

PURE MATHEMATICS
A level Practice Papers

PAPER Q
MARK SCHEME

1	Correctly factorises the denominator of the left-hand fraction: $\frac{6}{(2x+5)(2x-1)} + \frac{3x+1}{2x-1}$	M1
	Multiplies the right-hand fraction by $\frac{2x+5}{2x+5}$ For example: $\frac{6}{(2x+5)(2x-1)} + \frac{(3x+1)(2x+5)}{(2x-1)(2x+5)}$ is seen.	M1
	Makes an attempt to distribute the numerator of the right-hand fraction. For example: $\frac{6+6x^2+17x+5}{(2x+5)(2x-1)}$ is seen.	M1
	Fully simplified answer is seen. Accept either $\frac{6x^2+17x+11}{(2x+5)(2x-1)}$ or $\frac{(6x+11)(x+1)}{(2x+5)(2x-1)}$	A1
TOTAL: 4 marks		

2a	Uses $a_n = a + (n-1)d$ substituting $a = 5$ and $d = 3$ to get $a_n = 5 + (n-1)3$	M1
	Simplifies to state $a_n = 3n + 2$	A1
		(2 marks)
2b	Use the sum of an arithmetic series to state $\frac{k}{2}[10 + (k-1)3] = 948$	M1
	States correct final answer $3k^2 + 7k - 1896 = 0$	A1
		(2 marks)
TOTAL: 4 marks		

3a	Deduces from $3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1 = 0$ that $3\sin\left(\frac{x}{6}\right)^3 = \frac{1}{10}x + 1$	M1
	States $\left(\frac{x}{6}\right)^3 = \arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)$	M1
	Multiplies by 6^3 and then takes the cube root: $x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$	A1
		(3 marks)
3b	Attempts to use iterative procedure to find subsequent values.	M1
	Correctly finds: $x_1 = 4.716$ $x_2 = 4.802$ $x_3 = 4.812$ $x_4 = 4.814$	A1
		(2 marks)
	TOTAL: 5 marks	

NOTES: 3b

Award M1 if finds at least one correct answer.

4a	Shows that $2\cos 3\theta \approx 2\left(1 - \frac{9\theta^2}{2}\right) = 2 - 9\theta^2$	M1
	Shows that $2\cos 3\theta - 1 \approx 1 - 9\theta^2 = (1 - 3\theta)(1 + 3\theta)$	M1
	Shows $1 + \sin\theta + \tan 2\theta = 1 + \theta + 2\theta = 1 + 3\theta$	M1
	Recognises that $\frac{1 + \sin\theta + \tan 2\theta}{2\cos 3\theta - 1} \approx \frac{1 + 3\theta}{(1 - 3\theta)(1 + 3\theta)} = \frac{1}{1 - 3\theta}$	A1
		(4 marks)
4b	When θ is small, $\frac{1}{1 - 3\theta} \approx 1$	A1
		(1 mark)
	TOTAL: 5 marks	

5a	Writes out the first n terms of the arithmetic sequence in both ascending and descending form $S = a + (a + d) + (a + 2d) + \dots + (a + (n - 1)d)$ $S = (a + (n - 1)d) + (a + (n - 2)d) + (a + (n - 3)d) + \dots + a$	M1
	Attempts to add these two sequences $2S = (2a + (n - 1)d) \times n$	M1
	States $S = \frac{n}{2}(2a + (n - 1)d)$	A1
		(3 marks)
5b	Makes an attempt to find the sum. For example, $S = \frac{200}{2}(2 + 199(2))$ is seen.	M1
	States correct final answer. $S = 40\,000$	A1
		(2 marks)
	TOTAL: 5 marks	

NOTES: **5a** Do not award full marks for an incomplete proof.

5a Do award second method mark if student indicates that $(2a + (n - 1)d)$ appears n times.

6	Makes an attempt to set up a long division. For example: $x + 6 \overline{)x^3 + 8x^2 - 9x + 12}$ is seen.	M1
	$\begin{array}{r} x^2 + 2x - 21 \\ x + 6 \overline{)x^3 + 8x^2 - 9x + 12} \\ \underline{x^3 + 6x^2} \\ 2x^2 - 9x \\ \underline{2x^2 + 12x} \\ -21x + 12 \\ \underline{-21x - 126} \\ 138 \end{array}$ <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> Award 1 accuracy mark for each of the following: x^2 seen, $2x$ seen, -21 seen. For the final accuracy mark either $D = 138$ or $\frac{138}{x + 6}$ or the remainder is 138 must be seen. </div>	A4
	TOTAL: 5 marks	

NOTES:

This question can be solved by first writing $(Ax^2 + Bx + C)(x + 6) + D \equiv x^3 + 8x^2 - 9x + 12$ and then solving for A , B , C and D . Award 1 mark for the setting up the problem as described. Then award 1 mark for each correct coefficient found. For example:

Equating the coefficients of x^3 : $A = 1$

Equating the coefficients of x^2 : $6 + B = 8$, so $B = 2$

Equating the coefficients of x : $12 + C = -9$, so $C = -21$

Equating the constant terms: $-126 + D = 12$, so $D = 138$

7	Begins the proof by assuming the opposite is true. Assumption: there is a finite amount of prime numbers.'	B1
	Considers what having a finite amount of prime numbers means by making an attempt to list them: Let all the prime numbers exist be $p_1, p_2, p_3, \dots, p_n$	M1
	Consider a new number that is one greater than the product of all the existing prime numbers: Let $N = (p_1 \times p_2 \times p_3 \times \dots \times p_n) + 1$	M1
	Understands the implication of this new number is that division by any of the existing prime numbers will leave a remainder of 1. So none of the existing prime numbers is a factor of N .	M1
	Concludes that either N is prime or N has a prime factor that is not currently listed.	B1
	Recognises that either way this leads to a contradiction, and therefore there is an infinite number of prime numbers.	B1
TOTAL: 6 marks		

NOTES: If N is prime, it is a new prime number separate to the finite list of prime numbers, $p_1, p_2, p_3, \dots, p_n$.

If N is divisible by a previously unknown prime number, that prime number is also separate to the finite list of prime numbers.

8	Makes an attempt to differentiate $y = \ln 3x - e^{-2x}$ using the chain rule, or otherwise.	M1
	Differentiates $y = \ln 3x - e^{-2x}$ to obtain $\frac{dy}{dx} = \frac{1}{x} + 2e^{-2x}$	A1
	Evaluates $\frac{dy}{dx}$ at $x = 1$ $\frac{dy}{dx} = 1 + \frac{2}{e^2} = \frac{e^2 + 2}{e^2}$	A1
	Evaluates $y = \ln 3x - e^{-2x}$ at $x = 1$ $y = \ln 3 - e^{-2} = \ln 3 - \frac{1}{e^2}$	M1
	Attempts to substitute values into $y - y_1 = m(x - x_1)$ E.g. $y - \ln 3 + \frac{1}{e^2} = \left(\frac{e^2 + 2}{e^2}\right)(x - 1)$ is seen.	M1 ft
	Shows logical progression to simplify algebra, arriving at: $y = \left(\frac{e^2 + 2}{e^2}\right)x - \left(\frac{e^2 + 3}{e^2}\right) + \ln 3$	A1
TOTAL: 6 marks		

NOTES: Award ft marks for a correct attempt to substitute into the formula using incorrect values.

9a	Clearly states that $\int \frac{6}{x} dx = 6 \ln x$	A1
	Makes an attempt to integrate the remaining two terms. Raising a power by 1 would constitute an attempt.	M1
	States the fully correct answer $6 \ln x - \frac{3}{x} - 2x^{\frac{7}{2}} + C$	A1
		(3 marks)
9b	Makes an attempt to substitute the limits into the expression. For example, $\left(6 \ln 9 - \frac{3}{9} - 2(2187)\right) - \left(6 \ln 4 - \frac{3}{4} - 2(128)\right)$ is seen.	M1
	Begins to simplify this expression. For example, $6 \ln \frac{9}{4} + \frac{5}{12} - 4118$ is seen.	M1
	States the fully correct answer $-\frac{49411}{12} + 6 \ln \frac{9}{4}$ or states $m = -\frac{49411}{12}$, $n = 6$ and $p = \frac{9}{4}$ Also accept $-\frac{49411}{12} + 12 \ln \frac{3}{2}$ or equivalent.	A1
		(3 marks)
	TOTAL: 6 marks	

10a	Correctly states $\cos(5x + 2x) \equiv \cos 5x \cos 2x - \sin 5x \sin 2x$	M1
	Correctly states $\cos(5x - 2x) \equiv \cos 5x \cos(-2x) - \sin 5x \sin(-2x)$ or states $\cos(5x - 2x) \equiv \cos 5x \cos(2x) + \sin 5x \sin(2x)$	M1
	Adds the two above expressions and states $\cos 7x + \cos 3x \equiv 2 \cos 5x \cos 2x$	A1
		(3 marks)
10b	States that $\int (\cos 5x \cos 2x) dx = \frac{1}{2} \int (\cos 7x + \cos 3x) dx$	M1
	Makes an attempt to integrate. Changing cos to sin constitutes an attempt.	M1
	Correctly states the final answer $\frac{1}{14} \sin 7x + \frac{1}{6} \sin 3x + C$ o.e.	A1
		(3 marks)
	TOTAL: 6 marks	

NOTES: 10b

Student does not need to state '+C' to be awarded the first method mark.

Must be stated in the final answer.

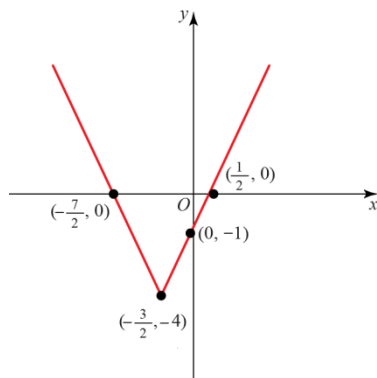
<p>Understands that integration is required to solve the problem.</p> <p>For example, writes $\int_{\frac{\pi}{2}}^{\pi} (x \sin^2 x) dx$</p>	M1
<p>Uses the trigonometric identity $\cos 2x \equiv 1 - 2\sin^2 x$</p> <p>to rewrite $\int_{\frac{\pi}{2}}^{\pi} x \sin^2 x dx$ as $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2}x - \frac{1}{2}x \cos 2x \right) dx$ o.e.</p>	M1
<p>Shows $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}x dx = \left[\frac{1}{4}x^2 \right]_{\frac{\pi}{2}}^{\pi}$</p>	A1
<p>Demonstrates an understanding of the need to find $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}x \cos 2x dx$ using integration by parts.</p> <p>For example, $u = x, \frac{du}{dx} = 1$</p> <p>$\frac{dv}{dx} = \cos 2x, v = \frac{1}{2} \sin 2x$ o.e. is seen.</p>	M1
<p>States fully correct integral</p> $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2}x - \frac{1}{2}x \cos 2x \right) dx = \left[\frac{1}{4}x^2 - \frac{1}{4}x \sin 2x - \frac{1}{8} \cos 2x \right]_{\frac{\pi}{2}}^{\pi}$	A1
<p>Makes an attempt to substitute the limits</p> $\left(\frac{\pi^2}{4} - \frac{1}{4}(0) - \frac{1}{8}(1) \right) - \left(\frac{\pi^2}{16} - \frac{1}{4}(0) - \frac{1}{8}(-1) \right)$	M1
<p>States fully correct answer:</p> <p>either $\frac{3\pi^2}{16} - \frac{1}{4}$ or $\frac{3\pi^2 - 4}{16}$ o.e.</p>	A1
TOTAL: 7 marks	

12a	Writes $(\sin 3\theta + \cos 3\theta)^2 \equiv (\sin 3\theta + \cos 3\theta)(\sin 3\theta + \cos 3\theta)$ $\equiv \sin^2 3\theta + 2\sin 3\theta \cos 3\theta + \cos^2 3\theta$	M1
Uses $\sin^2 3\theta + \cos^2 3\theta \equiv 1$ and $2\sin 3\theta \cos 3\theta \equiv \sin 6\theta$ to write: $(\sin 3\theta + \cos 3\theta)^2 \equiv 1 + \sin 6\theta$ Award one mark for each correct use of a trigonometric identity.		A2
		(3 marks)
12b	States that: $1 + \sin 6\theta = \frac{2 + \sqrt{2}}{2}$	B1
Simplifies this to write: $\sin 6\theta = \frac{\sqrt{2}}{2}$		M1
Correctly finds $6\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ Additional answers might be seen, but not necessary in order to award the mark.		M1
States $\theta = \frac{\pi}{24}, \frac{3\pi}{24}$ Note that $\theta \neq \frac{9\pi}{24}, \frac{11\pi}{24}$ For these values 3θ lies in the third quadrant, therefore $\sin 3\theta$ and $\cos 3\theta$ are both negative and cannot be equal to a positive surd.		A1
		(4 marks)
TOTAL: 7 marks		

NOTES: 12b

Award all 4 marks if correct final answer is seen, even if some of the 6θ angles are missing in the preceding step.

13a



Graph has a distinct V-shape.

M1

Labels vertex $\left(-\frac{3}{2}, -4\right)$

A1

Finds intercept with the y-axis.

M1

Makes attempt to find x-intercept, for example states that $|2x + 3| - 4 = 0$

M1

Successfully finds both x-intercepts.

A1

(5 marks)

13b

Recognises that there are two solutions.

M1

For example, writing $2x + 3 = -\frac{1}{4}x + 2$ and $-(2x + 3) = -\frac{1}{4}x + 2$

Makes an attempt to solve both questions for x, by manipulating the algebra.

M1

Correctly states $x = -\frac{4}{9}$ or $x = -\frac{20}{7}$. Must state both answers.

A1

Makes an attempt to substitute to find y.

M1

Correctly finds y and states both sets of coordinates correctly $\left(-\frac{4}{9}, -\frac{17}{9}\right)$ and $\left(-\frac{20}{7}, -\frac{9}{7}\right)$

A1

(5 marks)

TOTAL: 10 marks

14a	Demonstrates an attempt to find the vectors \overrightarrow{KL} , \overrightarrow{LM} and \overrightarrow{KM}	M1
	Finds $\overrightarrow{KL} = (3, 0, -6)$, $\overrightarrow{LM} = (2, 5, 4)$ and $\overrightarrow{KM} = (5, 5, -2)$	A1
	Demonstrates an attempt to find $ \overrightarrow{KL} $, $ \overrightarrow{LM} $ and $ \overrightarrow{KM} $	M1
	Finds $ \overrightarrow{KL} = \sqrt{(3)^2 + (0)^2 + (-6)^2} = \sqrt{45}$ Finds $ \overrightarrow{LM} = \sqrt{(2)^2 + (5)^2 + (4)^2} = \sqrt{45}$ Finds $ \overrightarrow{KM} = \sqrt{(5)^2 + (5)^2 + (-2)^2} = \sqrt{54}$	A1
	Demonstrates an understanding of the need to use the Law of Cosines. Either $c^2 = a^2 + b^2 - 2ab \times \cos C$ (or variation) is seen, or attempt to substitute into formula is made $(\sqrt{54})^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})\cos\theta$	M1 ft
	Makes an attempt to simplify the above equation. For example, $-36 = -90\cos\theta$ is seen.	M1 ft
	Shows a logical progression to state $\theta = 66.4^\circ$	B1
		(7 marks)
14b	States or implies that ΔKLM is isosceles.	M1
	Makes an attempt to find the missing angles $\angle LKM = \angle LMK = \frac{180 - 66.421...}{2}$	M1
	States $\angle LKM = \angle LMK = 56.789...^\circ$. Accept awrt 56.8°	A1
		(3 marks)
TOTAL: 10 marks		

NOTES: 14b

Award ft marks for a correct answer to part **a** using their incorrect answer from earlier in part **a**.

15a	Shows or implies that if $y = 0$, $t = 1$	M1
	Finds the coordinates of P . $t = 1 \Rightarrow x = 3$ $P(3, 0)$	A1
		(2 marks)
15b	Attempts to find a cartesian equation of the curve. For example, $t = x - 2$ is substituted into $y = \frac{t-1}{t+2}$	M1
	Correctly finds the cartesian equation of the curve $y = \frac{x-3}{x}$ Accept any equivalent answer. For example, $y = 1 - \frac{3}{x}$	A1
		(2 marks)
15c	Finds $\frac{dy}{dx} = 3x^{-2} = \frac{3}{x^2}$	M1
	Substitutes $t = -1$ to find $x = 1$ and $\frac{dy}{dx} = \frac{3}{(1)^2} = 3$	M1
	Finds the gradient of the normal $m_N = -\frac{1}{3}$	M1
	Substitutes $t = -1$ to find $x = 1$ and $y = -2$	A1
	Makes an attempt to find the equation of the normal. For example, $y + 2 = -\frac{1}{3}(x - 1)$ is seen.	M1
	States fully correct answer $x + 3y + 5 = 0$	A1
		(6 marks)
15d	Substitutes $x = t + 2$ and $y = \frac{t-1}{t+2}$ into $x + 3y + 5 = 0$ obtaining $t + 2 + 3\left(\frac{t-1}{t+2}\right) + 5 = 0$	M1 ft
	Manipulates and simplifies this equation to obtain $t^2 + 12t + 11 = 0$	M1 ft
	Factorises and solves to find $t = -1$ or $t = -11$	M1 ft
	Substitutes $t = -11$ to find $x = -9$ and $y = \frac{4}{3}$, i.e. $\left(-9, \frac{4}{3}\right)$	A1 ft
		(4 marks)
TOTAL: 14 marks		

NOTES: 15c Award ft marks for correct answer using incorrect values from part **b**.

(TOTAL: 100 MARKS)