PURE MATHEMATICS
A level Practice Papers

PAPER M
MARK SCHEME

1 Makes an attempt to substitute any of n = 1, 2, 3, 4, 5 or 6 into $\frac{n(n+1)}{2}$	**M1**
Successfully substitutes n = 1, 2, 3, 4, 5 **and** 6 into $\frac{n(n+1)}{2}$ $1=\frac{(1)(2)}{2}$ $1+2=\frac{(2)(3)}{2}$ $1+2+3=\frac{(3)(4)}{2}$ $1+2+3+4=\frac{(4)(5)}{2}$ $1+2+3+4+5=\frac{(5)(6)}{2}$ $1+2+3+4+5+6=\frac{(6)(7)}{2}$	**A1**
Draws the conclusion that as the statement is true for all numbers from 1 to 6 inclusive, it has been proved by exhaustion.	**B1**
TOTAL: 3 marks	

2 Uses the double-angle formulae to write: $6\sin\theta\cos60+6\cos\theta\sin60=8\sqrt{3}\cos\theta$	**M1**
Uses the fact that $\cos60=\frac{1}{2}$ and $\sin60=\frac{\sqrt{3}}{2}$ to write: $3\sin\theta+3\sqrt{3}\cos\theta=8\sqrt{3}\cos\theta$	**M1**
Simplifies this expression to $\tan\theta=\frac{5\sqrt{3}}{3}$	**M1**
Correctly solves to find $\theta=70.9°, 250.9°$	**A1**
TOTAL: 4 marks	

3a Makes an attempt to substitute $t = 0$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$ For example, $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^0$ or $\mathrm{T}(t) = T_R + (90 - T_R)$ is seen.	**M1**
Concludes that the T_R terms will always cancel at $t = 0$, therefore the room temperature does not influence the initial coffee temperature.	**B1**
	(2 marks)
3b Makes an attempt to substitute $T_R = 20$ and $t = 10$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$ For example, $\mathrm{T}(10) = 20 + (90 - 20)\mathrm{e}^{-\frac{1}{20}(10)}$ is seen.	**M1**
Finds $\mathrm{T}(10) = 62.457...°\mathrm{C}$. Accept awrt 62.5°.	**A1**
	(2 marks)
TOTAL: 4 marks	

4 Makes an attempt to find $\int (10 - 2x)^4 \, \mathrm{d}x$. Raising the power by 1 would constitute an attempt.	**M1**
Correctly states $\int (10 - 2x)^4 \, \mathrm{d}x = -\frac{1}{10}(10 - 2x)^5$	**A1**
States $-\frac{1}{10}(2)^5 + \frac{1}{10}(10 - 2a)^5 = \frac{211}{10}$	**M1 ft**
Makes an attempt to solve this equation. For example, $\frac{1}{10}(10 - 2a)^5 = \frac{243}{10}$ or $(10 - 2a)^5 = 243$ is seen.	**M1 ft**
Solves to find $a = \frac{7}{2}$	**A1 ft**
TOTAL: 5 marks	

NOTES:

Student does not need to state '+C' in an answer unless it is the final answer to an indefinite integral.

Award ft marks for a correct answer using an incorrect initial answer.

5	Begins the proof by assuming the opposite is true. 'Assumption: there exist positive integer solutions to the statement $x^2 - y^2 = 1$'	**B1**
	Sets up the proof by factorising $x^2 - y^2$ and stating $(x-y)(x+y)=1$	**M1**
	States that there is only one way to multiply to make 1: $1 \times 1 = 1$ and concludes this means that: $x - y = 1$ and $x + y = 1$	**M1**
	Solves this pair of simultaneous equations to find the values of x and y: $x = 1$ and $y = 0$	**M1**
	Makes a valid conclusion. $x = 1, y = 0$ are not both positive integers, which is a contradiction to the opening statement. Therefore there do not exist positive integers x and y such that $x^2 - y^2 = 1$	**B1**
	TOTAL: 5 marks	

6	States that: $A(x-4)(3x+1) + B(3x+1) + C(x-4)(x-4) \equiv 18x^2 - 98x + 78$	**M1**
	Further states that: $A(3x^2 - 11x - 4) + B(3x+1) + C(x^2 - 8x + 16) \equiv 18x^2 - 98x + 78$	**M1**
	Equates the various terms. Equating the coefficients of x^2: $3A + C = 18$ Equating the coefficients of x: $-11A + 3B - 8C = -98$ Equating constant terms: $-4A + B + 16C = 78$	**M1**
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1**
	Finds the correct value of any one variable: either $A = 4$, $B = -2$ or $C = 6$	**A1**
	Finds the correct value of all three variables: $A = 4$, $B = -2$, $C = 6$	**A1**
	TOTAL: 6 marks	

Alternative method

Uses the substitution method, having first obtained this equation:

$A(x-4)(3x+1) + B(3x+1) + C(x-4)(x-4) \equiv 18x^2 - 98x + 78$

Substitutes $x = 4$ to obtain $13B = -26$

Substitutes $x = -\frac{1}{3}$ to obtain $\frac{169}{9}C = \frac{338}{3} \Rightarrow C = \frac{1014}{169} = 6$

Equates the coefficients of x^2: $3A + C = 18$

Substitutes the found value of C to obtain $3A = 12$

7a	States or implies that $\mathrm{pq}(x) = (5-2x)^2$	**M1**
	States or implies that $\mathrm{qp}(x) = 5-2x^2$	**M1**
	Makes an attempt to solve $(5-2x)^2 = 5-2x^2$ For example, $25-20x+4x^2 = 5-2x^2$ or $6x^2-20x+20=0$ is seen.	**M1**
	States that $3x^2-10x+10=0$. Must show all steps and a logical progression.	**A1**
		(4 marks)
7b	$b^2-4ac = 100-4(3)(10) = -20 < 0$	**M1***
	States that as $b^2-4ac<0$ there are no real solutions to the equation.	**B1***
		(2 marks)
	TOTAL: 6 marks	

NOTES: 7b Alternative Method

M1: Uses the method of completing the square to show that $3\left(x-\frac{5}{3}\right)^2+\frac{65}{9}=0$ or $3\left(x-\frac{5}{3}\right)^2=-\frac{65}{9}$

B1: Concludes that this equation will have no real solutions.

8a	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+12x-12$	**M1**
	Finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x+12$	**M1**
	States that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x+12 \le 0$ for all -5 Ñ x Ñ -3 and concludes this implies C is concave over the given interval.	**B1**
		(3 marks)
8b	States or implies that a point of inflection occurs when $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	**M1**
	Finds $x=-2$	**A1**
	Substitutes $x=-2$ into $y=x^3+6x^2-12x+6$, obtaining $y=46$	**A1**
		(3 marks)
	TOTAL: 6 marks	

9a	Forms a pair of simultaneous equations, using the given values $a + 3d = 98$ $a + 10d = 56$	**M1**
	Correctly solves to find $d = -6$	**A1**
	Finds $a = 116$	**A1**
	Uses $a_n = a + (n-1)d$ to find $a_{20} = 116 + 19 \times (-6) = 2$	**A1**
		(4 marks)
9b	Uses the sum of an arithmetic series to form the equation $\frac{n}{2}\left[232 + (n-1)(-6)\right] = 78$	**M1 ft**
	Successfully multiplies out the brackets and simplifies. Fully simplified quadratic of $3n^2 - 119n + 78 = 0$ is seen or $6n^2 - 238n + 156 = 0$ is seen.	**M1 ft**
	Correctly factorises: $(3n-2)(n-39) = 0$	**M1 ft**
	States that $n = 39$ is the correct answer.	**A1**
		(4 marks)
	TOTAL: 8 marks	

NOTES:

9a Can use elimination or substitution to solve the simultaneous equations.

9b Award method marks for a correct attempt to solve the equation using their incorrect values from part **a**.

10a	Interprets the stone hitting the ground as when $8t-4.9t^2+10=0$	**M1**
	Makes an attempt to use the quadratic formula to find *t*. For example, $t=\dfrac{8\pm\sqrt{64-4(4.9)(-10)}}{2(4.9)}$ is seen	**M1**
	Finds $t=2.461...$	**M1**
	Deduces $x=10(2.461...)=24.61....\text{m}$. Accept awrt 24.6	**A1**
		(4 marks)
10b	Finds $\dfrac{\mathrm{d}y}{\mathrm{d}t}=-9.8t+8$	**M1**
	Demonstrates an understanding that the greatest height will occur when $\dfrac{\mathrm{d}y}{\mathrm{d}t}=0$ For example, $-9.8t+8=0$	**M1**
	Solves to find $t=\dfrac{40}{49}=0.816...$	**M1**
	Makes an attempt to find the greatest height by substituting $t=\dfrac{40}{49}$ into $y=8t-4.9t^2+10$ For example, $y=8\left(\dfrac{40}{49}\right)-4.9\left(\dfrac{40}{49}\right)^2+10$	**M1 ft**
	Finds $y=\dfrac{650}{49}$=13.265… m. Accept awrt 13.3 m	**A1 ft**
		(5 marks)
	TOTAL: 9 marks	

NOTES:

10b: $t=\dfrac{40}{49}$ can also be found using $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=0$. This is an acceptable method.

10b: Award ft marks for correct sketch using incorrect values from earlier in part **b**.

11	Finds $\overrightarrow{OP}$ via M $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \overrightarrow{OM} + \lambda\overrightarrow{ML} = \frac{1}{4}a + \lambda\overrightarrow{ML}$	**M1**
	Finds $\overrightarrow{OP}$ via N $\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP} = \overrightarrow{ON} + \mu\overrightarrow{NK} = \frac{3}{4}a + \mu\overrightarrow{NK}$	**M1**
	Finds $\overrightarrow{ML} = \overrightarrow{MO} + \overrightarrow{OB} + \overrightarrow{BE} + \overrightarrow{EL} = -\frac{1}{4}a + b + c + \frac{3}{4}a = \frac{1}{2}a + b + c$	**M1**
	Finds $\overrightarrow{NK} = \overrightarrow{NO} + \overrightarrow{OB} + \overrightarrow{BE} + \overrightarrow{EK} = -\frac{3}{4}a + b + c + \frac{1}{4}a = -\frac{1}{2}a + b + c$	**M1**
	Equates the two ways of moving from O to P. $\frac{1}{4}a + \lambda\left(\frac{1}{2}a + b + c\right) = \frac{3}{4}a + \mu\left(-\frac{1}{2}a + b + c\right)$	**M1**
	Equates coefficients of a: $\frac{1}{4} + \frac{1}{2}\lambda = \frac{3}{4} - \frac{1}{2}\mu \Rightarrow \lambda + \mu = 1$	**M1**
	Equates coefficients of b. $\lambda = \mu$ OR equates coefficients of c. $\lambda = \mu$	**M1**
	Solves to find $\lambda = \mu = \frac{1}{2}$	**A1**
	Concludes that at this value the lines intersect.	**B1**
	Concludes that the lines must bisect one another as $\lambda = \frac{1}{2} \Rightarrow \overrightarrow{MP} = \frac{1}{2}\overrightarrow{ML}$ and $\mu = \frac{1}{2} \Rightarrow \overrightarrow{NP} = \frac{1}{2}\overrightarrow{NK}$	**B1**
	TOTAL: 10 marks	

12a	States that $A(2x+3)+B(1-4x)\equiv 21-14x$	**M1**
	Equates the various terms. Equating xs $2A-4B=-14$ Equating numbers $3A+B=21$	**M1**
	Multiplies or or both of the equations in an effort to equate one of the two variables.	**M1**
	Finds $A = 5$	**A1**
	Find $B = 6$	**A1**
		(5 marks)
12b	Writes $\int_{-1}^{0}\left(\frac{5}{1-4x}+\frac{6}{2x+3}\right)\mathrm{d}x$ as $\int_{-1}^{0}\left(5(1-4x)^{-1}+6(2x+3)^{-1}\right)\mathrm{d}x$	**M1 ft**
	Makes an attempt to integrate the expression. Attempt would constitute the use of logarithms.	**M1 ft**
	Integrates the expression to find $\left[-\frac{5}{4}\ln(1-4x)+3\ln(2x+3)\right]_{-1}^{0}$	**A1 ft**
	Makes an attempt to substitute the limits $\left(-\frac{5}{4}\ln(1-4(0))+3\ln(2(0)+3)\right)-\left(-\frac{5}{4}\ln(1-4(-1))+3\ln(2(-1)+3)\right)$	**M1 ft**
	Simplifies to find $\ln 27+\frac{5}{4}\ln 5$ o.e.	**A1 ft**
		(5 marks)
	TOTAL: 10 marks	

NOTES:

Award ft marks for a correct answer to part **b** using incorrect values from part **a**.

13a	States that the local maximum occurs when $p'(t) = 0$	**B1**
	Makes an attempt to differentiate $p(t)$	**M1**
	Correctly finds $p'(t) = \frac{1}{10(t+1)} + \frac{1}{2}\sin\left(\frac{t}{2}\right) + \frac{3}{20}t^{\frac{1}{2}}$	**A1**
	Finds $p'(8.5) = 0.000353...$ and $p'(8.6) = -0.00777...$	**M1**
	Change of sign and continuous function in the interval $[8.5, 8.6]$ Therefore the gradient goes from positive to negative and so the function has reached a maximum.	**A1**
		(5 marks)
13b	States that the local minimum occurs when $p'(t) = 0$	**B1**
	Makes an attempt to differentiate $p'(t)$	**M1**
	Correctly finds $p''(t) = -\frac{1}{10(t+1)^2} + \frac{1}{4}\cos\left(\frac{t}{2}\right) + \frac{3}{40}t^{-\frac{1}{2}}$	**A1**
	Finds $p'(9.9) = -0.00481...$ and $p''(9.9) = 0.0818...$	**M1**
	Attempts to find t_1 $t_1 = t_0 - \frac{p'(t_0)}{p''(t_0)} \Rightarrow t_1 = 9.9 - \frac{-0.0048...}{0.0818...}$	**M1**
	Finds $t_1 = 9.959$	**A1**
		(6 marks)
	TOTAL: 11 marks	

NOTES:

13a

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

14a Makes an attempt to set up a long division. For example, $2x^2 - x - 1 \overline{)4x^2 - 4x - 9}$ is seen.	**M1**
Long division completed so that a 2 is seen in the quotient and a remainder of $-2x - 7$ is also seen. $$\begin{array}{r} 2 \\ 2x^2 - x - 1 \overline{)4x^2 - 4x - 9} \\ \underline{4x^2 - 2x - 2} \\ -2x - 7 \end{array}$$	**M1**
States $B(x-1) + C(2x+1) \equiv -2x - 7$	**M1**
Either equates variables or makes a substitution in an effort to find B or C.	**M1**
Finds $B = 4$	**A1**
Finds $C = -3$	**A1**
	(6 marks)
14b Correctly writes $4(2x+1)^{-1}$ or $4(1+2x)^{-1}$ as $4\left(1 + (-1)(2x) + \frac{(-1)(-2)(2)^2 x^2}{2} + \ldots\right)$	**M1 ft**
Simplifies to obtain $4 - 8x + 16x^2 + \ldots$	**A1 ft**
Correctly writes $\frac{-3}{x-1}$ as $\frac{3}{1-x}$	**M1 ft**
Correctly writes $3(1-x)^{-1}$ as $3\left(1 + (-1)(-x) + \frac{(-1)(-2)(-1)^2(-x)^2}{2} + \ldots\right)$	**M1 ft**
Simplifies to obtain $3 + 3x + 3x^2 + \ldots$	**A1 ft**
States the correct final answer: $9 - 5x + 19x^2$	**A1 ft**
	(6 marks)
14c The expansion is only valid for $\lvert x \rvert < \frac{1}{2}$	**B1**
	(1 mark)
TOTAL: 13 marks	

NOTES:

14a

Writes the RHS as a single fraction.

Obtains $4x^2 - 4x - 9 = A(2x+1)(x-1) + B(x-1) + C(2x+1)$

Substitutes $x = 1$ to obtain $C = -3$

Substitutes $x = -\frac{1}{2}$ to obtain $B = 4$

Compares coefficients of x^2 to obtain $A = 2$

14b

Award all 6 marks for a correct answer using their incorrect values of A, B and/or C from part **a**.

(TOTAL: 100 MARKS)