NAME:

Date to be handed in:

MARK (out of 100):

PAPER Q

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

<u>Questions to revise</u>:

1 Express $\frac{6}{4x^2+8x-5}+\frac{3x+1}{2x-1}$ as a single fraction in its simplest form. **(4 marks)**

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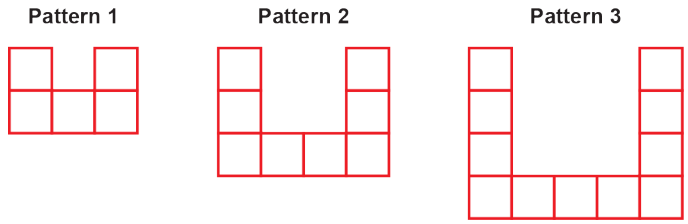
2 Jacob is making some patterns out of squares. The first 3 patterns in the sequence are shown.



a Find an expression, in terms of n, for the number of squares required to make pattern n. **(2 marks)**

Jacob uses a total of 948 squares in constructing the first k patterns.

b Show that $3k^2+7k-1896=0$ **(2 marks)**

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3 $g(x)=3\sin\left(\frac{x}{6}\right)^3-\frac{1}{10}x-1$, $-40<x<20$, x is in radians.

a Show that the equation $g(x)=0$ can be written as $x=6\left(\sqrt[3]{\arcsin\left(\frac{1}{3}+\frac{1}{30}x\right)}\right)$ **(3 marks)**

b Using the formula $x_{n+1}=6\left(\sqrt[3]{\arcsin\left(\frac{1}{3}+\frac{1}{30}x_n\right)}\right)$, $x_0=4$ find, to 3 decimal places, the values of x_1, x_2 and x_3. **(2 marks)**

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4 **a** When θ is small, show that the equation $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta-1}$ can be written as $\frac{1}{1-3\theta}$ **(4 marks)**

b Hence write down the value of $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta-1}$ when θ is small. **(1 mark)**

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5 **a** Prove that the sum of the first n terms of an arithmetic series is $S=\frac{n}{2}\left(2a+(n-1)d\right)$ **(3 marks)**

b Hence, or otherwise, find the sum of the first 200 odd numbers. **(2 marks)**

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6 $\frac{x^3+8x^2-9x+12}{x+6}=Ax^2+Bx+C+\frac{D}{x+6}$

Find the values of the constants A, B, C and D. **(5 marks)**

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7 Prove by contradiction that there are infinitely many prime numbers. **(6 marks)**

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8 A curve has the equation $y = \ln 3x - e^{-2x}$

Show that the equation of the tangent at the point with an x-coordinate of 1 is

$$y = \left(\frac{e^2 + 2}{e^2}\right)x - \left(\frac{e^2 + 3}{e^2}\right) + \ln 3$$ **(6 marks)**

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9 $f(x) = \frac{6}{x} + \frac{3}{x^2} - 7x^{\frac{5}{2}}$

a Find $\int f(x)\,dx$ **(3 marks)**

b Evaluate $\int_4^9 f(x)\,dx$, giving your answer in the form $m + n\ln p$, where m, n and p are rational numbers. **(3 marks)**

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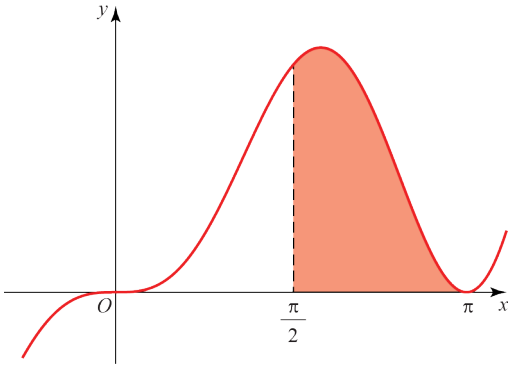
10 **a** Show that $\cos 7x + \cos 3x = 2\cos 5x \cos 2x$ by expanding $\cos(5x + 2x)$ and $\cos(5x - 2x)$ using the compound-angle formulae. **(3 marks)**

b Hence find $\int (\cos 5x \cos 2x)\,dx$ **(3 marks)**

11



The diagram shows part of the curve with equation $y = x\sin^2 x$.

The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the x-axis is shown shaded in the diagram.

Find the area of the shaded region. **(7 marks)**

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12 **a** Prove that $(\sin 3\theta + \cos 3\theta)^2 \equiv 1 + \sin 6\theta$ **(3 marks)**

b Use the result to solve, for $0 \le \theta \le \frac{\pi}{2}$, the equation $(\sin 3\theta + \cos 3\theta) = \sqrt{\frac{2+\sqrt{2}}{2}}$

Give your answer in terms of π. Check for extraneous solutions. **(4 marks)**

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13 $f(x) = |2x + 3| - 4, x \in \mathbb{R}$

a Sketch the graph of $y = f(x)$, labelling its vertex and any points of intersection with the coordinate axes. **(5 marks)**

b Find the coordinates of the points of intersection of $y = |2x + 3| - 4$ and $y = -\frac{1}{4}x + 2$ **(5 marks)**

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14 **a** Show that in ΔKLM with $\overrightarrow{KL} = 3\mathbf{i} + 0\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{LM} = 2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, $\angle KLM = 66.4°$ to one decimal place. **(7 marks)**

b Hence find $\angle LKM$ and $\angle LMK$. **(3 marks)**

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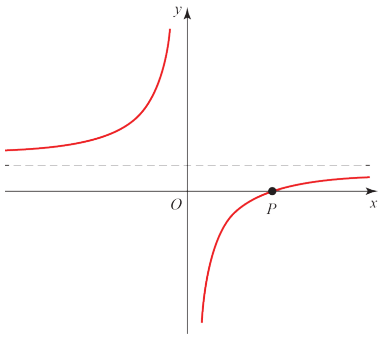
15 The diagram shows the curve C with parametric equations $x = t + 2$, $y = \frac{t-1}{t+2}$, $t \neq -2$.

The curve passes through the x-axis at P.



a Find the coordinate of P. **(2 marks)**

b Find the cartesian equation of the curve. **(2 marks)**

c Find the equation of the normal to the curve at the point $t = -1$.
Give your answer in the form $ax + by + c = 0$ **(6 marks)**

d Find the coordinates of the point where the normal meets C. **(4 marks)**

(TOTAL: 100 MARKS)