1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there exists a product of two odd numbers that is even.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Defines two odd numbers. Can choose any two different variables. 'Let $2m + 1$ and $2n + 1$ be our two odd numbers.'	**B1**	2.2a	
	Successfully multiplies the two odd numbers together: $(2m+1)(2n+1) \equiv 4mn+2m+2n+1$	**M1**	1.1b	
	Factors the expression and concludes that this number must be odd. $4mn+2m+2n+1 \equiv 2(2mn+m+n)+1$ $2(2mn+m+n)$ is even, so $2(2mn+m+n)+1$ must be odd.	**M1**	1.1b	
	Makes a valid conclusion. This contradicts the assumption that the product of two odd numbers is even, therefore the product of two odd numbers is odd.	**B1**	2.4	
				(5 marks)

Notes

Alternative method

Assume the opposite is true: there exists a product of two odd numbers that is even. **(B1)**

If the product is even then 2 is a factor. **(B1)**

So 2 is a factor of at least one of the two numbers. **(M1)**

So at least one of the two numbers is even. **(M1)**

This contradicts the statement that both numbers are odd. **(B1)**

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes out the first n terms of the arithmetic sequence in both ascending and descending form $S = a + (a+d) + (a+2d) + ... + (a+(n-1)d)$ $S = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + ... + a$	**M1**	2.4	5th Understand the proof of the Sn formula for arithmetic series.
	Attempts to add these two sequences $2S = (2a+(n-1)d) \times n$	**M1**	2.4	
	States $S = \frac{n}{2}(2a+(n-1)d)$	**A1**	1.1b	
		(3)		
(b)	Makes an attempt to find the sum. For example, $S = \frac{200}{2}(2+199(2))$ is seen.	**M1**	2.2a	4th Understand simple arithmetic series.
	States correct final answer. S = 40 000	**A1**	1.1b	
		(2)		
				(5 marks)

Notes

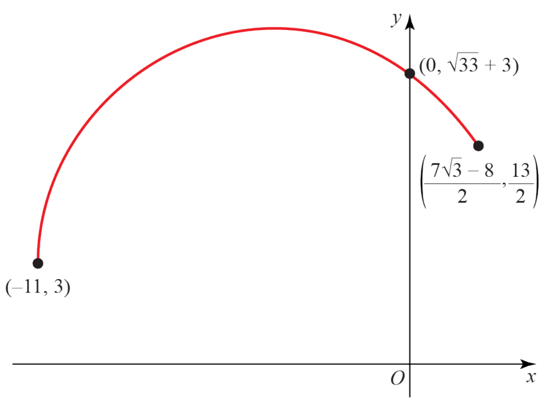
(a) Do not award full marks for an incomplete proof.

(a) Do award second method mark if student indicates that $(2a + (n - 1)d$ appears n times.

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to differentiate $y = \ln 3x$ using the chain rule, or otherwise.	**M1**	2.2a	6th Differentiate sums and differences of functions involving trigonometric, logarithmic and exponential functions.
	Differentiates $y = \ln 3x - e^{-2x}$ to obtain $\frac{dy}{dx} = \frac{1}{x} + 2e^{-2x}$	**A1**	1.1b	
	Evaluates $\frac{dy}{dx}$ at $x = 1$ $\frac{dy}{dx} = 1 + \frac{2}{e^2} = \frac{e^2 + 2}{e^2}$	**A1**	1.1b	
	Evaluates $y = \ln 3x - e^{-2x}$ at $x = 1$ $y = \ln 3 - e^{-2} = \ln 3 - \frac{1}{e^2}$	**M1**	1.1b	
	Attempts to substitute values into $y - y_1 = m(x - x_1)$ For example, $y - \ln 3 + \frac{1}{e^2} = \left(\frac{e^2 + 2}{e^2}\right)(x - 1)$ is seen.	**M1 ft**	2.2a	
	Shows logical progression to simplify algebra, arriving at: $y = \left(\frac{e^2 + 2}{e^2}\right)x - \left(\frac{e^2 + 3}{e^2}\right) + \ln 3$	**A1**	2.4	
				(6 marks)

Notes

Award ft marks for a correct attempt to substitute into the formula using incorrect values.

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $\sin t = \frac{x+4}{7}$ and $\cos t = \frac{y-3}{7}$	M1	1.1b	6th Convert between parametric equations and cartesian forms using trigonometry.
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	M1	2.2a	
	Makes the substitution to find $(x+4)^2 + (y-3)^2 = 7^2$	A1	1.1b	
		(3)		
(b)	States or implies that the curve is a circle with centre (−4, 3) and radius 7	M1 ft	2.2a	6th Sketch graphs of parametric functions.
	Substitutes $t = -\frac{\pi}{2}$ to find $x = -11$ and $y = 3$ (−11, 3) Substitutes $t = \frac{\pi}{3}$ to find $x \approx 2.06$ and $y = 6.5$ (2.06, 6.5) Could also substitute $t = 0$ to find $x = -4$ and $y = 10$ (−4, 10)	M1 ft	1.1b	
	Figure 1  Draws fully correct curve.	A1 ft	1.1b	
		(3)		
(c)	Makes an attempt to find the length of the curve by recognising that the length is part of the circumference. Must at least attempt to find the circumference to award method mark. $C = 2 \times \pi \times 7 = 14\pi$	M1 ft	1.1b	6th Sketch graphs of parametric functions.
	Uses the fact that the arc is $\frac{5}{12}$ of the circumference to write arc length $= \frac{35}{6}\pi$	A1 ft	1.1b	
		(2)		

(8 marks)

Notes

(b) Award ft marks for correct sketch using incorrect values from part **a**.

(c) Award ft marks for correct answer using incorrect values from part **a**.

(c) Alternative method: use $s = r\theta$, with $r = 7$ and $\theta = \frac{5\pi}{6}$. Award one mark for the attempt and one for the correct answer.

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $5^2+6^2+(k-10)^2=(5\sqrt{5})^2$	**M1**	2.2a	5th Find the magnitude of a vector in 3 dimensions.
	Makes an attempt to solve the equation. For example, $(k-10)^2=64$ is seen.	**M1**	1.1b	
	States $k=2$ and $k=18$	**A1**	1.1b	
		(3)		
(b)	Finds the vector $\overrightarrow{OA}=(-1,7,18)$	**M1 ft**	1.1b	5th Find the magnitude of a vector in 3 dimensions.
	Finds $\lvert\overrightarrow{OA}\rvert=\sqrt{(-1)^2+(7)^2+(18)^2}=\sqrt{374}$	**M1 ft**	1.1b	
	States the unit vector $\frac{1}{\sqrt{374}}(-\mathbf{i}+7\mathbf{j}+18\mathbf{k})$	**A1 ft**	1.1b	
		(3)		
				(6 marks)

Notes

(b) Award ft marks for a correct answer to part **b** using their incorrect answer from part **a.**

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to find $\int\left(\frac{e^{2x}}{e^{2x}-1}\right)dx$ Writing $\int\frac{f'(x)}{f(x)}dx=\ln\left[f(x)\right]$ or writing $\ln\left(e^{2x}-1\right)$ constitutes an attempt.	**M1**	2.2a	6th Integrate using the reverse chain rule.
	Correctly states $\int\left(\frac{e^{2x}}{e^{2x}-1}\right)dx=\frac{1}{2}\ln\left(e^{2x}-1\right)(+C)$	**A1**	2.2a	
	Makes an attempt to substitute the limits $x = \ln b$ and $x = \ln 2$ into $\frac{1}{2}\ln\left(e^{2x}-1\right)$ For example, $\frac{1}{2}\ln\left(e^{2\ln b}-1\right)$ and $\frac{1}{2}\ln\left(e^{2\ln 2}-1\right)$ is seen.	**M1 ft**	1.1b	
	Uses laws of logarithms to begin to simplify the expression. Either $\frac{1}{2}\ln\left(b^2-1\right)$ or $\frac{1}{2}\ln\left(2^2-1\right)$ is seen.	**M1 ft**	2.2a	
	Correctly states the two answers as $\frac{1}{2}\ln\left(b^2-1\right)$ and $\frac{1}{2}\ln 3$	**A1 ft**	1.1b	
	States that $\frac{1}{2}\ln\left(b^2-1\right)-\frac{1}{2}\ln 3=\ln 4$	**M1 ft**	2.2a	
	Makes an attempt to solve this equation. For example, $\ln\left(\frac{b^2-1}{3}\right)=2\ln 4$ is seen.	**M1 ft**	1.1b	
	Correctly states the final answer $b = 7$	**A1 ft**	1.1b	
				(8 marks)

Notes

Student does not need to state '+C' in an answer unless it is the final answer to an indefinite integral.

Award ft marks for a correct answer using an incorrect initial answer.

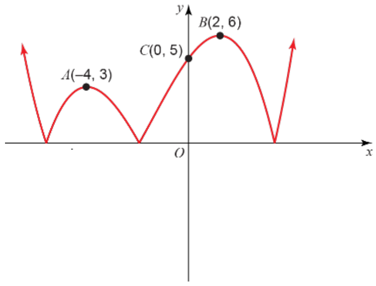
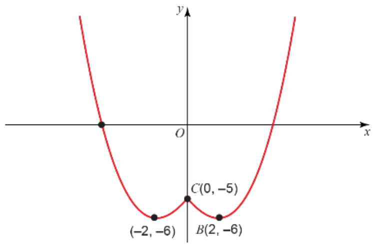
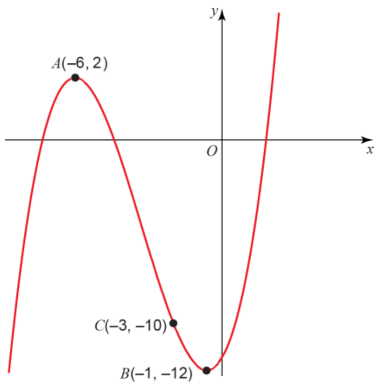
7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States that $x_2 = 4p - 9$	**A1**	1.1b	5th Work with sequences defined by simple recurrence relations.
	Attempts to substitute x_2 into x_3. $x_3 = p(4p - 9) - 9$ and simplifies to find $x_3 = 4p^2 - 9p - 9$	**A1**	1.1b	
		(2)		
(b)	States $4p^2 - 9p - 9 = 46$ or $4p^2 - 9p - 55 = 0$	**M1**	2.2a	5th Work with sequences defined by simple recurrence relations.
	Factorises to get $(4p + 11)(p - 5) = 0$	**M1**	1.1b	
	States $p = 5$. May also state that $p \neq -\frac{11}{4}$, but mark can be awarded without that being seen.	**A1**	1.1b	
		(3)		
(c)	$x_4 = 5(46) - 9 = 221$ $x_5 = 5(221) - 9 = 1096$	**A1 ft**	1.1b	5th Work with sequences defined by simple recurrence relations.
		(1)		
				(6 marks)

Notes

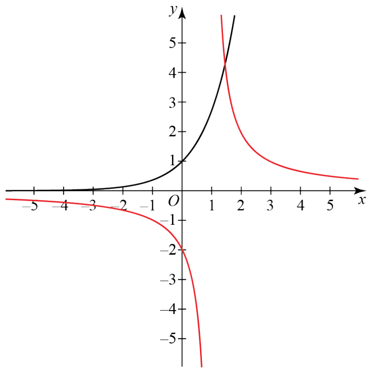
(c) Award mark for a correct answer using their value of *p* from part **b**.

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Correctly factorises the denominator of the left-hand fraction: $\frac{6}{(2x+5)(2x-1)}+\frac{3x+1}{2x-1}$	**M1**	2.2a	4th Add, subtract, multiply and divide algebraic fractions.
	Multiplies the right-hand fraction by $\frac{2x+5}{2x+5}$ For example: $\frac{6}{(2x+5)(2x-1)}+\frac{(3x+1)(2x+5)}{(2x-1)(2x+5)}$ is seen.	**M1**	1.1b	
	Makes an attempt to distribute the numerator of the right-hand fraction. For example: $\frac{6+6x^2+17x+5}{(2x+5)(2x-1)}$ is seen.	**M1**	1.1b	
	Fully simplified answer is seen. Accept either $\frac{6x^2+17x+11}{(2x+5)(2x-1)}$ or $\frac{(6x+11)(x+1)}{(2x+5)(2x-1)}$	**A1**	1.1b	
				(4 marks)

Notes

9	Scheme		Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	**Figure 2** 	Clear attempt to reflect the negative part of the original graph in the *x*-axis.	M1	2.2a	7th Sketch the graphs of the modulus function of unfamiliar non-linear functions.
		Labels all three points correctly.	A1	1.1b	
		Fully correct graph.	A1	1.1b	
			(3)		
(b)	**Figure 3** 	Clear attempt to reflect the positive *x* part of the original graph in the *y*-axis.	M1	2.2a	7th Sketch the graphs of the modulus function of unfamiliar non-linear functions.
		Labels all three points correctly.	A1	1.1b	
		Fully correct graph.	A1	1.1b	
			(3)		
(c)	**Figure 4** 	Clear attempt to move the graph to the left 3 spaces.	M1	2.2a	6th Combine two or more transformations, including modulus graphs.
		Clear attempt to stretch the graph vertically by a factor of 2.	M1	2.2a	
		Fully correct graph.	A1	1.1b	
			(3)		
					(9 marks)

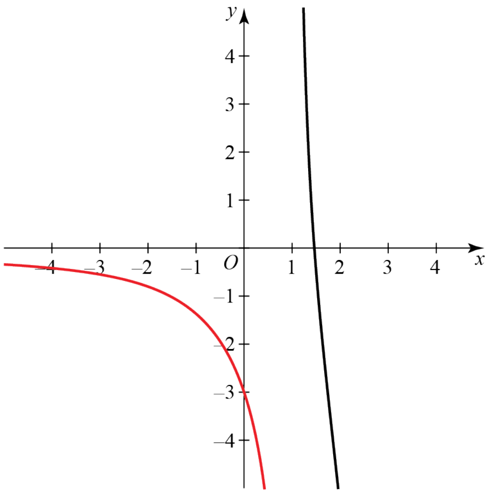
Notes

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	 Attempts to sketch both $y = \frac{2}{x-1}$ and $y = e^x$	M1	3.1a	5th Understand the concept of roots of equations.
	States that $y = \frac{2}{x-1}$ meets $y = e^x$ in just one place, therefore $\frac{2}{x-1} = e^x$ has just one root $\Rightarrow$ $g(x) = 0$ has just one root	A1	2.4	
		(2)		
(b)	Makes an attempt to rearrange the equation. For example, $\frac{2}{x-1} - e^x = 0 \Rightarrow xe^x - e^x = 2$	M1	1.1b	5th Understand the concept of roots of equations.
	Shows logical progression to state $x = 2e^{-x} + 1$ For example, $x = \frac{2+e^x}{e^x}$ is seen.	A1	1.1b	
		(2)		
(c)	Attempts to use iterative procedure to find subsequent values.	M1	1.1b	6th Solve equations approximately using the method of iteration.
	Correctly finds: $x_1 = 1.4463$ $x_2 = 1.4709$ $x_3 = 1.4594$ $x_4 = 1.4647$	A1	1.1b	
		(2)		

(d)	Correctly finds $g'(x) = -\dfrac{2}{(x-1)^2} - e^x$	**A1**	2.2a	6th Solve equations approximately using the Newton–Raphson method.
	Finds $g(1.5) = -0.4816...$ and $g'(1.5) = -12.4816...$	**M1**	1.1b	
	Attempts to find x_1 : $x_1 = x_0 - \dfrac{g(x_0)}{g'(x_0)} \Rightarrow x_1 = 1.5 - \dfrac{-0.4816...}{-12.4816...}$	**M1**	1.1b	
	Finds $x_1 = 1.461$	**A1**	1.1b	
		(4)		
				(10 marks)

Notes

(a) Uses their graphing calculator to sketch $g(x) = \dfrac{2}{x-1} - e^x$ (**M1**)



States that as g(x) only intersects the x-axis in one place, there is only one solution. (**A1**)

(c) Award M1 if finds at least one correct answer.

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes: $\frac{1+x}{\sqrt{1-2x}}$ as $(1+x)(1-2x)^{-\frac{1}{2}}$	M1	2.2a	6th Understand the binomial theorem for rational n.
	Uses the binomial expansion to write: $(1-2x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2}+\ldots$	M1	2.2a	
	Simplifies to obtain: $(1+x)(1-2x)^{-\frac{1}{2}}=(1+x)\left(1+x+\frac{3}{2}x^2\right)$	M1	1.1b	
	Writes the correct final answer: $1+2x+\frac{5}{2}x^2\ldots$	A1 ft	1.1b	
		(4)		
(b)	Either states $\|x\|<\frac{1}{2}$ or states $-\frac{1}{2}<x<\frac{1}{2}$	B1	3.2b	6th Understand the conditions for validity of the binomial theorem for rational n.
		(1)		
(c)	Makes an attempt to substitute $x=\frac{1}{100}$ into $\frac{1+x}{\sqrt{1-2x}}$ For example $\frac{1+\frac{1}{100}}{\sqrt{1-2\left(\frac{1}{100}\right)}}=\frac{\frac{101}{100}}{\sqrt{\frac{98}{100}}}$	M1	1.1b	6th Understand the binomial theorem for rational n.
	Continues to simplify the expression: $\frac{101}{100}\times\frac{\sqrt{100}}{\sqrt{98}}$ And states the correct final answer: $\frac{101\sqrt{2}}{140}$	A1	1.1b	
		(2)		

(d)	Substitutes $x=\frac{1}{100}$ into $1+2x+\frac{5}{2}x^2$ Obtains: $1+2\left(\frac{1}{100}\right)+\frac{5}{2}\left(\frac{1}{100}\right)^2\approx 1.02025$	**M1 ft**	2.2a	6th Understand the binomial theorem for rational n.
	States that $1.02025\approx\frac{101\sqrt{2}}{140}$	**M1 ft**	1.1b	
	Deduces that $\sqrt{2}\approx\frac{140\times 1.02025}{101}\approx 1.41421$	**A1 ft**	1.1b	
		(3)		
				(10 marks)

Notes

(a) Award 3 marks if a student has used an incorrect expansion but worked out all the other steps correctly.

(d) Award all three marks if a student provided an incorrect answer in part **a**, but accurately works out an approximation for root 2 consistent with this incorrect answer.

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly substitutes $x = 1.5$ into $y = \frac{1}{2}x^3\sqrt{4-x^2}$ and obtains 2.2323…	**A1**	1.1b	5th Understand and use the trapezium rule.
		(1)		
(b)	States or implies formula for the trapezium rule $A = \frac{h}{2}\left(y_0 + 2\left(y_1 + y_2 + y_3\right) + y_4\right)$	**M1**	2.2a	5th Understand and use the trapezium rule.
	Makes an attempt to substitute into the formula $A = \frac{0.5}{2}\left(0 + 2\left(0.12103 + 0.86603 + 2.23235\right) + 0\right)$	**M1**	1.1b	
	States correct final answer 1.610 (4 s.f.)	**A1**	1.1b	
		(3)		

(c)	Recognises the need to make a substitution. **Method 1** $u = 4 - x^2$ is seen.	Recognises the need to make a substitution. **Method 2** $u = \left(4 - x^2\right)^{\frac{1}{2}}$ is seen.	**M1**	2.2a	6th Integrate functions by substitution.
	Correctly states $\frac{du}{dx} = -2x$ and finds new limits $x = 0 \Rightarrow u = 4$ and $x = 2 \Rightarrow u = 0$	States $u^2 = 4 - x^2$ and finds $2u\frac{du}{dx} = -2x$ and finds new limits $x = 0 \Rightarrow u = 2$ and $x = 2 \Rightarrow u = 0$	**M1**	1.1b	
	Correctly transforms the integral $\int_{x=0}^{x=2}\left(\frac{1}{2}x^3\sqrt{4 - x^2}\right)dx$ into $-\frac{1}{4}\int_{u=4}^{u=0}\left(4u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$	Correctly transforms the integral $\int_{x=0}^{x=2}\left(\frac{1}{2}x^3\sqrt{4 - x^2}\right)dx$ into $\frac{1}{2}\int_{u=2}^{u=0}\left(-4u^2 + u^4\right)du$	**M1**	2.2a	
	Correctly finds the integral $-\frac{1}{4}\left[\frac{8}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_4^0$	Correctly finds the integral $\frac{1}{2}\left[-\frac{4}{3}u^3 + \frac{1}{5}u^5\right]_2^0$	**M1**	1.1b	
	Makes an attempt to substitute the limits $-\frac{1}{4}\left[\left(\frac{8}{3}(0)^{\frac{3}{2}} - \frac{2}{5}(0)^{\frac{5}{2}}\right) - \left(\frac{8}{3}(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}}\right)\right]$	Makes an attempt to substitute the limits $\frac{1}{2}\left[\left(-\frac{4}{3}(0)^3 + \frac{1}{5}(0)^5\right) - \left(-\frac{4}{3}(2)^3 + \frac{1}{5}(2)^5\right)\right]$	**M1**	1.1b	
	Correctly finds answer $\frac{32}{15}$	Correctly finds answer $\frac{32}{15}$	**A1**	1.1b	
			(6)		
(d)	Using more strips would improve the accuracy of the answer.		**B1**	3.5c	5th Understand and use the trapezium rule.
			(1)		
					(11 marks)

Notes

(c) Either method is acceptable.

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States: $R\cos(\theta+\alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ Or: $5\cos\theta - 8\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	**M1**	1.1b	6th Understand how to use identities to rewrite a cos*x* + b sin*x*.
	Deduces that: $5 = R\cos\alpha \qquad 8 = R\sin\alpha$	**M1**	1.1b	
	States that $R = \sqrt{89}$ Use of $\sin^2\theta + \cos^2\theta = 1$ might be seen, but is not necessary to award the mark.	**A1**	1.1b	
	Finds that $\alpha = 1.0122$ $\tan\alpha = \frac{8}{5}$ might be seen, but is not necessary to award the mark.	**A1**	1.1b	
		(4)		
(b)	Uses the maths from part **a** to deduce that $T_{max} = 1100 + \sqrt{89} = 1109.43°C$	**A1**	3.4	7th Solve problems involving a cos*x* + b sin*x*.
	Recognises that the maximum temperature occurs when $\cos\left(\frac{x}{3} + 1.0122\right) = 1$	**M1**	3.4	
	Solves this equation to find $\frac{x}{3} = 2\pi - 1.0122$	**M1**	1.1b	
	Finds x = 15.81 hours	**A1**	1.1b	
		(4)		

(c)	Deduces that $1097 = 1100 + \sqrt{89}\cos\left(\frac{x}{3} + 1.0122\right)$	**M1**	3.4	8th Use trigonometric functions and identities to solve problems in a range of unfamiliar contexts.
	Begins to solve the equation. For example, $\cos\left(\frac{x}{3} + 1.0122\right) = -\frac{3}{\sqrt{89}}$ is seen.	**M1**	1.1b	
	States that $\frac{x}{3} + 1.0122 = 1.8944,\ 2\pi - 1.8944,\ 2\pi + 1.8944$ Further values may be seen, but are not necessary in order to award the mark.	**M1**	1.1b	
	Finds that x = 2.65 hours, 10.13 hours, 21.50 hours	**A1**	1.1b	
		(4)		
				(12 marks)

Notes