

PURE MATHEMATICS
A level Practice Papers

PAPER P
MARK SCHEME

1	Recognises that two subsequent values will divide to give an equal ratio and sets up an appropriate equation. $\frac{2k^2}{4k} = \frac{4k}{k+2}$	M1
	Makes an attempt to solve the equation. For example, $2k^3 + 4k^2 = 16k^2$ or $2k^3 - 12k^2 = 0$	M1
	Factorises to get $2k^2(k-6) = 0$	M1
	States the correct solution: $k = 6$. $k \neq 0$ or $k = 0$ is trivial may also be seen, but is not required.	A1
TOTAL: 4 marks		

2	Recognises the need to use the chain rule to find $\frac{dV}{dt}$ For example $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dS} \times \frac{dS}{dt}$ is seen.	M1
	Finds $\frac{dV}{dr} = 4\pi r^2$ and $\frac{dS}{dr} = 8\pi r$	M1
	Makes an attempt to substitute known values. For example, $\frac{dV}{dt} = \frac{4\pi r^2}{1} \times \frac{1}{8\pi r} \times \frac{-12}{1}$	M1
	Simplifies and states $\frac{dV}{dt} = -6r$	A1
TOTAL: 4 marks		

3	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	M1
	States $\sin t = \frac{y}{2}$ or $\sin^2 t = \frac{y^2}{4}$ Also states $\cos^2 t = \frac{1}{x-1}$	M1
	Substitutes $\sin^2 t = \frac{y^2}{4}$ and $\cos^2 t = \frac{1}{x-1}$ into $\sin^2 t + \cos^2 t \equiv 1$ $\frac{y^2}{4} + \frac{1}{x-1} = 1 \Rightarrow \frac{y^2}{4} = \frac{x-2}{x-1}$	M1
	Solves to find $y = \sqrt{\frac{4x-8}{x-1}}$, accept $y = \sqrt{\frac{8-4x}{1-x}}$, $x < 1$ or $x \geq 2$	A1
TOTAL: 4 marks		

4	States that: $A(2x+5)+B(5x-1)\equiv 6x+42$	M1
	Equates the various terms. Equating the coefficients of x : $2A+5B=6$ Equating constant terms: $5A-B=42$	M1*
	Multiplies both of the equations in an effort to equate one of the two variables.	M1*
	Finds $A=8$	A1
	Find $B=-2$	A1
	TOTAL: 5 marks	

Alternative method

Uses the substitution method, having first obtained this equation: $A(2x+5)+B(5x-1)\equiv 6x+42$

Substitutes $x=-\frac{5}{2}$ to obtain $-\frac{27}{2}B=27$ (M1)

Substitutes $x=\frac{1}{5}$ to obtain $\frac{27}{5}A=43.2$ (M1)

5	Differentiates 4^x to obtain $4^x \ln 4$	M1
	Differentiates $2xy$ to obtain $2x \frac{dy}{dx} + 2y$	M1
	Rearranges $4^x \ln 4 = 2x \frac{dy}{dx} + 2y$ to obtain $\frac{dy}{dx} = \frac{4^x \ln 4 - 2y}{2x}$	A1
	Makes an attempt to substitute (2, 4)	M1
	States fully correct final answer: $4 \ln 4 - 2$ Accept $\ln 256 - 2$	A1
	TOTAL: 5 marks	

6	Equating the coefficients of x^4 : $A = 5$	A1
	Equating the coefficients of x^3 : $B = -4$	A1
	Equating the coefficients of x^2 : $2A + C = 17, C = 7$	A1
	Equating the coefficients of x : $2B + D = -5, D = 3$	A1
	Equating constant terms: $2C + E = 7, E = -7$	A1
TOTAL: 5 marks		

7a	Correctly states that $(1+ax)^{-2} = 1 + (-2)(ax) + \frac{(-2)(-3)(ax)^2}{2} + \frac{(-2)(-3)(-4)(ax)^3}{6} + \dots$	M1
	Simplifies to obtain $(1+ax)^{-2} = 1 - 2ax + 3a^2x^2 - 4a^3x^3 \dots$	M1
	Deduces that $3a^2 = 75$	M1
	Solves to find $a = \pm 5$	A1
		(4 marks)
7b	$a = 5 \Rightarrow -4(125)x^3 = -500x^3$. Award mark for -500 seen.	A1
	$a = -5 \Rightarrow -4(-125)x^3 = 500x^3$. Award mark for 500 seen.	A1
		(2 marks)
TOTAL: 6 marks		

8a	States that $x_2 = 4p - 9$	A1
	Attempts to substitute x_2 into x_3 . $x_3 = p(4p - 9) - 9$ and simplifies to find $x_3 = 4p^2 - 9p - 9$	A1
		(2 marks)
8b	States $4p^2 - 9p - 9 = 46$ or $4p^2 - 9p - 55 = 0$	M1
	Factorises to get $(4p + 11)(p - 5) = 0$	M1
	States $p = 5$. May also state that $p \neq -\frac{11}{4}$, but mark can be awarded without that being seen.	A1
		(3 marks)
8c	$x_4 = 5(46) - 9 = 221$ $x_5 = 5(221) - 9 = 1096$	A1 ft
		(1 mark)
	TOTAL: 6 marks	

NOTES: 8c: Award mark for a correct answer using their value of p from part **b**.

9a	States $5^2 + 6^2 + (k - 10)^2 = (5\sqrt{5})^2$	M1
	Makes an attempt to solve the equation. For example, $(k - 10)^2 = 64$ is seen.	M1
	States $k = 2$ and $k = 18$	A1
		(3 marks)
9b	Finds the vector $\overrightarrow{OA} = (-1, 7, 18)$	M1 ft
	Finds $ \overrightarrow{OA} = \sqrt{(-1)^2 + (7)^2 + (18)^2} = \sqrt{374}$	M1 ft
	States the unit vector $\frac{1}{\sqrt{374}}(-\mathbf{i} + 7\mathbf{j} + 18\mathbf{k})$	A1 ft
		(3 marks)
	TOTAL: 6 marks	

NOTES: 9b

Award ft marks for a correct answer to part **b** using their incorrect answer from part **a**.

10a	Makes an attempt to find $fg(x)$. For example, writing $fg(x) = e^{2\ln(x+1)} + 4$	M1
	Uses the law of logarithms to write $fg(x) = e^{\ln(x+1)^2} + 4$	M1
	States that $fg(x) = (x+1)^2 + 4$	A1
	States that the range is $y > 4$ or $fg(x) > 4$	B1
		(4 marks)
10b	States that $(x+1)^2 + 4 = 85$	M1
	Makes an attempt to solve for x , including attempting to take the square root of both sides of the equation. For example, $x + 1 = \pm 9$	M1
	States that $x = 8$. Does not need to state that $x \neq -10$, but do not award the mark if $x = -10$ is stated.	A1
		(3 marks)
	TOTAL: 7 marks	

11a	Rearranges $x^4 - 8x^2 + 2 = 0$ to find $x^2 = \frac{x^4 + 2}{8}$	M1
	States $x = \sqrt{\frac{x^4 + 2}{8}}$ and therefore $a = \frac{1}{8}$ and $b = \frac{1}{4}$ or states $x = \sqrt{\frac{1}{8}x^4 + \frac{1}{4}}$	A1
		(2 marks)
	Attempts to use iterative procedure to find subsequent values.	M1
11b	Correctly finds: $x_1 = 0.9396$ $x_2 = 0.5894$ $x_3 = 0.5149$ $x_4 = 0.5087$	A1
		(2 marks)
11c	Demonstrates an understanding that the two values of $f(x)$ to be calculated are for $x = -2.7815$ and $x = -2.7825$.	M1*
	Finds $f(-2.7815) = -0.0367\dots$ and $f(-2.7825) = (+)0.00485\dots$	M1
	Change of sign and continuous function in the interval $[-2.7825, -2.7815] \Rightarrow$ root	A1
		(3 marks)
TOTAL: 7 marks		

NOTES:

11b

Award M1 if finds at least one correct answer.

11c

Any two numbers that produce a change of sign, where one is greater than -2.782 and one is less than -2.782 , and both numbers round to -2.782 to 3 decimal places, are acceptable.

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

12a	Recognises the need to write $\tan^4 x \equiv \tan^2 x \tan^2 x$	M1
	Recognises the need to write $\tan^2 x \tan^2 x \equiv (\sec^2 x - 1)\tan^2 x$	M1
	$(\sec^2 x - 1)\tan^2 x$ Multiplies out the bracket and makes a further substitution $\equiv \sec^2 x \tan^2 x - \tan^2 x$ $\equiv \sec^2 x \tan^2 x - (\sec^2 x - 1)$	M1
	States the fully correct final answer $\sec^2 x \tan^2 x + 1 - \sec^2 x$	A1
		(4 marks)
12b	States or implies that $\int \sec^2 x \, dx = \tan x$	M1
	States fully correct integral $\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x + x - \tan x + C$	M1
	Makes an attempt to substitute the limits. For example, $\left[\frac{1}{3} \tan^3 x + x - \tan x \right]_0^{\frac{\pi}{4}} = \left(\frac{1}{3} \left(\tan \frac{\pi}{4} \right)^3 + \frac{\pi}{4} - \tan \frac{\pi}{4} \right) - (0)$ is seen.	M1 ft
	Begins to simplify the expression $\frac{1}{3} + \frac{\pi}{4} - 1$	M1 ft
	States the correct final answer $\frac{3\pi - 8}{12}$	A1 ft
		(5 marks)
	TOTAL: 9 marks	

NOTES:

12b

Student does not need to state '+C' to be awarded the second method mark.

12b

Award ft marks for a correct answer using an incorrect initial answer.

13a	Begins the proof by assuming the opposite is true.	B1
	‘Assumption: there exists a number n such that n^2 is even and n is odd.’	
	Defines an odd number (choice of variable is not important) and successfully calculates n^2 Let $2k + 1$ be an odd number. $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$	M1
	Factors the expression and concludes that this number must be odd. $4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, so n^2 is odd.	M1
	Makes a valid conclusion. This contradicts the assumption n^2 is even. Therefore if n^2 is even, n must be even.	B1
		(4 marks)
13b	Begins the proof by assuming the opposite is true.	B1
	‘Assumption: $\sqrt{2}$ is a rational number.’	
	Defines the rational number: $\sqrt{2} = \frac{a}{b}$ for some integers a and b , where a and b have no common factors.	M1
	Squares both sides and concludes that a is even: $\sqrt{2} = \frac{a}{b} \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$ From part a: a^2 is even implies that a is even.	M1
	Further states that if a is even, then $a = 2c$. Choice of variable is not important.	M1
	Makes a substitution and works through to find $b^2 = 2c^2$, concluding that b is also even. $a^2 = 2b^2 \Rightarrow (2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2$ From part a: b^2 is even implies that b is even.	M1
	Makes a valid conclusion. If a and b are even, then they have a common factor of 2, which contradicts the statement that a and b have no common factors. Therefore $\sqrt{2}$ is an irrational number.	B1
		(6 marks)
	TOTAL: 10 marks	

14a	States $\frac{dV}{dt} = -kV$	M1
	Separates the variables $\int \frac{1}{V} dV = \int -k dt$	M1
	Finds $\ln V = -kt + C$	A1
	Shows clearly progression to state $V = V_0 e^{-kt}$ For example, $V = e^{-kt+C} = e^{-kt} e^C$ is seen. May also explain the $V_0 = e^C$ where e^C is a constant.	A1
		(4 marks)
14b	States $\frac{1}{5}V_0 = V_0 e^{-kt}$	M1
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln \frac{1}{5} = -kt$	M1
	States that $k = -\frac{1}{10} \ln \frac{1}{5}$	A1
		(3 marks)
14c	States $\frac{1}{20}V_0 = V_0 e^{-kt}$	M1
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln \frac{1}{20} = t \left(\frac{1}{10} \ln \frac{1}{5} \right)$	M1
	Finds $t = 18.613\dots$ years. Accept 18.6 years.	A1
		(3 marks)
TOTAL: 10 marks		

15a	States: $R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ Or: $5\cos\theta - 8\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	M1
	Deduces that: $5 = R\cos\alpha$ $8 = R\sin\alpha$	M1
	States that $R = \sqrt{89}$ Use of $\sin^2\theta + \cos^2\theta = 1$ might be seen, but is not necessary to award the mark.	A1
	Finds that $\alpha = 1.0122$ $\tan\alpha = \frac{8}{5}$ might be seen, but is not necessary to award the mark.	A1
		(4 marks)
15b	Uses the maths from part a to deduce that $T_{\max} = 1100 + \sqrt{89} = 1109.43^\circ\text{C}$	A1
	Recognises that the maximum temperature occurs when $\cos\left(\frac{x}{3} + 1.0122\right) = 1$	M1
	Solves this equation to find $\frac{x}{3} = 2\pi - 1.0122$	M1
	Finds $x = 15.81$ hours	A1
		(4 marks)
15c	Deduces that $1097 = 1100 + \sqrt{89} \cos\left(\frac{x}{3} + 1.0122\right)$	M1
	Begins to solve the equation. For example, $\cos\left(\frac{x}{3} + 1.0122\right) = -\frac{3}{\sqrt{89}}$ is seen.	M1
	States that $\frac{x}{3} + 1.0122 = 1.8944, 2\pi - 1.8944, 2\pi + 1.8944$ Further values may be seen, but are not necessary in order to award the mark.	M1
	Finds that $x = 2.65$ hours, 10.13 hours, 21.50 hours	A1
		(4 marks)
	TOTAL: 12 marks	

(TOTAL: 100 MARKS)