Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	Makes an attempt to substitute any of n = 1, 2, 3, 4, 5 or 6 into $\frac{n(n+1)}{2}$	M1	1.1b	5th Complete proofs by exhaustion.
	Successfully substitutes n = 1, 2, 3, 4, 5 **and** 6 into $\frac{n(n+1)}{2}$ $1=\frac{(1)(2)}{2}$ $1+2=\frac{(2)(3)}{2}$ $1+2+3=\frac{(3)(4)}{2}$ $1+2+3+4=\frac{(4)(5)}{2}$ $1+2+3+4+5=\frac{(5)(6)}{2}$ $1+2+3+4+5+6=\frac{(6)(7)}{2}$	A1	1.1b	
	Draws the conclusion that as the statement is true for all numbers from 1 to 6 inclusive, it has been proved by exhaustion.	B1	2.4	
				(3 marks)

Notes

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Shows that $2\cos 3\theta \approx 2\left(1-\frac{9\theta^2}{2}\right) = 2-9\theta^2$	M1	2.1	6th Understand small-angle approximations for sin, cos and tan (angle in radians).
	Shows that $2\cos 3\theta - 1 \approx 1-9\theta^2 = (1-3\theta)(1+3\theta)$	M1	1.1b	
	Shows $1+\sin\theta+\tan 2\theta = 1+\theta+2\theta = 1+3\theta$	M1	2.1	
	Recognises that $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta-1} \approx \frac{1+3\theta}{(1-3\theta)(1+3\theta)} = \frac{1}{1-3\theta}$	A1	1.1b	
		(4)		
(b)	When θ is small, $\frac{1}{1-3\theta} \approx 1$	A1	1.1b	7th Use small-angle approximations to solve problems.
		(1)		
				(5 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Interprets the stone hitting the ground as when $8t - 4.9t^2 + 10 = 0$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Makes an attempt to use the quadratic formula to find t. For example, $t = \dfrac{8 \pm \sqrt{64 - 4(4.9)(-10)}}{2(4.9)}$ is seen.	M1	2.2a	
	Finds $t = 2.461...$	M1	1.1b	
	Deduces $x = 10(2.461...) = 24.61....$m. Accept awrt 24.6	A1	3.2a	
		(4)		
(b)	Finds $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -9.8t + 8$	M1	2.2a	8th Use parametric equations in modelling in a variety of contexts.
	Demonstrates an understanding that the greatest height will occur when $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 0$. For example, $-9.8t + 8 = 0$	M1	3.1a	
	Solves to find $t = \dfrac{40}{49} = 0.816...$	M1	1.1b	
	Makes an attempt to find the greatest height by substituting $t = \dfrac{40}{49}$ into $y = 8t - 4.9t^2 + 10$ For example, $y = 8\left(\dfrac{40}{49}\right) - 4.9\left(\dfrac{40}{49}\right)^2 + 10$	M1 ft	3.2a	
	Finds $y = \dfrac{650}{49}$=13.265… m. Accept awrt 13.3 m.	A1 ft	1.1b	
		(5)		
				(9 marks)

Notes

(b) $t = \dfrac{40}{49}$ can also be found using $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = 0$. This is an acceptable method.

(b) Award ft marks for correct sketch using incorrect values from earlier in part **b**.

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Differentiates $x = \sec 4y$ to obtain $\frac{dx}{dy} = 4\sec 4y \tan 4y$	M1	1.1b	6th Differentiate reciprocal and inverse trigonometric functions.
	Writes $\frac{dy}{dx} = \frac{1}{4\sec 4y \tan 4y}$	A1	1.1b	
		(2)		
(b)	Use the identity $\tan^2 A + 1 = \sec^2 A$ to write $\tan 4y = \sqrt{\sec^2 4y - 1} = \sqrt{x^2 - 1}$	M1	2.2a	6th Differentiate reciprocal and inverse trigonometric functions.
	Attempts to substitute $\sec 4y = x$ and $\tan 4y = \sqrt{x^2 - 1}$ into $\frac{dy}{dx} = \frac{1}{4\sec 4y \tan 4y}$	M1	2.2a	
	Correctly substitutes to find $\frac{dy}{dx} = \frac{1}{4x\sqrt{x^2 - 1}}$ and states $k = \frac{1}{4}$	A1	1.1b	
		(3)		
				(5 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Clearly states that $\int \frac{6}{x}\,dx = 6\ln x$	A1	1.1b	5th Integrate $\frac{1}{x}$
	Makes an attempt to integrate the remaining two terms. Raising a power by 1 would constitute an attempt.	M1	1.1b	
	States the fully correct answer $6\ln x - \frac{3}{x} - 2x^{\frac{7}{2}} + C$	A1	1.1b	
		(3)		
(b)	Makes an attempt to substitute the limits into the expression. For example, $\left(6\ln 9 - \frac{3}{9} - 2(2187)\right) - \left(6\ln 4 - \frac{3}{4} - 2(128)\right)$ is seen.	M1	1.1b	5th Integrate $\frac{1}{x}$
	Begins to simplify this expression. For example, $6\ln\frac{9}{4} + \frac{5}{12} - 4118$ is seen.	M1	1.1b	
	States the fully correct answer $-\frac{49\,411}{12} + 6\ln\frac{9}{4}$ or states $m = -\frac{49\,411}{12}$, $n = 6$ and $p = \frac{9}{4}$ Also accept $-\frac{49\,411}{12} + 12\ln\frac{3}{2}$ or equivalent.	A1	1.1b	
		(3)		
				(6 marks)

Notes

6	**Scheme**	**Marks**	**AOs**	**Pearson Progression Step and Progress descriptor**
(a)	States the range is y Ö -5 or $\mathrm{f}(x)$ Ö -5	**B1**	3.2b	5th Find the domain and range for a variety of familiar functions.
		(1)		
(b)	Recognises that $3(x-4)-5=-\frac{1}{3}x+k$ and $-3(x-4)-5=-\frac{1}{3}x+k$	**M1**	2.2a	7th Solve problems involving the modulus function in unfamiliar contexts.
	Makes an attempt to solve both of these equations.	**M1**	1.1b	
	Correctly states $\frac{10}{3}x=k+17$. Equivalent version is acceptable.	**A1**	1.1b	
	Correctly states $-\frac{8}{3}x=k-7$. Equivalent version is acceptable.	**A1**	1.1b	
	Makes an attempt to substitute one equation into the other in an effort to solve for *k*. For example, $x=\frac{3}{10}(k+17)$ and $-\left(\frac{8}{3}\right)\left(\frac{3}{10}\right)(k+17)=k-7$ is seen.	**M1 ft**	2.2a	
	Correctly solves to find $k=-\frac{11}{3}$	**A1 ft**	1.1b	
	States the correct range for *k*. $k>-\frac{11}{3}$	**B1**	3.2b	
		(7)		
				(8 marks)

Notes

(b) Award ft marks for a correct method using an incorrect answer from earlier in the question.

Alternative Method

Student draws the line with gradient $-\frac{1}{3}$ passing through the vertex and calculates that $k = -\frac{11}{3}$, so answer is $k > -\frac{11}{3}$

M1: States the x-coordinate of the vertex of the graph is 4

M1: States the y-coordinate of the vertex of the graph is −5

M1: Writes down the gradient of $-\frac{1}{3}$ or implies it later in the question.

M1: Attempts to use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (4, -5)$ and $m = -\frac{1}{3}$

A1: Finds $y = -\frac{1}{3}x - \frac{11}{3}$ o.e.

B1: States the correct range for k: $k > -\frac{11}{3}$

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to set up a long division. For example: $9x^2+0x-16\overline{)9x^2+25x+16}$ is seen. The '0x' being seen is not necessary to award the mark.	**M1**	2.2a	5th Decompose algebraic fractions into partial fractions − two linear factors.
	Long division completed so that a '1' is seen in the quotient and a remainder of $25x + 32$ is also seen. $\begin{array}{r} 1 \\ 9x^2+0x-16\overline{)9x^2+25x+16} \\ \underline{9x^2+\ \ 0x-16} \\ 25x+32 \end{array}$	**M1**	1.1b	
	States $B(3x+4)+C(3x-4)\equiv 25x-32$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x: $3B+3C=25$ Equating constant terms: $4B-4C=32$	**M1**	2.2a	
	Multiplies one or both of the equations in an effort to equate one of the two variables.	**M1**	1.1b	
	Finds $B=\frac{49}{6}$	**A1**	1.1b	
	Finds $C=\frac{1}{6}$	**A1**	1.1b	
				(7 marks)

Notes

Alternative method

Writes $A+\frac{B}{3x-4}+\frac{C}{3x+4}$ as $\frac{A(3x-4)(3x+4)}{9x^2-16}+\frac{B(3x+4)}{9x^2-16}+\frac{C(3x-4)}{9x^2-16}$

States $A(3x-4)(3x+4)+B(3x+4)+C(3x-4)\equiv 9x^2+25x+16$

Substitutes $x=\frac{4}{3}$ to obtain: $8B=\frac{196}{3}\Rightarrow B=\frac{49}{6}$

Substitutes $x=-\frac{4}{3}$ to obtain: $-8C=-\frac{4}{3}\Rightarrow C=\frac{1}{6}$

Equating the coefficients of x^2: $9A=9\Rightarrow A=1$

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States the recurrence relation. Choice of variable is not important. $x_n = 0.7x_{n-1}$ or $x_{n+1} = 0.7x_n$	M1	3.1a	5th Work with sequences defined by simple recurrence relations.
	Defines the first value. Accept either use of x_1 or x_0. $x_1 = 80$ or $x_0 = 80$	M1	3.1a	
		(2)		
(b)	Makes an attempt to find the height, for example 80×0.7^5 is seen.	M1	3.1a	5th Work with the *n*th term formula for geometric sequences.
	States that the maximum height would be 13.445… cm. Accept awrt 13.4	A1	1.1b	
		(2)		
(c)	Attempts to make use of the sum to infinity. For example, $\frac{56}{1-0.7}$ or $\frac{80}{1-0.7}$ is seen.	M1	3.1a	6th Use geometric sequences and series in context.
	Understands that the ball travels upwards and then downwards, so multiplies by 2. $2\left(\frac{56}{1-0.7}\right)$ or $2\left(\frac{80}{1-0.7}\right)$ is seen.	M1	3.1a	
	Recognises that when the ball is dropped, it initially only travels downwards. Either $80+2\left(\frac{56}{1-0.7}\right)$ or $2\left(\frac{80}{1-0.7}\right)-80$ is seen or implied.	M1	3.1a	
	States a fully correct answer of $453\frac{1}{3}$ cm. Accept awrt 453.3 cm.	A1	1.1b	
		(4)		

(d)	'It is very unlikely that the ball will not bounce vertically' or 'This model assumes the ball will continue to bounce forever'.	**B1**	3.5	6th Understand convergent geometric series and the sum to infinity.
		(1)		
				(9 marks)

Notes

(c) Award first method mark for an understanding that the sum to infinity formula is required.

Award second method mark for an understanding that the sum to infinity formula will need adjusting, i.e. the ball goes down and then up.

Award third method mark for an understanding that the ball only goes down initially as it is dropped.

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Uses the double-angle formulae to write: $6\sin\theta\cos 60 + 6\cos\theta\sin 60 = 8\sqrt{3}\cos\theta$	**M1**	2.2a	6th Use the double-angle formulae for sin, cos and tan.
	Uses the fact that $\cos 60 = \frac{1}{2}$ and $\sin 60 = \frac{\sqrt{3}}{2}$ to write: $3\sin\theta + 3\sqrt{3}\cos\theta = 8\sqrt{3}\cos\theta$	**M1**	1.1b	
	Simplifies this expression to $\tan\theta = \frac{5\sqrt{3}}{3}$	**M1**	1.1b	
	Correctly solves to find $\theta = 70.9°, 250.9°$	**A1**	1.1b	
				(4 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
10	Begins the proof by assuming the opposite is true. 'Assumption: there exists a rational number $\frac{a}{b}$ such that $\frac{a}{b}$ is the greatest positive rational number.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Makes an attempt to consider a number that is clearly greater than $\frac{a}{b}$: 'Consider the number $\frac{a}{b}+1$, which must be greater than $\frac{a}{b}$'	**M1**	2.2a	
	Simplifies $\frac{a}{b}+1$ and concludes that this is a rational number. $\frac{a}{b}+1 \equiv \frac{a}{b}+\frac{b}{b} \equiv \frac{a+b}{b}$ By definition, $\frac{a+b}{b}$ is a rational number.	**M1**	1.1b	
	Makes a valid conclusion. This contradicts the assumption that there exists a greatest positive rational number, so we can conclude that there is not a greatest positive rational number.	**B1**	2.4	
				(4 marks)

Notes

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes $(a+bx)^{\frac{1}{3}}$ as $a^{\frac{1}{3}}\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}$	M1	2.2a	6th Understand the binomial theorem for rational n.
	Expands $\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}$ $\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}=1+\left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2x^2}{2}+\ldots$	M1	1.1b	
	Simplifies: $a^{\frac{1}{3}}\left(1+\left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2x^2}{2}+\ldots\right)$ $=a^{\frac{1}{3}}\left(1+\frac{b}{3a}x-\frac{b^2}{9a^2}x^2\ldots\right)$ $=a^{\frac{1}{3}}+\frac{b}{3a^{\frac{2}{3}}}x-\frac{b^2}{9a^{\frac{5}{3}}}x^2\ldots$ Award mark even if x^2 term is not seen.	M1	1.1b	
	Uses $a^{\frac{1}{3}}=4$ to write $a = 64$.	A1	1.1b	
	Uses $\frac{b}{3a^{\frac{2}{3}}}=-\frac{1}{8}$ to write $b = -6$.	A1	1.1b	
		(5)		
(b)	States expansion is valid for $\left\lvert -\frac{6}{64}x\right\rvert<1$	B1 ft	3.2b	6th Understand the conditions for validity of the binomial theorem for rational n.
	Solves to state $-\frac{32}{3}<x<\frac{32}{3}$	A1 ft	1.1b	
		(2)		

(c)	Substitutes $a = 64$ and $b = -6$ into $-\dfrac{b^2}{9a^{\frac{5}{3}}}$	**M1 ft**	1.1b	6th Understand the binomial theorem for rational n.
	Finds $c = -\dfrac{1}{256}$	**A1 ft**	1.1b	
		(2)		
				(9 marks)

Notes

(a) Note x^2 term is not necessary to answer part **a**, so is not required. Will be needed to answer part **c**.

(b) Award marks for a correct conclusion using incorrect values of a and b from part **a**.

(c) Award marks for a correct answer using incorrect values of a and b from part **a**.

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Finds $\overrightarrow{OP}$ via M $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \overrightarrow{OM} + \lambda\overrightarrow{ML} = \frac{1}{4}a + \lambda\overrightarrow{ML}$	M1	3.1a	6th Solve geometric problems using vectors in 3 dimensions.
	Finds $\overrightarrow{OP}$ via N $\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP} = \overrightarrow{ON} + \mu\overrightarrow{NK} = \frac{3}{4}a + \mu\overrightarrow{NK}$	M1	3.1a	
	Finds $\overrightarrow{ML} = \overrightarrow{MO} + \overrightarrow{OB} + \overrightarrow{BE} + \overrightarrow{EL} = -\frac{1}{4}a + b + c + \frac{3}{4}a = \frac{1}{2}a + b + c$	M1	3.1a	
	Finds $\overrightarrow{NK} = \overrightarrow{NO} + \overrightarrow{OB} + \overrightarrow{BE} + \overrightarrow{EK} = -\frac{3}{4}a + b + c + \frac{1}{4}a = -\frac{1}{2}a + b + c$	M1	3.1a	
	Equates the two ways of moving from O to P. $\frac{1}{4}a + \lambda\left(\frac{1}{2}a + b + c\right) = \frac{3}{4}a + \mu\left(-\frac{1}{2}a + b + c\right)$	M1	2.2a	
	Equates coefficients of a: $\frac{1}{4} + \frac{1}{2}\lambda = \frac{3}{4} - \frac{1}{2}\mu \Rightarrow \lambda + \mu = 1$	M1	2.2a	
	Equates coefficients of b. $\lambda = \mu$ OR equates coefficients of c. $\lambda = \mu$	M1	1.1b	
	Solves to find $\lambda = \mu = \frac{1}{2}$	A1	1.1b	
	Concludes that at this value the lines intersect.	B1	2.1	
	Concludes that the lines must bisect one another as $\lambda = \frac{1}{2} \Rightarrow \overrightarrow{MP} = \frac{1}{2}\overrightarrow{ML}$ and $\mu = \frac{1}{2} \Rightarrow \overrightarrow{NP} = \frac{1}{2}\overrightarrow{NK}$	B1	2.1	
				(10 marks)

Notes

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $\frac{dV}{dt} = -kV$	**M1**	3.3	8th Solve differential equations in a range of contexts.
	Separates the variables $\int \frac{1}{V}\, dV = \int -k\, dt$	**M1**	2.2a	
	Finds $\ln V = -kt + C$	**A1**	1.1b	
	Shows clearly progression to state $V = V_0 e^{-kt}$ For example, $V = e^{-kt+C} = e^{-kt}e^{C}$ is seen. May also explain the $V_0 = e^{C}$ where e^{C} is a constant.	**A1**	2.1	
		(4)		
(b)	States $\frac{1}{5}V_0 = V_0 e^{-kt}$	**M1**	3.3	8th Solve differential equations in a range of contexts.
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln\frac{1}{5} = -kt$	**M1**	2.2a	
	States that $k = -\frac{1}{10}\ln\frac{1}{5}$	**A1**	1.1b	
		(3)		
(c)	States $\frac{1}{20}V_0 = V_0 e^{-kt}$	**M1**	3.3	8th Solve differential equations in a range of contexts.
	Simplifies the expression by cancelling V_0 and then taking the natural log of both sides $\ln\frac{1}{20} = t\left(\frac{1}{10}\ln\frac{1}{5}\right)$	**M1**	2.2a	
	Finds t = 18.613… years. Accept 18.6 years.	**A1**	1.1b	
		(3)		
				(10 marks)

Notes

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States that the local maximum occurs when $p'(t)=0$	B1	3.1a	7th Use numerical methods to solve problems in context.
	Makes an attempt to differentiate $p(t)$	M1	2.2a	
	Correctly finds $p'(t)=\frac{1}{10(t+1)}+\frac{1}{2}\sin\left(\frac{t}{2}\right)+\frac{3}{20}t^{\frac{1}{2}}$	A1	1.1b	
	Finds $p'(8.5)=0.000353...$ and $p'(8.6)=-0.00777...$	M1	1.1b	
	Change of sign and continuous function in the interval $[8.5, 8.6]$ Therefore the gradient goes from positive to negative and so the function has reached a maximum.	A1	2.4	
		(5)		
(b)	States that the local minimum occurs when $p'(t)=0$	B1	3.1a	7th Use numerical methods to solve problems in context.
	Makes an attempt to differentiate $p'(t)$	M1	2.2a	
	Correctly finds $p''(t)=-\frac{1}{10(t+1)^2}+\frac{1}{4}\cos\left(\frac{t}{2}\right)+\frac{3}{40}t^{-\frac{1}{2}}$	A1	1.1b	
	Finds $p'(9.9)=-0.00481...$ and $p''(9.9)=0.0818...$	M1	1.1b	
	Attempts to find t_1 $t_1=t_0-\frac{p'(t_0)}{p''(t_0)}\Rightarrow t_1=9.9-\frac{-0.0048...}{0.0818...}$	M1	1.1b	
	Finds $t_1=9.959$	A1	1.1b	
		(6)		
				(11 marks)

Notes

(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.