9 \times 17^{13}$			

Product of Prime Factors	Square Number	Cube Number	Neither
2 × 3			
3 × 3			
3 ²			
3 ³			
$3^3 \times 7$			
$3^3 \times 7^2$			
$3^3 \times 7^3$			
$3^2 \times 7^2$			
$5^2 \times 7^2$			
$2 \times 5^2 \times 7^2$			
$2^2 \times 5^2 \times 7^2$			
$2^3 \times 5^2 \times 7^2$			
$2^3 \times 5^3 \times 7^3$			

Product of Prime Factors	Square Number	Cube Number	Neither
$2^4 \times 5^4 \times 7^4$			
$2^5 \times 5^5 \times 7^5$			
$2^6 \times 5^6 \times 7^6$			
$2^7 \times 5^7 \times 7^7$			
$2^8 \times 5^8 \times 7^8$			
$2^9 \times 5^9 \times 7^9$			
$2^9 \times 5^9 \times 7^6$			
$2^2 \times 5^9 \times 7^6$			
$2^3 \times 5^9 \times 7^6$			
$2^6 \times 5^{18} \times 7^{12}$			
$2^6 \times 5^{18} \times 7^{12} \times 11$			
$2^6 \times 5^{18} \times 7^{12} \times 11^2$			
$2^6 \times 5^{18} \times 7^{12} \times 11^3$			

1.6 Using Prime Factorisation to Simplify Fractions

Worked Example	Your Turn									
Simplify $\frac{693}{1925}$	Simplify $\frac{693}{1155}$									

1.7 Using Prime Factorisation to Find Roots

	١	No	rke	ed	Exa	am	ple	е	Your Turn										
a) b)	a) Find $\sqrt{784}$ b) Find $\sqrt[3]{216}$										a) Find $\sqrt{324}$ b) Find $\sqrt[3]{512}$								

1.8 Number of Factors

To get the number of factors of a number in prime factorised form, add one to each power and times the powers together.

	Worked Example										Your Turn								
a)	a) How many factors does 36 have?										a) How many factors does 72 have?								
b)	b) How many factors does 37									b) How many factors does 73									
c)	c) How many factors does 38								c)	Но	we: wr	mar	ıy fa	icto	rs d	oes	74		
	ha	ver									ha	ive :							

2 Probability

2.1 Probability Scale

- Probability is a numerical measure of how likely or unlikely an event is to occur.
- Probabilities are usually written as fractions, but can be written in any form equivalent to that fraction, e.g., $\frac{3}{4} = 0.75 = 75\%$
- Probabilities can be anywhere between 0 (impossible) and 1 (certain):



Worked Example		Your Turn						
Place a probability of $\frac{3}{8}$ on a line and state how likely it is.	ne, Pla and	Place a probability of $\frac{6}{8}$ on a line, and state how likely it is.						

	V	No	rke	ed	Exa	am	ple	9	Your Turn											
 Describe using impossible, unlikely, even chance, likely or certain the probability that: a) You will walk to Mars. b) The day after Monday is Tuesday. c) You roll a three on a fair die. d) You flip a tails on a fair coin. 											 Four Turn Describe using impossible, unlikely, even chance, likely or certain the probability that: a) You roll an even number on a fair die. b) The day after Monday is Wednesday. c) You roll a number between 1 and 6 on a fair die. 									
										d)	You will go to space in your life.									

2.2 Probability of Single Events
Worked Example	Your Turn									
 A bag of sweets contains only 4 red sweets, 2 yellow sweets and 4 green sweets. a) What is the probability of choosing a red sweet? b) What is the probability of choosing a red or yellow sweet? 	 A bag of sweets contains only 8 red sweets, 4 yellow sweets and 8 green sweets. a) What is the probability of choosing a red sweet? b) What is the probability of choosing a red or yellow sweet? 									
c) What is the probability of choosing a mint?	c) What is the probability of choosing a mint?									



2.3 Mutually Exclusive Events

Frayer Model – Mutually Exclusive Events

Definition	Characteristics
Examples	Non-Examples

2.4 Exhaustive Events

Worked Example	Your Turn											
There are green, red and blue counters in a bag.	It is either raining or not raining.											
The chance of picking a green counter is 0.1.	The chance that it rains tomorrow is 0.55.											
The chance of picking a red counter is 0.3.	What is the chance that it does not rain?											
What is the chance of picking a blue counter?	Image:											

Wo	Worked Example										Your Turn											
The sides of colours. The shows the pr will land on e purple or blu twice as likel	a spin spinne obabi each o ie. The y as fo	ner are er is bia lity that f the co e probal or Purpl	The tha rec pro for cou	e tab It a c I, gre Dbabi Red unter	le si oun en, ility . Wo tak	ter ta yello for G ork o en a	s each aken ow, bl Green ut the t rand	n of t at ra ue or is tw e pro dom	he j ndo r bla vice bab will	proba m wi ack. T as lik vility t be re	biliti ll be he ely a hat a	es s										
	probability the spinner will land on black.										Green	Yello	w	Blue	Black							
Colour	Red	Black	Purp	ole	Blu	e		robabili	ty			0.15	5	0.25	0.3							
Probability	0.1				0.3	;																
	1																					
											_											

2.5 Expectation

Wo	orke	d Ex	amp	ble	Your Turn									
The table probabili taken at black, ye	shov ties tl randc llow c	ws ea hat a om wi or pur	ch of count Il be v rple.	the ter white,	The sides of a spinner are labelled with colours. The spinner is biased. The table shows the probability that the									
Colour Probability Amelia se	White 0.2 elects	Black 0.2	Yellow 0.2 unter	Purple 0.4	spinner will land on each of the colours black, blue, red or white.									
random s estimate times the	50 tim for th e cour	າes. V າe tot າter v	Vork o :al nui vill be	out ar mber e white	Harry spins the spinner 60 times. Work out an estimate for the total number of times the spinner will land on red.									
					Image: state stat									
						Image: Second								

Worked Example	Your Turn									
Keith designs a game. It costs £1.60 to play the game.	Bob designs a game. It costs $50p$ to play the game.									
The probability of winning the game is $\frac{2}{5}$	The probability of winning the game is $\frac{1}{4}$									
The prize for each win is £3	The prize for each win is ± 1.50									
80 people play the game.	100 people play the game.									
Work out an estimate of the profit that Keith should expect to make.	Work out an estimate of the profit that Bob should expect to make.									

2.6 Relative Frequency

Worked Example	Your Turn									
A coin is flipped 30 times. The results are:	A coin is flipped 20 times. The results are:									
нтнннннтттннтт	тнтттттннн									
тттнннттннтнтнн	ннтнтннннн									
 a) What are the relative frequencies for heads and tails? 	 a) What are the relative frequencies for heads and tails? 									
 b) The coin is flipped 300 times. Estimate how many times the coin will land on tails. 	 b) The coin is flipped 100 times. Estimate how many times the coin will land on tails. 									

2.7 Listing Outcomes

Outcomes and Events

When we roll a six-sided die there are 6 outcomes. The outcomes are the results we could get from rolling the die.

Outcomes: 1, 2, 3, 4, 5, 6.

An event is one or more outcomes.

Event	Outcome(s)
A square number	
A 3	
A factor of 8	
A multiple of 3	

Here is a spinner numbered 1 to 8. List the outcome(s) for each event.



Event	Outcome(s)
An even number	
A number less than 3	
A number greater than 7	
An integer	
A 5	
A 2-digit number	

Worked Example									Your Turn										
List all the ways of arranging the letters in the word: CAT								ne	List all the ways of arranging the letters in the word: DOG										

Worked Example									Your Turn										
I flip a coin and then roll a six- sided die. List the possible outcomes.									I flip a coin and then roll a 4- sided die. List the possible outcomes.										

Worked Example										Your Turn									
The first five positive integers are 1, 2, 3, 4, 5. I choose two numbers from this list without repeats. Write down all possible combinations of two numbers I can choose.									The four square numbers are 1, 4, 9, 16. I choose two numbers from this list without repeats. Write down all possible combinations of two numbers I can choose.										

2.8 Sample Space Diagrams



Activity



Worked Example										Your Turn									
Elisa throws a spinner with faces labelled R, G, B and Y and a four- sided dice numbered -1, -2, -3 and -4 at the same time. Draw a sample space diagram.									Elisa throws a spinner with faces labelled R, G, B and Y and a four- sided dice numbered 2, 4, 6 and 8 at the same time. Draw a sample space diagram.									es ur- id	

Worked Example										Your Turn									
Noel throws a four-sided dice and a four-sided dice at the same time and adds up the scores. Draw a sample space diagram.										Kayleigh throws a four-sided dice numbered 2, 4, 6 and 8 and a four-sided dice at the same time and adds up the scores. Draw a sample space diagram.									nd

Worked Example	Your Turn							
Carolina throws a four-sided dice numbered 1, 3, 5 and 7 and a four-sided dice numbered -1, - 2, -3 and -4 at the same time and multiplies the scores.	Carolina throws a six-sided dice and a four-sided dice numbered -1, -2, -3 and -4 at the same time and multiplies the scores. Draw a sample space diagram.							
Draw a sample space diagram.								

Wo	rked		Your Turn												
Paul thrownumbered four-sided and 7 at t the difference scores.	ws a for d 2, 4, 6 d dice r he sam ence be	ur-side 5 and 8 number ne time etween	5 d	Kayleigh throws a four-sided dice and a six-sided dice at the same time and find the difference between the scores. Draw a sample space diagram.											
Draw a sa	mple s	pace di													

Worked Example

Your Turn

Lisa throws a four-sided dice and a six-sided dice at the same time and find the difference between the scores. The sample space diagram below shows all the possible outcomes.

	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0
5	4	3	2	1
6	5	4	3	2

Find the probability that Lisa gets a difference which is 3 or more.

Joel throws a four-sided dice numbered 2, 4, 6 and 8 and a four-sided dice at the same time and find the difference between the scores. The sample space diagram below shows all the possible outcomes.

	2	4	6	8
1	1	3	5	7
2	0	2	4	6
3	1	1	3	5
4	2	0	2	4

Find the probability that Joel gets a difference which is 4 or less.

Worked Example	Your Turn							
I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 4.	I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 4.							



Fill in the Gaps

Sample	Spa	ce	Probability Questions				
A fair four-sided	1	1	2	3	4	Find the probability that the total score is 7.	
1 to 4. The spinner is spun twice, and the	2	2	4	4 5	6	Find the probability that the total score is greater than 4.	
two scores added together.	3 4	4 5	5 6	6 7	7 8	Find the probability that the total score is a multiple of 3.	
A fair four-sided dice	0	2	3	4	5	Find the probability that the total score is 8.	
and 5. The spinner is spun twice, and the	3	4				Find the probability that the total score is less than 7.	
two scores added together.	4 5				10	Find the probability that the total score is a multiple of 4.	
A fair four-sided dice		1	2	3	4	Find the probability that the total score is even.	
and 4. The spinner is spun twice, and the	1	1			8	Find the probability that the total score is greater than 6.	
two scores multiplied together.	3 4		6			Find the probability that the total score is prime.	
A fair four-sided		2	3	5	7	Find the probability that the difference is zero.	
2, 3, 5 and 7. The spinner is spun and	2 3	0		2		Find the probability that the difference is odd.	
between the two scores recorded.	5 7	5				Find the probability that the difference is two or more.	
Two fair four-sided spinners are spun,		1	2	3	4	Find the probability that the total score is 10.	
and the scores added together. The first spinner is numbered	3 5					$\frac{5}{16}$	
1, 2, 3 and 4 and the second spinner is numbered 3, 5, 7 and 9.	7 9					$\frac{10}{\frac{3}{2}}$	
						0	

3 Expanding Single Brackets

3.1 Distributive Law

Frayer Model – Distributive Law								
Definition	Characteristics							
Examples	Non-Examples							

	Worked Example										Your Turn								
Use the distributive property to calculate: a) $7 \times (80 + 3)$ b) $(70 + 8) \times 3$ c) $8 \times (30 - 7)$											Use the distributive property to calculate: a) $3 \times (80 + 7)$ b) $(30 + 8) \times 7$ c) $-8 \times (70 - 3)$								

3.2 Expanding Single Brackets without Powers

	Worked Example										Your Turn											
Ex a) b)	Expand: a) $2(x-3)$ b) $-2(x-3)$											Expand: a) $2(3-x)$ b) $-2(3-x)$										

3.3 Expanding Single Brackets with Powers

Worked Example	Your Turn										
Expand: a) $2x(x-3)$ b) $-2x(x-3)$ b)	Expand: a) $2x(3-x)$ b) $-2x(3-x)$										

3.4 Expanding Single Brackets with Index Laws
Worked Example	Your Turn									
Expand and simplify: a) $a^{3}bc(10b^{2}c^{2} + 9a^{2})$ b) $4a^{5}b^{2}(3a^{4}b^{4} - 5b^{2})$	Expand and simplify: a) $a^{3}b^{5}(3a^{3}b + 7ab^{4}c)$ b) $7x^{5}y^{4}(6x^{2}y + 5x^{4}y)$									

3.5 Expanding and Simplifying Single Brackets

Worked Example										Your Turn												
Ex	Expand and simplify:									Expand and simplify:												
a) b)		4 + 7(6x - 5) 8x + 6 + 7(5x + 9)										b) $6z + 3 + 5(7z + 2)$										



Worked Example										Your Turn											
Expand and simplify: a) $2x(x-1) - 3x(x-4)$ b) $2x(x-1) - 3(x-4)$											Expand and simplify: a) $2x(x-1) - 5x(x-4)$ b) $2x(x-1) - 5(x-4)$										

Worked Example										Your Turn												
Ex	Expand and simplify: a) $5 - (2 - 3x)$											Expand and simplify: a) $5x - (x - 6)$										
b)		3x(5x+4) - (2x-1)										b) $5(1-5x) - (x+3)$										

Worked Example	Your Turn								
Find an expression for the area of the shape below.	Find an expression for the area of the shape below.								
$2x + 6$ $2 \qquad 8x + 7$ 4	2 $7x + 5$ 4								
Give your answer in its simplest form.	Give your answer in its simplest form.								