1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(2x+5)+B(5x-1)\equiv 6x+42$	**M1**	2.2a	5th Decompose algebraic fractions into partial fractions − two linear factors.
	Equates the various terms. Equating the coefficients of x: $2A+5B=6$ Equating constant terms: $5A-B=42$	**M1***	2.2a	
	Multiplies both of the equations in an effort to equate one of the two variables.	**M1***	1.1b	
	Finds $A = 8$	**A1**	1.1b	
	Find $B = -2$	**A1**	1.1b	
				(5 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation: $A(2x+5)+B(5x-1)\equiv 6x+42$

Substitutes $x=-\frac{5}{2}$ to obtain $-\frac{27}{2}B = 27$ (**M1**)

Substitutes $x=\frac{1}{5}$ to obtain $\frac{27}{5}A = 43.2$ (**M1**)

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there do exist integers a and b such that $25a+15b=1$'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Understands that $25a+15b=1 \Rightarrow 5a+3b=\frac{1}{5}$ 'As both 25 and 15 are multiples of 5, divide both sides by 5 to leave $5a+3b=\frac{1}{5}$'	**M1**	2.2a	
	Understands that if a and b are integers, then $5a$ is an integer, $3b$ is an integer and $5a + 3b$ is also an integer.	**M1**	1.1b	
	Recognises that this contradicts the statement that $5a+3b=\frac{1}{5}$, as $\frac{1}{5}$ is not an integer. Therefore there do not exist integers a and b such that $25a+15b=1$'	**B1**	2.4	
				(4 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\frac{dx}{dt} = -2\sin 2t$ and $\frac{dy}{dt} = \cos t$	M1	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
	Writes $-2\sin 2t = -4\sin t\cos t$	M1	2.2a	
	Calculates $\frac{dy}{dx} = \frac{\cos t}{-4\sin t\cos t} = -\frac{1}{4}\operatorname{cosec} t$	A1	1.1b	
		(3)		
(b)	Evaluates $\frac{dy}{dx}$ at $t = -\frac{5\pi}{6}$ $\frac{dy}{dx} = \frac{-1}{4\sin\left(-\frac{5\pi}{6}\right)} = \frac{1}{2}$	A1 ft	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
	Understands that the gradient of the tangent is $\frac{1}{2}$, and then the gradient of the normal is -2.	M1 ft	1.1b	
	Finds the values of x and y at $t = -\frac{5\pi}{6}$ $x = \cos\left(2\times -\frac{5\pi}{6}\right) = \frac{1}{2}$ and $y = \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$	M1 ft	1.1b	
	Attempts to substitute values into $y - y_1 = m(x - x_1)$ For example, $y + \frac{1}{2} = -2\left(x - \frac{1}{2}\right)$ is seen.	M1 ft	2.2a	
	Shows logical progression to simplify algebra, arriving at: $y = -2x + \frac{1}{2}$ or $4x + 2y - 1 = 0$	A1	2.4	
		(5)		
				(8 marks)

Notes

(b) Award ft marks for a correct answer using an incorrect answer from part **a**.

4	**Scheme**	**Marks**	**AOs**	**Pearson Progression Step and Progress descriptor**
	States that $\cot 3x = \frac{\cos 3x}{\sin 3x}$	**M1**	2.2a	6th Integrate using trigonometric identities.
	Makes an attempt to find $\int \left(\frac{\cos 3x}{\sin 3x}\right) \mathrm{d}x$ Writing $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)} \mathrm{d}x = \ln\left[\mathrm{f}(x)\right]$ or writing ln (sin x) constitutes an attempt.	**M1**	2.2a	
	States a fully correct answer $\frac{1}{3}\ln\lvert\sin 3x\rvert + C$	**A1**	1.1b	
				(3 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Demonstrates an attempt to find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$	**M1**	2.2a	5th Find the magnitude of a vector in 3 dimensions.
	Finds $\overrightarrow{AB} = (0, 4, -2)$, $\overrightarrow{AC} = (5, 4, 8)$ and $\overrightarrow{BC} = (5, 0, 10)$	**A1**	1.1b	
	Demonstrates an attempt to find $\|\overrightarrow{AB}\|$, $\|\overrightarrow{AC}\|$ and $\|\overrightarrow{BC}\|$	**M1**	2.2a	
	Finds $\|\overrightarrow{AB}\| = \sqrt{(0)^2 + (4)^2 + (-2)^2} = \sqrt{20}$ Finds $\|\overleftarrow{AC}\| = \sqrt{(5)^2 + (4)^2 + (8)^2} = \sqrt{105}$ Finds $\|\overrightarrow{BC}\| = \sqrt{(5)^2 + (0)^2 + (10)^2} = \sqrt{125}$	**A1**	1.1b	
	States or implies in a right-angled triangle $c^2 = a^2 + b^2$	**M1**	2.2a	
	States that $\|\overrightarrow{AB}\|^2 + \|\overrightarrow{AC}\|^2 = \|\overrightarrow{BC}\|^2$	**B1**	2.1	
				(6 marks)

Notes

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States or implies that $\text{pq}(x) = (5-2x)^2$	M1	2.2a	5th Find composite functions.
	States or implies that $\text{qp}(x) = 5-2x^2$	M1	2.2a	
	Makes an attempt to solve $(5-2x)^2 = 5-2x^2$. For example, $25-20x+4x^2 = 5-2x^2$ or $6x^2-20x+20=0$ is seen.	M1	1.1b	
	States that $3x^2-10x+10=0$. Must show all steps and a logical progression.	A1	1.1b	
		(4)		
(b)	$b^2-4ac = 100-4(3)(10) = -20 < 0$	M1*	2.2a	5th Find the domain and range of composite functions.
	States that as $b^2-4ac<0$ there are no real solutions to the equation.	B1*	3.2b	
		(2)		
				(6 marks)

Notes

(b) Alternative Method

M1: Uses the method of completing the square to show that $3\left(x-\frac{5}{3}\right)^2+\frac{65}{9}=0$ or $3\left(x-\frac{5}{3}\right)^2=-\frac{65}{9}$

B1: Concludes that this equation will have no real solutions.

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there is a finite amount of prime numbers.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Considers what having a finite amount of prime numbers means by making an attempt to list them: Let all the prime numbers exist be $p_1, p_2, p_3, \ldots p_n$	**M1**	2.2a	
	Consider a new number that is one greater than the product of all the existing prime numbers: Let $N = (p_1 \times p_2 \times p_3 \times \ldots \times p_n) + 1$	**M1**	1.1b	
	Understands the implication of this new number is that division by any of the existing prime numbers will leave a remainder of 1. So none of the existing prime numbers is a factor of N.	**M1**	1.1b	
	Concludes that either N is prime or N has a prime factor that is not currently listed.	**B1**	2.4	
	Recognises that either way this leads to a contradiction, and therefore there is an infinite number of prime numbers.	**B1**	2.4	
				(6 marks)

Notes

If N is prime, it is a new prime number separate to the finite list of prime numbers, $p_1, p_2, p_3, \ldots p_n$.

If N is divisible by a previously unknown prime number, that prime number is also separate to the finite list of prime numbers.

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Attempts to write a differential equation. For example, $\frac{\mathrm{d}F}{\mathrm{d}t} \propto F$ or $\frac{\mathrm{d}F}{\mathrm{d}t} \propto -F$ is seen.	**M1**	3.1a	7th Construct simple differential equations.
	States $\frac{\mathrm{d}F}{\mathrm{d}t} = -kF$	**A1**	3.1a	
				(2 marks)

Notes

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Recognises that it is a geometric series with a first term $a = 100$ and common ratio $r = 1.05$	M1	3.1a	6th Use geometric sequences and series in context.
	Attempts to use the sum of a geometric series. For example, $S_9 = \frac{100\left(1-1.05^9\right)}{1-1.05}$ or $S_9 = \frac{100\left(1.05^9-1\right)}{1.05-1}$ is seen.	M1*	2.2a	
	Finds $S_9 = £1102.66$	A1	1.1b	
		(3)		
(b)	States $\frac{100\left(1.05^n-1\right)}{1.05-1} > 6000$ or $\frac{100\left(1-1.05^n\right)}{1-1.05} > 6000$	M1	3.1a	5th Use arithmetic sequences and series in context.
	Begins to simplify. $1.05^n > 4$ or $-1.05^n < -4$	M1	1.1b	
	Applies law of logarithms correctly $n\log 1.05 > \log 4$ or $-n\log 1.05 < -\log 4$	M1	2.2a	
	States $n > \frac{\log 4}{\log 1.05}$	A1	1.1b	
		(4)		
(c)	Uses the sum of an arithmetic series to state $\frac{29}{2}\left[100+(28)d\right] = 6000$	M1	3.1a	5th Use arithmetic sequences and series in context.
	Solves for d. $d = £11.21$	A1	1.1b	
		(2)		
				(9 marks)

Notes

M1

Award mark if attempt to calculate the amount of money after 1, 2, 3,….,8 and 9 months is seen.

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Selects $\cos 2x \equiv 2\cos^2 x - 1$ as the appropriate trigonometric identity.	**M1**	2.2a	6th Integrate using trigonometric identities.
	Manipulates the identity to the question: $\cos 12x \equiv 2\cos^2 6x - 1$	**M1**	1.1b	
	States that $\int \left(\cos^2 6x\right) \mathrm{d}x = \frac{1}{2}\int \left(1 + \cos 12x\right) \mathrm{d}x$	**M1**	1.1b	
	Makes an attempt to integrate the expression, x and $\sin x$ are seen.	**M1**	1.1b	
	Correctly states $\frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right) + C$	**A1**	1.1b	
				(5 marks)

Notes

Student does not need to state '+C' to be awarded the third method mark. Must be stated in the final answer.

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes tan$x$ and sec$x$ in terms of sin$x$ and cos$x$. For example, $\frac{\tan x - \sec x}{1-\sin x} = \frac{\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right)}{\left(\frac{1-\sin x}{1}\right)}$	**M1**	2.1	5th Understand the functions sec, cosec and cot.
	Manipulates the expression to find $\left(\frac{\sin x - 1}{\cos x}\right) \times \left(\frac{1}{1-\sin x}\right)$	**M1**	1.1b	
	Simplifies to find $-\frac{1}{\cos x} = -\sec x$	**A1**	1.1b	
		(3)		
(b)	States that $-\sec x = \sqrt{2}$ or $\sec x = -\sqrt{2}$	**B1**	2.2a	6th Use the functions sec, cosec and cot to solve simple trigonometric problems.
	Writes that $\cos x = -\frac{1}{\sqrt{2}}$ or $x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	**M1**	1.1b	
	Finds $x = \frac{3\pi}{4}, \frac{5\pi}{4}$	**A1**	1.1b	
		(3)		
				(6 marks)

Notes

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Rearranges $x = 8(t+10)$ to obtain $t = \frac{x-80}{8}$	M1	1.1b	8th Use parametric equations in modelling in a variety of contexts.
	Substitutes $t = \frac{x-80}{8}$ into $y = 100 - t^2$ For example, $y = 100 - \left(\frac{x-80}{8}\right)^2$ is seen.	M1	1.1b	
	Finds $y = -\frac{1}{64}x^2 + \frac{5}{2}x$	A1	1.1b	
		(3)		
(b)	Deduces that the width of the arch can be found by substituting $t = \pm 10$ into $x = 8(t+10)$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Finds x = 0 and x = 160 and deduces the width of the arch is 160 m.	A1	3.2a	
		(2)		
(c)	Deduces that the greatest height occurs when $\frac{\mathrm{d}y}{\mathrm{d}t} = 0 \Rightarrow -2t = 0 \Rightarrow t = 0$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Deduces that the height is 100 m.	A1	3.2a	
		(2)		
				(7 marks)

Notes

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to set up a long division. For example: $x+6\overline{)x^3+8x^2-9x+12}$ is seen.	**M1**	2.2a	5th Divide polynomials by linear expressions with a remainder.
	Award 1 accuracy mark for each of the following: x^2 seen, $2x$ seen, −21 seen. For the final accuracy mark either D = 138 or $\frac{138}{x+6}$ or the remainder is 138 must be seen. $x^2+2x-21$ $x+6\overline{)x^3+8x^2-9x+12}$ $\underline{x^3+6x^2}$ $2x^2-9x$ $\underline{2x^2+12x}$ $-21x+12$ $\underline{-21x-126}$ 138	**A4**	1.1b	
				(5 marks)

Notes

This question can be solved by first writing $(Ax^2+Bx+C)(x+6)+D\equiv x^3+8x^2-9x+12$ and then solving for A, B, C and D. Award 1 mark for the setting up the problem as described. Then award 1 mark for each correct coefficient found. For example:

Equating the coefficients of x^3: $A = 1$

Equating the coefficients of x^2: $6 + B = 8$, so $B = 2$

Equating the coefficients of x: $12 + C = -9$, so $C = -21$

Equating the constant terms: $-126 + D = 12$, so $D = 138$.

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises the need to use the chain rule to find $\frac{dV}{dt}$ For example $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dS} \times \frac{dS}{dt}$ is seen.	**M1**	3.1a	8th Construct differential equations in a range of contexts.
	Finds $\frac{dV}{dr} = 4\pi r^2$ and $\frac{dS}{dr} = 8\pi r$	**M1**	2.2a	
	Makes an attempt to substitute known values. For example, $\frac{dV}{dt} = \frac{4\pi r^2}{1} \times \frac{1}{8\pi r} \times \frac{-12}{1}$	**M1**	1.1b	
	Simplifies and states $\frac{dV}{dt} = -6r$	**A1**	1.1b	
				(4 marks)

Notes

	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
15	Recognises the need to write $\sin^3 x \equiv \sin x\left(\sin^2 x\right)$	M1	2.2a	6th Integrate using trigonometric identities.
	Selects the correct trigonometric identity to write $\sin x\left(\sin^2 x\right) \equiv \sin x\left(1-\cos^2 x\right)$. Could also write $\sin x - \sin x\cos^2 x$	M1	2.2a	
	Makes an attempt to find $\int\left(\sin x - \sin x\cos^2 x\right)\mathrm{d}x$	M1	1.1b	
	Correctly states answer $-\cos x + \frac{1}{3}\cos^3 x + C$	A1	1.1b	
				(4 marks)

Notes

16	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\text{h}(19.3) = (+)0.974...$ and $\text{h}(19.4) = -0.393...$	**M1**	3.1a	7th Use numerical methods to solve problems in context.
	Change of sign and continuous function in the interval $\left[19.3, 19.4\right] \Rightarrow \text{root}$	**A1**	2.4	
		(2)		
(b)	Makes an attempt to differentiate $\text{h}(t)$	**M1**	2.2a	7th Use numerical methods to solve problems in context.
	Correctly finds $\text{h}'(t) = \dfrac{40}{t+1} + 8\cos\left(\dfrac{t}{5}\right) - \dfrac{1}{2}t$	**A1**	1.1b	
	Finds $\text{h}(19.35) = 0.2903...$ and $\text{h}'(19.35) = -13.6792...$	**M1**	1.1b	
	Attempts to find x_1 $x_1 = x_0 - \dfrac{\text{h}(x_0)}{\text{h}'(x_0)} \Rightarrow x_1 = 19.35 - \dfrac{0.2903...}{-13.6792...}$	**M1**	1.1b	
	Finds $x_1 = 19.371$	**A1**	1.1b	
		(5)		
(c)	Demonstrates an understanding that $x = 19.3705$ and $x = 19.3715$ are the two values to be calculated.	**M1**	2.2a	7th Use numerical methods to solve problems in context.
	Finds $\text{h}(19.3705) = (+)0.0100...$ and $\text{h}(19.3715) = -0.00366...$	**M1**	1.1b	
	Change of sign and continuous function in the interval $\left[19.3705, 19.3715\right] \Rightarrow \text{root}$	**A1**	2.4	
		(3)		
				(10 marks)

Notes

(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

17	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Demonstrates an attempt to find the vectors $\overrightarrow{KL}$, $\overrightarrow{LM}$ and $\overrightarrow{KM}$	M1	2.2a	6th Solve geometric problems using vectors in 3 dimensions.
	Finds $\overrightarrow{KL}=(3,0,-6)$, $\overrightarrow{LM}=(2,5,4)$ and $\overrightarrow{KM}=(5,5,-2)$	A1	1.1b	
	Demonstrates an attempt to find $\lvert\overrightarrow{KL}\rvert$, $\lvert\overrightarrow{LM}\rvert$ and $\lvert\overrightarrow{KM}\rvert$	M1	2.2a	
	Finds $\lvert\overrightarrow{KL}\rvert=\sqrt{(3)^2+(0)^2+(-6)^2}=\sqrt{45}$ Finds $\lvert\overrightarrow{LM}\rvert=\sqrt{(2)^2+(5)^2+(4)^2}=\sqrt{45}$ Finds $\lvert\overrightarrow{KM}\rvert=\sqrt{(5)^2+(5)^2+(-2)^2}=\sqrt{54}$	A1	1.1b	
	Demonstrates an understanding of the need to use the Law of Cosines. Either $c^2=a^2+b^2-2ab\times\cos C$ (or variation) is seen, or attempt to substitute into formula is made $\left(\sqrt{54}\right)^2=\left(\sqrt{45}\right)^2+\left(\sqrt{45}\right)^2-2\left(\sqrt{45}\right)\left(\sqrt{45}\right)\cos\theta$	M1 ft	2.2a	
	Makes an attempt to simplify the above equation. For example, $-36=-90\cos\theta$ is seen.	M1 ft	1.1b	
	Shows a logical progression to state $\theta=66.4°$	B1	2.4	
		(7)		
(b)	States or implies that ΔKLM is isosceles.	M1	2.2a	6th Solve geometric problems using vectors in 3 dimensions.
	Makes an attempt to find the missing angles $\angle LKM=\angle LMK=\dfrac{180-66.421...}{2}$	M1	1.1b	
	States $\angle LKM=\angle LMK=56.789...°$. Accept awrt 56.8°	A1	1.1b	
		(3)		
				(10 marks)

Notes

(b) Award ft marks for a correct answer to part **a** using their incorrect answer from earlier in part **a**.