NAME:

Date to be handed in:

MARK (out of 100):

PAPER L

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1 It is suggested that the sequence $a_k = 2^k + 1, k$ Ö 1 produces only prime numbers.

a Show that a_1, a_2 and a_4 produce prime numbers. **(2 marks)**

b Prove by counter example that the sequence does not always produce a prime number. **(2 marks)**

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2 Find $\int \sin^3 x \, dx$ **(4 marks)**

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3 Use proof by contradiction to show that, given a rational number a and an irrational number b, $a - b$ is irrational. **(4 marks)**

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4 Given that $x = \sec 4y$, find:

a $\frac{dy}{dx}$ in terms of y **(2 marks)**

b Show that $\frac{dy}{dx} = \frac{k}{x\sqrt{x^2 - 1}}$ where k is a constant which should be found. **(3 marks)**

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5 **a** Prove that $\frac{\tan x - \sec x}{1 - \sin x} \equiv -\sec x, \ x \neq (2n+1)\frac{\pi}{2}$ **(3 marks)**

b Hence solve, in the interval $0 \leq x \leq 2\pi$, the equation $\frac{\tan x - \sec x}{1 - \sin x} = \sqrt{2}$ **(3 marks)**

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6 **a** Given that $f(x) = \sin x$, show that $f'(x) = \lim_{h \to 0}\left(\left(\frac{\cos h - 1}{h}\right)\sin x + \frac{\sin h}{h}\cos x\right)$ **(4 marks)**

b Hence prove that $f'(x) = \cos x$ **(2 marks)**

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7 $\quad \mathrm{f}(x)=\dfrac{x^4+2x^3-29x^2-47x+77}{x^2-2x-15}$

Show that $\mathrm{f}(x)$ can be written as $Px^2+Qx+R+\dfrac{V}{x+3}+\dfrac{W}{x-5}$ and find the values of P, Q, R, V and W.

(7 marks)

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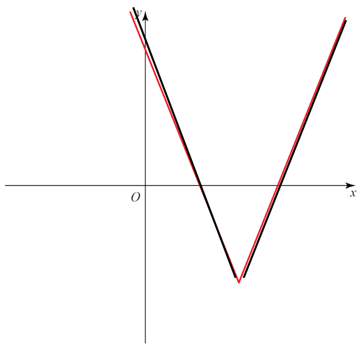
8 The diagram shows a sketch of part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x)=3|x-4|-5$



a State the range of f. **(1 mark)**

b Given that $\mathrm{f}(x)=-\dfrac{1}{3}x+k$, where k is a constant has two distinct roots, state the possible values of k.

(7 marks)

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9 C has parametric equations $x=\dfrac{1+4t}{1-t}$, $y=\dfrac{2+bt}{1-t}$, -1 Ñ t Ñ 0

a Show that the cartesian equation of C is $y=\left(\dfrac{2+b}{5}\right)x+\left(\dfrac{8-b}{5}\right)$,over an appropriate domain.

(4 marks)

Given that C is a line segment and that the gradient of the line is -1,

b show that the length of the line segment is $a\sqrt{2}$, where a is a rational number to be found.

(4 marks)

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10 Given that $\displaystyle\int_{\ln 2}^{\ln b}\left(\frac{\mathrm{e}^{2x}}{\mathrm{e}^{2x}-1}\right)\mathrm{d}x=\ln 4$, find the value of b showing each step in your working.

(8 marks)

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11 At the beginning of each month Kath places £100 into a bank account to save for a family holiday. Each subsequent month she increases her payments by 5%.

Assuming the bank account does not pay interest, find

a the amount of money in the account after 9 months. **(3 marks)**

Month n is the first month in which there is more than £6000 in the account.

b Show that $n > \dfrac{\log 4}{\log 1.05}$ **(4 marks)**

Maggie begins saving at the same time as Kath. She initially places £50 into the same account and plans to increase her payments by a constant amount each month.

c Given that she would like to reach a total of £6000 in 29 months, by how much should Maggie increase her payments each month? **(2 marks)**

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12 A particle of mass 3 kg is acted on by three forces, $F_1 = (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})\text{N}$, $F_2 = (7\mathbf{i} + 8\mathbf{k})\text{N}$ and $F_3 = (-3\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})\text{N}$.

a Find the resultant force R acting on the particle. **(2 marks)**

b Find the acceleration of the particle, giving your answer in the form $(p\mathbf{i} + q\mathbf{j} + r\mathbf{k})\text{ms}^{-2}$ **(2 marks)**

c Find the magnitude of the acceleration. **(2 marks)**

d Given that the particle starts at rest, find the exact distance travelled by the particle in the first 10 s. **(3 marks)**

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13 $\text{h}(t) = 40\ln(t+1) + 40\sin\left(\dfrac{t}{5}\right) - \dfrac{1}{4}t^2, \; t \geq 0$

The graph $y = \text{h}(t)$ models the height of a rocket t seconds after launch.

a Show that the rocket returns to the ground between 19.3 and 19.4 seconds after launch. **(2 marks)**

b Using $t_0 = 19.35$ as a first approximation to α, apply the Newton–Raphson procedure once to $\text{h}(t)$ to find a second approximation to α, giving your answer to 3 decimal places. **(5 marks)**

c By considering the change of sign of $\text{h}(t)$ over an appropriate interval, determine if your answer to part **b** is correct to 3 decimal places. **(3 marks)**

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14 $\text{f}(x) = \dfrac{6}{2+3x} - \dfrac{4}{3-5x}, \; |x| < \dfrac{3}{5}$

a Show that the first three terms in the series expansion of $\text{f}(x)$ can be written as $\dfrac{5}{3} - \dfrac{121}{18}x + \dfrac{329}{108}x^2$ **(7 marks)**

b Find the exact value of $\text{f}(0.01)$. Round your answer to 7 decimal places. **(2 marks)**

c Find the percentage error made in using the series expansion in part **a** to estimate the value of $\text{f}(0.01)$. Give your answer to 2 significant figures. **(3 marks)**

(TOTAL: 100 MARKS)