Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations.
Calculators must not have the facility for algebraic manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are
  clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers
  without working may not gain full credit.

Questions to revise:

1 Showing all steps, find $\int \cot 3x \, dx$ **(3 marks)**

2 Use proof by contradiction to show that there is no greatest positive rational number. **(4 marks)**

3 Find $\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sin 4x (1 - \cos 4x)^3 \, dx$ **(4 marks)**

4 Given that $\dfrac{x^2 - 36}{x^2 - 11x + 30} \times \dfrac{25 - x^2}{Ax^2 + Bx + C} \times \dfrac{6x^2 + 7x - 3}{3x^2 + 17x - 6} \equiv \dfrac{x + 5}{6 - x}$,

find the values of the constants A, B and C, where A, B and C are integers. **(5 marks)**

5 Prove by contradiction that if n is odd, $n^3 + 1$ is even. **(5 marks)**

6 An infinite geometric series has first four terms $1 - 4x + 16x^2 - 64x^3 + \ldots$
The series is convergent.

    a Find the set of possible values of x for which the series converges. **(2 marks)**

   **b** Given that $\sum_{r=1}^{\infty} (-4x)^{r-1} = 4$, calculate the value of x. **(3 marks)**

7 Given that $(b - a)\mathbf{i} - 2abc\mathbf{j} + 2\mathbf{k} = 10\mathbf{i} - 96\mathbf{j} + (7a + 5b)\mathbf{k}$, find the values of a, b and c.

(6 marks)

8 The function g(x) is defined by g(x) = $x^2 - 8x + 7$, $x \in \mathbb{R}$, $x > 4$.
Find g$^{-1}(x)$ and state its domain and range.

(6 marks)

9 f(x) $\equiv \dfrac{9x^2 + 25x + 16}{9x^2 - 16}$

Show that f(x) can be written in the form $A + \dfrac{B}{3x - 4} + \dfrac{C}{3x + 4}$, where A, B and C are constants
to be found. **(7 marks)**

10   The diagram shows the right-angled triangles ΔABC, ΔABD and ΔBCD, with $AB = 1$ and $\angle BAD = \theta$.
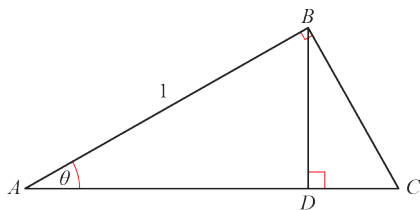
Prove that $1 + \tan^2\theta = \sec^2\theta$ **(8 marks)**

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11   The curve C has parametric equations $x = 7\sin t - 4$, $y = 7\cos t + 3$, $-\dfrac{\pi}{2} \leqslant t \leqslant \dfrac{\pi}{3}$

  a Show that the cartesian equation of C can be written as $(x+a)^2 + (y+b)^2 = c$,
      where a, b and c are integers which should be stated.   **(3 marks)**

  b Sketch the curve C on the given domain, clearly stating the endpoints of the curve.   **(3 marks)**

  c Find the length of C. Leave your answer in terms of π.   **(2 marks)**

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12   A curve has parametric equations $x = \cos 2t$, $y = \sin t$, $-\pi \leqslant t \leqslant \pi$.

  a Find an expression for $\dfrac{dy}{dx}$ in terms of t.
      Leave your answer as a single trigonometric ratio.   **(3 marks)**

  b Find an equation of the normal to the curve at the point A where $t = -\dfrac{5\pi}{6}$ **(5 marks)**

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13 $g(x) = \dfrac{2}{x-1} - e^x$

  a By drawing an appropriate sketch, show that there is only one solution to the equation $g(x) = 0$

                                                                                   (2 marks)

  b Show that the equation $g(x) = 0$ may be written in the form $x = 2e^{-x} + 1$

                                                                                   (2 marks)

  c Let $x_0 = 1.5$. Use the iterative formula $x_{n+1} = 2e^{-x_n} + 1$ to find to 4 decimal places the values
      of x_1, x_2, x_3 and x_4.   **(2 marks)**

  d Using $x_0 = 1.5$ as a first approximation, apply the Newton–Raphson procedure once to $g(x)$
      to find a second approximation to α, giving your answer to 4 decimal places.   **(4 marks)**

14 **a** Find the binomial expansion of $\dfrac{1+x}{\sqrt{1-2x}}$ in ascending powers of x up to and including the x^2 term, simplifying each term. **(4 marks)**

b State the set of values of x for which the expansion is valid. **(1 mark)**

c Show that when $x = \dfrac{1}{100}$, the exact value of $\dfrac{1+x}{\sqrt{1-2x}}$ is $\dfrac{101\sqrt{2}}{140}$ **(2 marks)**

d Substitute $x = \dfrac{1}{100}$ into the binomial expansion in part **a** and hence obtain an approximation to $\sqrt{2}$

Give your answer to 5 decimal places. **(3 marks)**

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15 The diagram shows the curve with equation $y = \dfrac{1}{2} x^3 \sqrt{4 - x^2}$
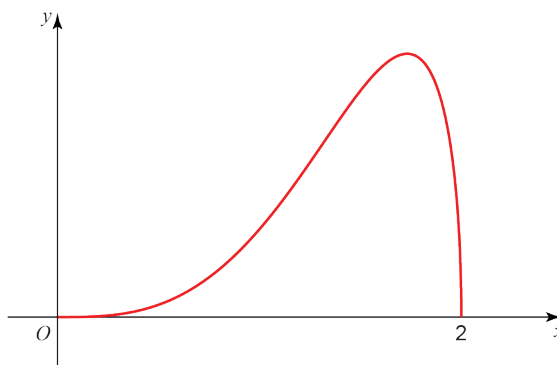
a Complete the table with the value of y corresponding to $x = 1.5$.
Give your answer correct to 5 decimal places. **(1 mark)**

x	0	0.5	1	1.5	2
y	0	0.12103	0.86603		0

b Given that $I = \displaystyle\int_0^2 \left(\dfrac{1}{2} x^3 \sqrt{4 - x^2} \right) dx$, use the trapezium rule with 4 equal width strips to find an approximate value of I, giving your answer to 4 significant figures. **(3 marks)**

c By using an appropriate substitution, or otherwise, find the exact value of $\displaystyle\int_0^2 \left(\dfrac{1}{2} x^3 \sqrt{4 - x^2} \right) dx$, leaving your answer as a rational number in its simplest form. **(6 marks)**

d Suggest one way in which your estimate using a trapezium rule could be improved. **(1 mark)**

(TOTAL: 100 MARKS)