PURE MATHEMATICS A level Practice Papers	PAPER N MARK SCHEME

1 Attempts to write a differential equation. For example, $\frac{\mathrm{d}F}{\mathrm{d}t} \propto F$ or $\frac{\mathrm{d}F}{\mathrm{d}t} \propto -F$ is seen.	**M1**
States $\frac{\mathrm{d}F}{\mathrm{d}t} = -kF$	**A1**
TOTAL: 2 marks	

2 Finds $\lvert a \rvert = \sqrt{(4)^2 + (-1)^2 + (3)^2} = \sqrt{26}$	**M1**
States $\cos\theta_y = -\frac{1}{\sqrt{26}}$	**M1**
Solves to find $\theta_y = 101.309...°$. Accept awrt 101.3°	**A1**
TOTAL: 3 marks	

3 Begins the proof by assuming the opposite is true. 'Assumption: there do exist integers a and b such that $25a + 15b = 1$'	**B1**
Understands that $25a + 15b = 1 \Rightarrow 5a + 3b = \frac{1}{5}$ 'As both 25 and 15 are multiples of 5, divide both sides by 5 to leave $5a + 3b = \frac{1}{5}$'	**M1**
Understands that if a and b are integers, then $5a$ is an integer, $3b$ is an integer and $5a + 3b$ is also an integer.	**M1**
Recognises that this contradicts the statement that $5a + 3b = \frac{1}{5}$, as $\frac{1}{5}$ is not an integer. Therefore there do not exist integers a and b such that $25a + 15b = 1$'	**B1**
TOTAL: 4 marks	

4	Selects $\cos 2x \equiv 2\cos^2 x - 1$ as the appropriate trigonometric identity.	**M1**
	Manipulates the identity to the question: $\cos 12x \equiv 2\cos^2 6x - 1$	**M1**
	States that $\int \left(\cos^2 6x\right) \mathrm{d}x = \frac{1}{2}\int \left(1 + \cos 12x\right) \mathrm{d}x$	**M1**
	Makes an attempt to integrate the expression, x and $\sin x$ are seen.	**M1**
	Correctly states $\frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right) + C$	**A1**
	TOTAL: 5 marks	

NOTES:

Student does not need to state '+C' to be awarded the third method mark.

Must be stated in the final answer.

5	Begins the proof by assuming the opposite is true. 'Assumption: there exists a product of two odd numbers that is even.'	**B1**
	Defines two odd numbers. Can choose any two different variables. 'Let 2m + 1 and 2n + 1 be our two odd numbers.'	**B1**
	Successfully multiplies the two odd numbers together: $(2m+1)(2n+1) \equiv 4mn + 2m + 2n + 1$	**M1**
	Factors the expression and concludes that this number must be odd. $4mn + 2m + 2n + 1 \equiv 2(2mn + m + n) + 1$ $2(2mn + m + n)$ is even, so $2(2mn + m + n) + 1$ must be odd.	**M1**
	Makes a valid conclusion. This contradicts the assumption that the product of two odd numbers is even, therefore the product of two odd numbers is odd.	**B1**
	TOTAL: 5 marks	

NOTES: Alternative method

Assume the opposite is true: there exists a product of two odd numbers that is even. **(B1)**

If the product is even then 2 is a factor. **(B1)**

So 2 is a factor of at least one of the two numbers. **(M1)**

So at least one of the two numbers is even. **(M1)**

This contradicts the statement that both numbers are odd. **(B1)**

6	States that: $A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$	**M1**
	Further states that: $A(-x^2-x+20)+B(x^2+2x-15)+C(-x^2+7x-12)\equiv 4x^2+x-23$	**M1**
	Equates the various terms. Equating the coefficients of x^2: $-A+B-C=4$ Equating the coefficients of x: $-A+2B+7C=1$ Equating constant terms: $20A-15B-12C=-23$	**M1***
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1***
	Finds the correct value of any one variable: either $A = 2$, $B = 5$ or $C = -1$	**A1***
	Finds the correct value of all three variables: $A = 2$, $B = 5$, $C = -1$	**A1**
	TOTAL: 6 marks	

NOTES: Alternative method

Uses the substitution method, having first obtained this equation:

$A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$ (**M3 as above**)

Substitutes $x = 4$ to obtain $9B = 45$ (**M1**)

Substitutes $x = 3$ to obtain $8A = 16$ (**M1**)

Substitutes $x = -5$ to obtain $-72C = 72$ (**A1**)

7	Demonstrates an attempt to find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$	**M1**
	Finds $\overrightarrow{AB}=(0,4,-2)$, $\overrightarrow{AC}=(5,4,8)$ and $\overrightarrow{BC}=(5,0,10)$	**A1**
	Demonstrates an attempt to find $\lvert\overrightarrow{AB}\rvert$, $\lvert\overrightarrow{AC}\rvert$ and $\lvert\overrightarrow{BC}\rvert$	**M1**
	Finds $\lvert\overrightarrow{AB}\rvert=\sqrt{(0)^2+(4)^2+(-2)^2}=\sqrt{20}$ Finds $\lvert\overrightarrow{AC}\rvert=\sqrt{(5)^2+(4)^2+(8)^2}=\sqrt{105}$ Finds $\lvert\overrightarrow{BC}\rvert=\sqrt{(5)^2+(0)^2+(10)^2}=\sqrt{125}$	**A1**
	States or implies in a right-angled triangle $c^2=a^2+b^2$	**M1**
	States that $\lvert\overrightarrow{AB}\rvert^2+\lvert\overrightarrow{AC}\rvert^2=\lvert\overrightarrow{BC}\rvert^2$	**B1**
	TOTAL: 6 marks	

8a	Rearranges $x=8(t+10)$ to obtain $t=\dfrac{x-80}{8}$	**M1**
	Substitutes $t=\dfrac{x-80}{8}$ into $y=100-t^2$ For example, $y=100-\left(\dfrac{x-80}{8}\right)^2$ is seen.	**M1**
	Finds $y=-\dfrac{1}{64}x^2+\dfrac{5}{2}x$	**A1**
		(3 marks)
8b	Deduces that the width of the arch can be found by substituting $t=\pm10$ into $x=8(t+10)$	**M1**
	Finds x = 0 and x = 160 and deduces the width of the arch is 160 m.	**A1**
		(2 marks)
8b	Deduces that the greatest height occurs when $\dfrac{dy}{dt}=0\Rightarrow-2t=0\Rightarrow t=0$	**M1**
	Deduces that the height is 100 m.	**A1**
		(2 marks)
	TOTAL: 7 marks	

9a	Finds $f(1.9) = -0.2188...$ and $f(2.0) = (+)0.1606...$	**M1**
	Change of sign and continuous function in the interval $[1.9, 2.0] \Rightarrow$ root	**A1**
		(2 marks)
9b	Makes an attempt to differentiate $f(x)$	**M1**
	Correctly finds $f'(x) = -9\sin^2 x\cos x + \sin x$	**A1**
	Finds $f(1.95) = -0.0348...$ and $f'(1.95) = 3.8040...$	**M1**
	Attempts to find x_1: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 1.95 - \frac{-0.0348...}{3.8040...}$	**M1**
	Finds $x_1 = 1.959$	**A1**
		(5 marks)
	TOTAL: 7 marks	

NOTES: 9a

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval

10	Use Pythagoras' theorem to show that the length of $OB = 2\sqrt{3}$ or $OD = 2\sqrt{3}$ or states $BD = 4\sqrt{3}$	**M1**
	Makes an attempt to find $\angle DAB$ or $\angle DCB$. For example, $\cos\angle DAO = \frac{2}{4}$ is seen.	**M1**
	Correctly states that $\angle DAB = \frac{2\pi}{3}$ or $\angle DCB = \frac{2\pi}{3}$	**A1**
	Makes an attempt to find the area of the sector with a radius of 4 and a subtended angle of $\frac{2\pi}{3}$ For example, $A = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ is shown.	**M1**
	Correctly states that the area of the sector is $\frac{16\pi}{3}$	**A1**
	Recognises the need to subtract the sector area from the area of the rhombus in an attempt to find the shaded area. For example, $\frac{16\pi}{3} - 8\sqrt{3}$ is seen.	**M1**
	Recognises that to find the total shaded area this number will need to be multiplied by 2 For example, $2 \times \left(\frac{16\pi}{3} - 8\sqrt{3}\right)$	**M1**
	Using clear algebra, correctly manipulates the expression and gives a clear final answer of $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$	**A1**
	TOTAL: 8 marks	

11a Differentiates $u = 4t^{\frac{2}{3}}$ obtaining $\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{8}{3}t^{-\frac{1}{3}}$ and differentiates $v = t^2 + 1$ obtaining $\frac{\mathrm{d}v}{\mathrm{d}t} = 2t$	**M1**
Makes an attempt to substitute the above values into the product rule formula: $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}t} - u\frac{\mathrm{d}v}{\mathrm{d}t}}{v^2}$	**M1**
Finds $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\frac{8}{3}t^{\frac{5}{3}} + \frac{8}{3}t^{-\frac{1}{3}} - 8t^{\frac{5}{3}}}{\left(t^2+1\right)^2}$	**M1**
Fully simplfies using correct algebra to obtain $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2}$	**A1**
	(4 marks)
11b Makes an attempt to substitute $t = 2$ into $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2} = \frac{8\left(1-2(2)^2\right)}{3\sqrt[3]{2}\left(2^2+1\right)^2}$	**M1 ft**
Correctly finds $\frac{\mathrm{d}H}{\mathrm{d}t} = -0.592...$ and concludes that as $\frac{\mathrm{d}H}{\mathrm{d}t} < 0$ the toy soldier was decreasing in height after 2 seconds.	**B1 ft***
	(2 marks)
11c $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2} = 0$ or $8 - 16t^2 = 0$ at a turning point.	**M1 ft**
Solves $8 - 16t^2 = 0$ to find $t = \frac{1}{\sqrt{2}}$ Can also state $t \neq -\frac{1}{\sqrt{2}}$	**A1 ft**
	(2 marks)
TOTAL: 8 marks	

NOTES:

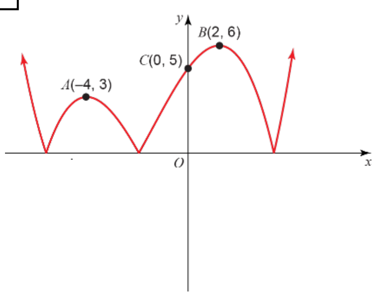
11b: Award ft marks for a correct answer using an incorrect answer from part **a**.

B1: Can also state $\frac{\mathrm{d}H}{\mathrm{d}t} < 0$ as the numerator of $\frac{\mathrm{d}H}{\mathrm{d}t}$ is negative and the denominator is positive.

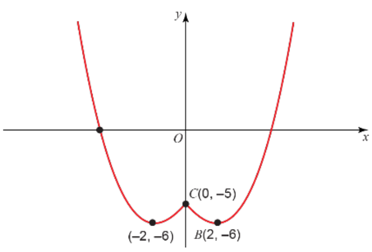
11c: Award ft marks for a correct answer using an incorrect answer from part **a**.

12a Clear attempt to reflect the negative part of the original graph in the x-axis.	**M1**
Labels all three points correctly.	**A1**
Fully correct graph. (see below)	**A1**
	(3 marks)
12b Clear attempt to reflect the positive x part of the original graph in the y-axis.	**M1**
Labels all three points correctly.	**A1**
Fully correct graph. (see below)	**A1**
	(3 marks)
12c Clear attempt to move the graph to the left 3 spaces.	
Clear attempt to stretch the graph vertically by a factor of 2.	
Fully correct graph. (see below)	
	(3 marks)
TOTAL: 9 marks	

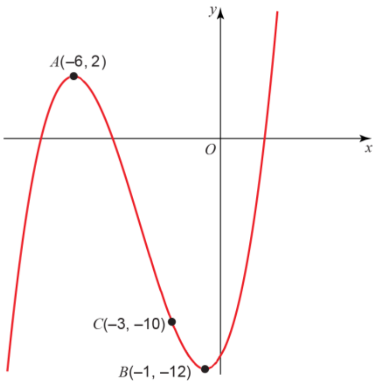
12a



12b



12c



13a	Recognises that it is a geometric series with a first term $a = 100$ and common ratio $r = 1.05$	**M1**
	Attempts to use the sum of a geometric series. For example, $S_9 = \frac{100(1-1.05^9)}{1-1.05}$ or $S_9 = \frac{100(1.05^9-1)}{1.05-1}$ is seen.	**M1***
	Finds $S_9 = £1102.66$	**A1**
		(3 marks)
13b	States $\frac{100(1.05^n-1)}{1.05-1} > 6000$ or $\frac{100(1-1.05^n)}{1-1.05} > 6000$	**M1**
	Begins to simplify. $1.05^n > 4$ or $-1.05^n < -4$	**M1**
	Applies law of logarithms correctly $n\log 1.05 > \log 4$ or $-n\log 1.05 < -\log 4$	**M1**
	States $n > \frac{\log 4}{\log 1.05}$	**A1**
		(4 marks)
13c	Uses the sum of an arithmetic series to state $\frac{29}{2}\left[100+(28)d\right] = 6000$	**M1**
	Solves for d. $d = £11.21$	**A1**
		(2 marks)
	TOTAL: 9 marks	

NOTES: 13a:

M1 Award mark if attempt to calculate the amount of money after 1, 2, 3,….,8 and 9 months is seen.

14a	Writes $(a+bx)^{\frac{1}{3}}$ as $a^{\frac{1}{3}}\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}$	**M1**
	Expands $\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}$ $\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}=1+\left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2x^2}{2}+\ldots$	**M1**
	Simplifies: $a^{\frac{1}{3}}\left(1+\left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2x^2}{2}+\ldots\right)$ $=a^{\frac{1}{3}}\left(1+\frac{b}{3a}x-\frac{b^2}{9a^2}x^2\ldots\right)$ Award mark even if x^2 term is not seen. $=a^{\frac{1}{3}}+\frac{b}{3a^{\frac{2}{3}}}x-\frac{b^2}{9a^{\frac{5}{3}}}x^2\ldots$	**M1**
	Uses $a^{\frac{1}{3}}=4$ to write $a = 64$.	**A1**
	Uses $\frac{b}{3a^{\frac{2}{3}}}=-\frac{1}{8}$ to write $b = –6$.	**A1**
		(5 marks)
14b	States expansion is valid for $\left\|-\frac{6}{64}x\right\|<1$	**B1 ft**
	Solves to state $-\frac{32}{3}<x<\frac{32}{3}$	**A1 ft**
		(2 marks)
14c	Substitutes $a = 64$ and $b = –6$ into $-\frac{b^2}{9a^{\frac{5}{3}}}$	**M1 ft**
	Finds $c=-\frac{1}{256}$	**A1 ft**
		(2 marks)
	TOTAL: 9 marks	

NOTES:

14a: Note x^2 term is not necessary to answer part **a**, so is not required. Will be needed to answer part **c**.

14b: Award marks for a correct conclusion using incorrect values of a and b from part **a**.

14c: Award marks for a correct answer using incorrect values of a and b from part **a**.

15a	States $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$	**M1**
	Deduces that $V = \pi r^2 h = 1600\pi h$	**M1**
	Finds $\frac{dV}{dh} = 1600\pi$ and/or $\frac{dh}{dV} = \frac{1}{1600\pi}$	**M1**
	States $\frac{dV}{dt} = 4000\pi - 50\pi h$	**M1**
	Makes an attempt to find $\frac{dh}{dt} = (4000\pi - 50\pi h) \times \frac{1}{1600\pi}$	**M1**
	Shows a clear logical progression to state $160\frac{dh}{dt} = 400 - 5h$	**A1**
		(6 marks)
15b	Separates the variables $\int \left(\frac{1}{400-5h}\right) dh = \int \frac{1}{160} dt$	**M1**
	Finds $-\frac{1}{5}\ln(400-5h) = \frac{t}{160} + C$	**A1**
	Uses the fact that $t = 0$ when $h = 50$ m to find C: $C = -\frac{1}{5}\ln(150)$	**M1**
	Substitutes $h = 60$ into the equation: $-\frac{1}{5}\ln(400-300) = \frac{t}{160} - \frac{1}{5}\ln(150)$	**M1**
	Uses law of logarithms to write: $\frac{1}{5}\ln(150) - \frac{1}{5}\ln(100) = \frac{t}{160}$ $\Rightarrow \frac{1}{5}\ln\left(\frac{150}{100}\right) = \frac{t}{160}$	**M1**
	States correct final answer: $t = 32\ln\left(\frac{3}{2}\right)$ minutes.	**A1**
		(6 marks)
	TOTAL: 12 marks	

(TOTAL: 100 MARKS)