PURE MATHEMATICS A level Practice Papers	PAPER O MARK SCHEME

(2 marks)

1	States that $\cot 3x = \dfrac{\cos 3x}{\sin 3x}$	**M1**
	Makes an attempt to find $\int \left(\dfrac{\cos 3x}{\sin 3x} \right) \mathrm{d}x$ Writing $\int \dfrac{\mathrm{f}'(x)}{\mathrm{f}(x)} \mathrm{d}x = \ln\left[\mathrm{f}(x)\right]$ or writing $\ln(\sin x)$ constitutes an attempt.	**M1**
	States a fully correct answer $\dfrac{1}{3}\ln\lvert\sin 3x\rvert + C$	**A1**
	TOTAL: 3 marks	

2	Begins the proof by assuming the opposite is true. 'Assumption: there exists a rational number $\dfrac{a}{b}$ such that $\dfrac{a}{b}$ is the greatest positive rational number.'	**B1**
	Makes an attempt to consider a number that is clearly greater than $\dfrac{a}{b}$: 'Consider the number $\dfrac{a}{b}+1$, which must be greater than $\dfrac{a}{b}$ '	**M1**
	Simplifies $\dfrac{a}{b}+1$ and concludes that this is a rational number. $\dfrac{a}{b}+1 \equiv \dfrac{a}{b}+\dfrac{b}{b} \equiv \dfrac{a+b}{b}$ By definition, $\dfrac{a+b}{b}$ is a rational number.	**M1**
	Makes a valid conclusion. This contradicts the assumption that there exists a greatest positive rational number, so we can conclude that there is not a greatest positive rational number.	**B1**
	TOTAL: 4 marks	

3		
	Makes an attempt to find $\int \sin 4x(1-\cos 4x)^3 \, dx$ Raising the power by 1 would constitute an attempt.	**M1**
	States a fully correct answe r $\int \sin 4x(1-\cos 4x)^3 \, dx = \frac{1}{16}(1-\cos 4x)^4 + C$	**M1**
	Makes an attempt to substitute the limits $\frac{1}{16}\left[(1-0)^4 - \left(1-\frac{1}{2}\right)^4\right]$	**M1 ft**
	Correctly states answer is $\frac{15}{256}$	**A1 ft**
	TOTAL: 4 marks	

4		
	Makes an attempt to factor all the quadratics on the left-hand side of the identity.	**M1**
	Correctly factors each expression on the left-hand side of the identity: $\frac{(x-6)(x+6)}{(x-5)(x-6)} \times \frac{(5-x)(5+x)}{Ax^2+Bx+C} \times \frac{(3x-1)(2x+3)}{(3x-1)(x+6)}$	**A1**
	Successfully cancels common factors: $\frac{(-1)(5+x)(2x+3)}{Ax^2+Bx+C} \equiv \frac{x+5}{(-1)(x-6)}$	**M1**
	States that $Ax^2+Bx+C \equiv (2x+3)(x-6)$	**M1**
	States or implies that $A=2$, $B=-9$ and $C=-18$	**A1**
	TOTAL: 5 marks	

NOTES: Alternative method

Makes an attempt to substitute $x = 0$	(**M1**)
Finds $C = -18$	(**A1**)
Substitutes $x = 1$ to give $A + B = -7$	(**M1**)
Substitutes $x = -1$ to give $A - B = 11$	(**M1**)
Solves to get $A = 2$, $B = -9$ and $C = -18$	(**A1**)

5	Begins the proof by assuming the opposite is true. 'Assumption: there exists a number n such that n is odd and $n^3 + 1$ is also odd.'	**B1**
	Defines an odd number: 'Let $2k + 1$ be an odd number.'	**B1**
	Successfully calculates $(2k+1)^3+1$ $(2k+1)^3+1 \equiv \left(8k^3+12k^2+6k+1\right)+1 \equiv 8k^3+12k^2+6k+2$	**M1**
	Factors the expression and concludes that this number must be even. $8k^3+12k^2+6k+2 \equiv 2\left(4k^3+6k^2+3k+1\right)$ $2\left(4k^3+6k^2+3k+1\right)$ is even.	**M1**
	Makes a valid conclusion. This contradicts the assumption that there exists a number n such that n is odd and $n^3 + 1$ is also odd, so if n is odd, then $n^3 + 1$ is even.	**B1**
	TOTAL: 5 marks	

NOTES: Alternative method

Assume the opposite is true: there exists a number n such that n is odd and $n^3 + 1$ is also odd. (**B1**)
If $n^3 + 1$ is odd, then n^3 is even. (**B1**)
So 2 is a factor of n^3. (**M1**)
This implies 2 is a factor of n. (**M1**)
This contradicts the statement n is odd. (**B1**)

6a	Understands that for the series to be convergent $\|r\|<1$ or states $\|-4x\|<1$	**M1**
	Correctly concludes that $\|x\|<\frac{1}{4}$. Accept $-\frac{1}{4}<x<\frac{1}{4}$	**A1**
		(2 marks)
6b	Understands to use the sum to infinity formula. For example, states $\frac{1}{1+4x}=4$	**M1**
	Makes an attempt to solve for x. For example, $4x=-\frac{3}{4}$ is seen.	**M1**
	States $x=-\frac{3}{16}$	**A1**
		(3 marks)
	TOTAL: 5 marks	

7	States $-a+b=10$ and $7a+5b=2$	**M1**
	Makes an attempt to solve the pair of simultaneous equations. Attempt could include making a substitution or multiplying the first equation by 5 or by 7.	**M1**
	Finds $a=-4$	**A1**
	Finds $b=6$	**A1**
	States $-2abc=-96$	**M1**
	Finds $c=-2$	**A1**
	TOTAL: 6 marks	

8	Understands the need to complete the square, and makes an attempt to do this. For example, $(x-4)^2$ is seen.	**M1**
	Correctly writes $\mathrm{g}(x)=(x-4)^2-9$	**A1**
	Demonstrates an understanding of the method for finding the inverse is to switch the x and y. For example, $x=(y-4)^2-9$ is seen.	**B1**
	Makes an attempt to rearrange to make y the subject. Attempt must include taking the square root.	**M1**
	Correctly states $\mathrm{g}^{-1}(x)=\sqrt{x+9}+4$	**A1**
	Correctly states domain is $x>-9$ and range is $y>4$	**B1**
	TOTAL: 6 marks	

9		
	Makes an attempt to set up a long division. For example: $9x^2+0x-16\overline{)9x^2+25x+16}$ is seen. The '$0x$' being seen is not necessary to award the mark.	**M1**
	Long division completed so that a '1' is seen in the quotient and a remainder of $25x + 32$ is also seen. $$\begin{array}{r} 1 \\ 9x^2+0x-16\overline{)9x^2+25x+16} \\ \underline{9x^2+\ \ 0x-16} \\ 25x+32 \end{array}$$	**M1**
	States $B(3x+4)+C(3x-4)\equiv 25x-32$	**M1**
	Equates the various terms. Equating the coefficients of x: $3B+3C=25$ Equating constant terms: $4B-4C=32$	**M1**
	Multiplies one or both of the equations in an effort to equate one of the two variables.	**M1**
	Finds $B=\dfrac{49}{6}$	**A1**
	Finds $C=\dfrac{1}{6}$	**A1**
	TOTAL: 7 marks	

NOTES: Alternative method

Writes $A+\dfrac{B}{3x-4}+\dfrac{C}{3x+4}$ as $\dfrac{A(3x-4)(3x+4)}{9x^2-16}+\dfrac{B(3x+4)}{9x^2-16}+\dfrac{C(3x-4)}{9x^2-16}$

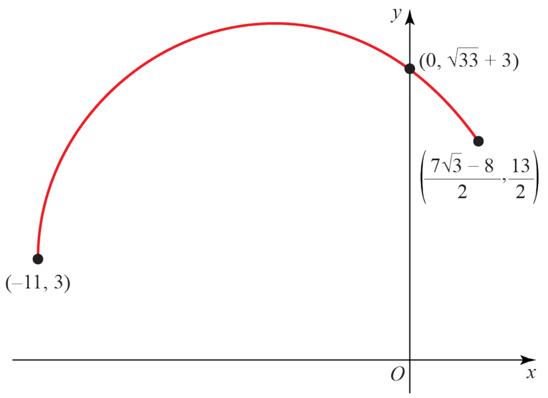
States $A(3x-4)(3x+4)+B(3x+4)+C(3x-4)\equiv 9x^2+25x+16$

Substitutes $x=\dfrac{4}{3}$ to obtain: $8B=\dfrac{196}{3}\Rightarrow B=\dfrac{49}{6}$

Substitutes $x=-\dfrac{4}{3}$ to obtain: $-8C=-\dfrac{4}{3}\Rightarrow C=\dfrac{1}{6}$

Equating the coefficients of x^2: $9A=9\Rightarrow A=1$

10	States that $\sin\theta = \dfrac{BD}{1}$ and concludes that $BD = \sin\theta$	**M1**
	States that $\cos\theta = \dfrac{AD}{1}$ and concludes that $AD = \cos\theta$	**M1**
	States that $\angle DBC = \theta$	**M1**
	States that $\tan\theta = \dfrac{DC}{\sin\theta}$ and concludes that $DC = \dfrac{\sin^2\theta}{\cos\theta}$ oe.	**M1**
	States that $\cos\theta = \dfrac{\sin\theta}{BC}$ and concludes that $BC = \tan\theta$ oe.	**M1**
	Recognises the need to use Pythagoras' theorem. For example, $AB^2 + BC^2 = AC^2$	**M1**
	Makes substitutions and begins to manipulate the equation: $1+\tan^2\theta = \left(\dfrac{\cos\theta}{1} + \dfrac{\sin^2\theta}{\cos\theta}\right)^2$ $1+\tan^2\theta = \left(\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right)^2$	**M1**
	Uses a clear algebraic progression to arrive at the final answer: $1+\tan^2\theta = \left(\dfrac{1}{\cos\theta}\right)^2$ $1+\tan^2\theta = \sec^2\theta$	**A1**
	TOTAL: 8 marks	

Question	Scheme	Marks
11a	States $\sin t = \dfrac{x+4}{7}$ and $\cos t = \dfrac{y-3}{7}$	M1
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	M1
	Makes the substitution to find $(x+4)^2 + (y-3)^2 = 7^2$	A1
		(3 marks)
11b	States or implies that the curve is a circle with centre (−4, 3) and radius 7	M1 ft
	Substitutes $t = -\dfrac{\pi}{2}$ to find $x = -11$ and $y = 3$ (−11, 3) Substitutes $t = \dfrac{\pi}{3}$ to find $x \approx 2.06$ and $y = 6.5$ (2.06, 6.5) Could also substitute $t = 0$ to find $x = -4$ and $y = 10$ (−4, 10)	M1 ft
	 Draws fully correct curve.	A1 ft
		(3 marks)
11c	Makes an attempt to find the length of the curve by recognising that the length is part of the circumference. Must at least attempt to find the circumference to award method mark. $C = 2 \times \pi \times 7 = 14\pi$	M1 ft
	Uses the fact that the arc is $\dfrac{5}{12}$ of the circumference to write: arc length $= \dfrac{35}{6}\pi$	A1 ft
		(2 marks)
	TOTAL: 8 marks	

NOTES:

11b Award ft marks for correct sketch using incorrect values from part **a**.

11c Award ft marks for correct answer using incorrect values from part **a**.

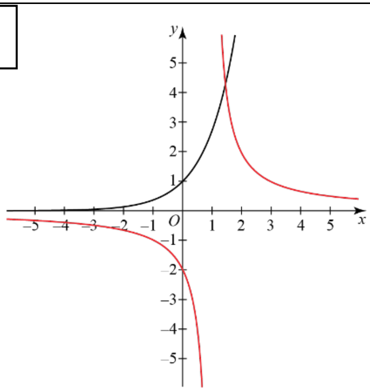
11c Alternative method: use $s = r\theta$, with $r = 7$ and $\theta = \dfrac{5\pi}{6}$

Award one mark for the attempt and one for the correct answer.

12a	Finds $\frac{dx}{dt} = -2\sin 2t$ and $\frac{dy}{dt} = \cos t$	**M1**
	Writes $-2\sin 2t = -4\sin t\cos t$	**M1**
	Calculates $\frac{dy}{dx} = \frac{\cos t}{-4\sin t\cos t} = -\frac{1}{4}\operatorname{cosec} t$	**A1**
		(3 marks)
12b	Evaluates $\frac{dy}{dx}$ at $t = -\frac{5\pi}{6}$ $\frac{dy}{dx} = \frac{-1}{4\sin\left(-\frac{5\pi}{6}\right)} = \frac{1}{2}$	**A1 ft**
	Understands that the gradient of the tangent is $\frac{1}{2}$, and then the gradient of the normal is -2	**M1 ft**
	Finds the values of x and y at $t = -\frac{5\pi}{6}$ $x = \cos\left(2\times -\frac{5\pi}{6}\right) = \frac{1}{2}$ and $y = \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$	**M1 ft**
	Attempts to substitute values into $y - y_1 = m(x - x_1)$ For example, $y + \frac{1}{2} = -2\left(x - \frac{1}{2}\right)$ is seen.	**M1 ft**
	Shows logical progression to simplify algebra, arriving at: $y = -2x + \frac{1}{2}$ or $4x + 2y - 1 = 0$	**A1**
		(5 marks)
	TOTAL: 8 marks	

NOTES: 12b

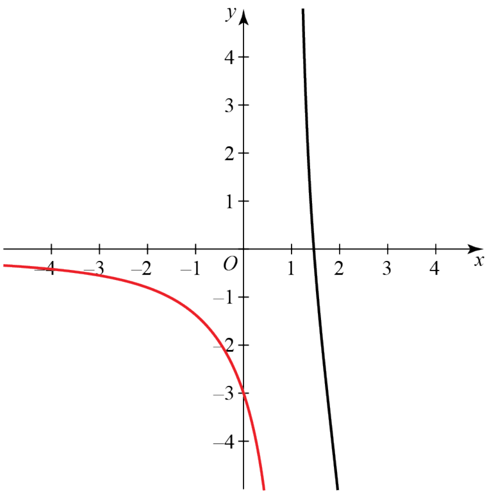
Award ft marks for a correct answer using an incorrect answer from part **a**.

13a  Attempts to sketch both $y=\frac{2}{x-1}$ and $y=e^x$	**M1**
States that $y=\frac{2}{x-1}$ meets $y=e^x$ in just one place, therefore $\frac{2}{x-1}=e^x$ has just one root $\Rightarrow$ $g(x)=0$ has just one root	**A1**
	(2 marks)
13b Makes an attempt to rearrange the equation. For example, $\frac{2}{x-1}-e^x=0 \Rightarrow xe^x-e^x=2$	**M1**
Shows logical progression to state $x=2e^{-x}+1$ For example, $x=\frac{2+e^x}{e^x}$ is seen.	**A1**
	(2 marks)
13c Attempts to use iterative procedure to find subsequent values.	**M1**
Correctly finds: $x_1 = 1.4463$ $x_2 = 1.4709$ $x_3 = 1.4594$ $x_4 = 1.4647$	**A1**
	(2 marks)
13d Correctly finds $g'(x)=-\frac{2}{(x-1)^2}-e^x$	**A1**
Finds $g(1.5)=-0.4816...$ and $g'(1.5)=-12.4816...$	**M1**
Attempts to find x_1: $x_1=x_0-\frac{g(x_0)}{g'(x_0)} \Rightarrow x_1=1.5-\frac{-0.4816...}{-12.4816...}$	**M1**
Finds $x_1=1.461$	**A1**
	(4 marks)
TOTAL: 10 marks	

NOTES:

13a

Uses their graphing calculator to sketch $g(x) = \dfrac{2}{x-1} - e^x$ **(M1)**



States that as g(x) only intersects the x-axis in one place, there is only one solution. **(A1)**

13c

Award M1 if finds at least one correct answer.

Question	Working	Marks
14a	Writes: $\frac{1+x}{\sqrt{1-2x}}$ as $(1+x)(1-2x)^{-\frac{1}{2}}$	**M1**
	Uses the binomial expansion to write: $(1-2x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2}+\ldots$	**M1**
	Simplifies to obtain: $(1+x)(1-2x)^{-\frac{1}{2}}=(1+x)\left(1+x+\frac{3}{2}x^2\right)$	**M1**
	Writes the correct final answer: $1+2x+\frac{5}{2}x^2\ldots$	**A1 ft**
		(4 marks)
14b	Either states $\lvert x\rvert<\frac{1}{2}$ or states $-\frac{1}{2}<x<\frac{1}{2}$	**B1**
		(1 mark)
14c	Makes an attempt to substitute $x=\frac{1}{100}$ into $\frac{1+x}{\sqrt{1-2x}}$ For example $\frac{1+\frac{1}{100}}{\sqrt{1-2\left(\frac{1}{100}\right)}}=\frac{\frac{101}{100}}{\sqrt{\frac{98}{100}}}$	**M1**
	Continues to simplify the expression: $\frac{101}{100}\times\frac{\sqrt{100}}{\sqrt{98}}$ **And** states the correct final answer: $\frac{101\sqrt{2}}{140}$	**A1**
		(2 marks)
14d	Substitutes $x=\frac{1}{100}$ into $1+2x+\frac{5}{2}x^2$ Obtains: $1+2\left(\frac{1}{100}\right)+\frac{5}{2}\left(\frac{1}{100}\right)^2\approx1.02025$	**M1 ft**
	States that $1.02025\approx\frac{101\sqrt{2}}{140}$	**M1 ft**
	Deduces that $\sqrt{2}\approx\frac{140\times1.02025}{101}\approx1.41421$	**A1 ft**
		(3 marks)
	TOTAL: 10 marks	

NOTES:

14a Award 3 marks if a student has used an incorrect expansion but worked out all the other steps correctly.

14d Award all three marks if a student provided an incorrect answer in part **a**, but accurately works out an approximation for root 2 consistent with this incorrect answer.

15a Correctly substitutes $x = 1.5$ into $y=\frac{1}{2}x^3\sqrt{4-x^2}$ and obtains 2.2323…		**A1**
		(1 mark)
15b States or implies formula for the trapezium rule: $A=\frac{h}{2}\left(y_0+2\left(y_1+y_2+y_3\right)+y_4\right)$		**M1**
Makes an attempt to substitute into the formula $A=\frac{0.5}{2}\left(0+2\left(0.12103+0.86603+2.23235\right)+0\right)$		**M1**
States correct final answer 1.610 (4 s.f.)		**A1**
		(3 marks)
15c Recognises the need to make a substitution. **<u>Method 1</u>** $u=4-x^2$ is seen.	**<u>Method 2</u>** $u=\left(4-x^2\right)^{\frac{1}{2}}$ is seen	**M1**
Correctly states $\frac{\mathrm{d}u}{\mathrm{d}x}=-2x$ and finds new limits $x=0\Rightarrow u=4$ and $x=2\Rightarrow u=0$	States $u^2=4-x^2$ and finds $2u\frac{\mathrm{d}u}{\mathrm{d}x}=-2x$ and finds new limits $x=0\Rightarrow u=2$ and $x=2\Rightarrow u=0$	**M1**
Correctly transforms the integral $\int_{x=0}^{x=2}\left(\frac{1}{2}x^3\sqrt{4-x^2}\right)\mathrm{d}x$ into $-\frac{1}{4}\int_{u=4}^{u=0}\left(4u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)\mathrm{d}u$	into $\frac{1}{2}\int_{u=2}^{u=0}\left(-4u^2+u^4\right)\mathrm{d}u$	**M1**
Correctly finds the integral $-\frac{1}{4}\left[\frac{8}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]_4^0$	$\frac{1}{2}\left[-\frac{4}{3}u^3+\frac{1}{5}u^5\right]_2^0$	**M1**
Makes an attempt to substitute the limits $-\frac{1}{4}\left[\begin{array}{l}\left(\frac{8}{3}(0)^{\frac{3}{2}}-\frac{2}{5}(0)^{\frac{5}{2}}\right)\\-\left(\frac{8}{3}(4)^{\frac{3}{2}}-\frac{2}{5}(4)^{\frac{5}{2}}\right)\end{array}\right]$	$\frac{1}{2}\left[\begin{array}{l}\left(-\frac{4}{3}(0)^3+\frac{1}{5}(0)^5\right)\\-\left(-\frac{4}{3}(2)^3+\frac{1}{5}(2)^5\right)\end{array}\right]$	**M1**
Correctly finds answer $\frac{32}{15}$	**15c** Either method is acceptable.	**A1**
		(6 marks)
15d Using more strips would improve the accuracy of the answer.		**B1**
		(1 mark)
TOTAL: 11 marks		

(TOTAL: 100 MARKS)