Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel GCE
A Level Mathematics (9MA0)
Paper 01

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General Comments

This was the second set of papers taken by the full cohort of candidates since improvements were made to the accessibility of questions. The aim was to improve the exam experience of candidates, and feedback regarding their experience was largely positive. There were 15 questions on the paper, which was one less than last year, however the greatest number of marks on any part of any question was 6.

With the improvements designed to focus on helping candidates to get off to a good start, it was pleasing that questions 2, 3 and 5 were appropriately placed in the first five. In fact, typically candidates scored full marks on Questions 1, 2, 4, 5, 6, 8, 9, 11 and 12 and usually candidates scored 2 out of 3 on Question 3. Question 12 was one of the most successfully answered questions on the paper and Question 8 on radians was a high scoring question for many. Question 1, however, seemed to prove more challenging that it was intended to be, often due to the approach taken by some candidates.

The questions had plenty of parts which were not reliant on each other, or later parts using an established result from a "show that" part of a question e.g. Question 8 part (c) could be answered with the aid of the established results in part (a) and part (b). Question 15 was the final question on the paper, and this was the most challenging (and longest) which provided suitable challenge to the most able candidates. This question was demanding, but it was pleasing to see many candidates able to make progress on part (a) and many of all abilities attempted parts (c) and (d) to a pleasing level of success. It was rare for candidates not to score at least one mark on Question 15. Time did not appear to be an issue for candidates to demonstrate what they could do over the whole paper.

The presentation of solutions was generally good, although a number of candidates still do not show all the steps in their working or indicate what they are trying to do. This is particularly an issue when their answer is incorrect but there is no method visible to determine whether the candidate had made a slip in their method or not. Alternatively, it can be an issue when there are many attempts, with a lot of incorrect approaches and no indication as to what should be marked.

Another increasing trend is the use of calculators to solve quadratic equations (e.g. Question 2) and simultaneous equations (e.g. Question 5). This is an acceptable approach and is encouraged. Candidates should be reminded, however, that they need to show sufficient steps in their method, if they are to gain full credit for their solutions as, for example, solving a cubic or quartic would likely require some factorising first to achieve a quadratic, which then the use of a calculator would be appropriate for the quadratic. The advice at the top of some questions indicates where it is even more important to show the full method to the solution, and the extent to which calculators can be used. It was noticed on several occasions that some candidates write down an incorrect quadratic formula, demonstrating a lack of understanding of a key process, but are then able to produce the correct solution via use of a calculator.

Comments on individual questions

Question 1

This was an accessible question for virtually all candidates. The fractional index had already been given to avoid any misconceptions. However, for the first question on the exam paper, this was frequently done poorly compared to previous years. Candidates may have anticipated a tougher start to the paper and wanted to demonstrate one of the more advanced integration techniques.

Candidates who spotted that you needed to expand the brackets usually went on to score full marks. The most common reason for losing the final answer mark was forgetting the constant of integration.

Many candidates failed to notice the simpler method in the main mark scheme and instead attempted to use integration by parts, with either part taken as *u*. If a candidate chose to set $u = 2x - 5$, they did not always expand the brackets in their integration formula to achieve a form which was ready to integrate where the indices had been combined correctly. Candidates who set $u = x^{\frac{1}{2}}$ were more successful, as there was less working to do to allow access to the marks. Candidates who chose to do integration by parts did not always combine the terms in their answer, losing the final mark. Many candidates would divide both their *u* and v' by 3, leading to final answers that were $\frac{1}{3}$ as large as they should have been.

Integration by substitution was occasionally seen, but rarely successfully completed. Most attempts involved setting $u = 2x - 5$, and this resulted in a tougher integral that required further work; candidates rarely progressed to a point which would have been worthy of a mark.

Across the various methods, a common error seen was initially to factorise out the $\frac{1}{3}$, work through the solution without it, but then to forget to reintroduce it. This error meant that neither of the two accuracy marks were available.

Question 2

This question was accessible and generally well answered, with a large number of candidates gaining full marks with concise solutions. Some, however, did not pay heed to the instruction to show all stages of their working.

The vast majority of candidates correctly identified that part (a) related to an application of the factor theorem, substituted in a and were able to reach the required result. However, many lost marks for not including "$= 0$" until the final line. In a small number of cases, "$= 0$" was omitted completely and, in these cases, candidates were awarded no marks. Amongst the unsuccessful candidates, the majority had attempted algebraic long division using $(x - a)$ or carried out an attempt at factorisation via inspection, but had often abandoned their attempts, presumably unsure of how this would lead to the required result.

Very few successfully attempted long division and set the remainder equal to 0. Factorisation using inspection then solving two simultaneous equations by equating like terms was rarely seen as a method.

In part (b)(i), most candidates had no problems solving the cubic, getting the values 0, $\frac{3}{4}$ and -2 for a. However, many did not take note of the fact that a was positive, and that the question asked for the **value** of a, so lost the mark for not rejecting $a = 0$ and $a = -2$. There was confusion as to the reasons for rejecting values. Some thought that a had to be an integer so rejected $\frac{3}{4}$. Some chose -2 as their solution giving spurious reasons such as "a is a real number."

Of those who did not identify $a = \frac{3}{4}$ as the only valid answer, many recovered by opting to use only $a = \frac{3}{4}$ in part (b)(ii).

Part (b)(ii) was well done by those who showed every step of their working, but some failed to reject the values of a that were not needed.

Once the cubic was obtained by substituting $a = \frac{3}{4}$ and setting $\mathrm{f}(x) = 3$, those who solved the cubic without factorising to $x(4x^2 + 5x - 10)$ lost the final A mark. Candidates were asked to show all stages of their working and not proceeding as far as the quadratic factor was penalised. Many gave the exact solutions followed by rounded decimal solutions, or even just the decimal solutions, so were not paying heed to the question asking for exact solutions.

There were some who were unable to solve the cubic or the quadratic, having taken out a factor x outside the brackets. These were mainly cases where they tried to complete the square or their value for *a* meant that they could make no further progress. Some did not collect terms on one side with 0 on the other side, and having taken out a factor of *x*, solved their quadratic ignoring the constant on the other side.

The solution $x = 0$ was omitted by many candidates due to division by *x* instead of using factorisation. Others rejected this particular solution, if it was found, possibly confusing the restriction that *a* was positive rather than *x*. Those who did reach the correct quadratic factor were usually successful in finding the two exact roots for the A1 mark.

A common incorrect approach was to find $\mathrm{f}(3)$ rather than setting $\mathrm{f}(x) = 3$. This approach gained no credit.

Question 3

This was another short question on two-dimensional vectors with many candidates able to score 2 out of the 3 marks.

Part (a) was generally answered accurately by all candidates. Sometimes the mark was lost because the candidate omitted the square root and wrote $|\overrightarrow{OA}| = 25+9+4$. Sometimes the notation was not made clear or **i**, **j** and **k** appeared.

In part (b), the most common approach was to proceed to $a^2 > 18$ or $a^2 = 18$, and the majority of candidates were able to achieve this. However, a significant number of candidates did not conclude that $a = 5$, with some claiming that e.g. $\sqrt{18}$ is an integer or missing the requirement that "*a*" had to be an integer. Some other incorrect answers included $a = \sqrt{19}$ following $a > \sqrt{18}$ and the answer $a \geq 5$ was not condoned, which lots the final mark for a number of candidates. A common processing error was the simplification of $\sqrt{20 + a^2}$ to $\sqrt{20} + a$. Very few candidates used the substitution method and some of those did not reach any conclusion in the end.

Question 4

This question testing the small angle approximation for cosine and finding the equation of a straight line was accessible to many candidates with a large number scoring full marks.

In part (a), some candidates formed and solved the equation but were not able to use the information that *x* was small to choose the correct solution. There was a very small minority of candidates who did not quote the correct small angle approximation, despite this being in the formula book. A slightly larger minority misunderstood that *x* being small meant that x was 0 and therefore did not include the $2x$ term in their solution attempt. Candidates should also pay closer attention to the question as several did not round to the required decimal places. It was also notable that some candidates at this level found it difficult to expand $\frac{1}{2}\left(1-\frac{x^2}{2}\right)$ when multiplying the fractions together.

Part (b) was mostly well answered with the majority scoring both marks, with most candidates gaining these marks efficiently without unnecessary calculations. Most candidates found the gradient of the tangent by substituting $x=0$ into the given derivative of $\mathrm{f}(x)$, however a small number of candidates took a longer approach by attempting to use the small angle approximation. Some candidates tried to substitute their answer from part (a) into the given derivative or found the gradient of the normal having originally found the correct gradient for tangent; these approaches lost both marks. A small number of candidates failed to realise the significance of the point (0,3) and therefore retained an algebraic gradient which they tried to substitute into $y=mx+c$. A number of responses were seen not giving their answer as an equation, just an expression $\frac{1}{2}x+3$, or forgetting to substitute their value of c found, giving their equation as $y=\frac{1}{2}x+c$; having been given the y intercept in the question, it was a requirement that for any marks to be scored that $c=3$.

Question 5

This was a lovely twist on a usually very standard question using the trapezium rule that was overall very well-answered. It was very rare to see a blank response to this question.

Candidates usually scored full marks in part (a). Where marks were lost the most common error was working out "h", the width of each strip, as $\frac{(4-3)}{6}=\frac{1}{6}$ where 6 was the number of ordinates given, rather than $\frac{(4-3)}{5}=\frac{1}{5}$, where 5 was the number of strips required. Very few candidates failed to give "h" a value. Most applied the standard trapezium rule correctly, although a minority failed to use the area of 17.59 to form an equation. Only a handful of solutions used the method of adding the areas of individual trapezia, which is much more time consuming and does not demonstrate as effectively an understanding the trapezium rule, which is provided in the formula booklet. The brackets were dealt with correctly in most cases although quite a number of candidates expanded the brackets before collecting terms, creating a more difficult equation to cope with. Others multiplied both sides by 10 which made the manipulation much easier. Several candidates had invisible brackets and recovered appropriately but lost the A mark as the answer was given. Although many values needed to be copied from the given table of *x* and *y* values, there were very few copying slips seen.

Part (b) was dealt with very well overall, with a variety of different methods used to solve the equations simultaneously, usually via the use of a calculator. Some arithmetic errors were seen in forming the second equation, $a+b=28$, which resulted in the wrong values for *a* and *b*. Again, despite requiring the copying of many values from the given table of *x* and *y* values to form the required equation, there were few transcription errors. It is also worth noting that nearly all candidates used the given equation in (a) to correctly solve for values of *a* and *b*. Although it was made possible for them to gain the method mark for using "their" equation, this was very infrequently utilised, or required. Furthermore, it was pleasing to see candidates who made no attempt at (a) still proceeded to use the given answer to attempt and generally gain full marks in (b).

Question 6

This was again another accessible question with a total of 6 marks split between 3 different sub-parts. However, this topic continues to be one which candidates struggle with and in all parts of this question. It was not uncommon to see candidates attempting to use rules of indices rather than logarithms, starting off with $2^a = x$, $2^b = x+8$ and attempting (usually incorrectly) to substitute these into the expressions given. Where this was successful candidates generally failed to remove $\log_2 2$ resulting in them losing one mark per part.

Part (a) was a B mark for achieving the correct answer. Several candidates got as far as $0.5\log_2 x$ without substituting *a* into this and so gained no credit. Others could not apply the index log law to the square root of *x* correctly and so ended up with the answer square root of *a* often written as $a^{\frac{1}{2}}$

Part (b) surprisingly seemed to have more success. It was generally answered well with most candidates realising that factorising the argument of the logarithm was needed before applying the addition law of logarithms. There were several candidates who, having factorised correctly, applied the addition law incorrectly, resulting in an answer of $a \times b$ and so lost both marks. Several candidates just wrote down the correct answer and gained both marks. If the correct answer followed the product of the relevant logs, $\log(x) \times \log(x+8)$ rather than the sum, then the answer was allowed to imply the correct method, but the final mark was withheld for this incorrect working seen. The main misconception was the sight of $\log x^2 + \log 8x$ which was awarded no marks, although writing $\log x + \log + \log x + \log 8$ (possibly seen as $2a+3$) gained the method mark as evidence of correct addition law used at some point. Some candidates attempted to work backwards from the given log expressions for *a* and *b* with very limited success.

Part (c) was the most challenging part of this question and clearly identified the stronger mathematicians of the cohort. Many responses to this part were blank, whilst the more able candidates provided very succinct solutions. For those who attempted with less success there were some rather dubious attempts at writing $8+\frac{64}{x}$ as a single fraction. Of those who managed this correctly and progressed to applying the laws of logarithms, quite a few left in the $\log_2 8$ rather than simplifying to its value of 3, which meant that only the first mark was available: marks had already been awarded for the power law and addition law in previous parts of the question and here we required an understanding of reaching an answer in its simplest form. This further demonstrated the lack of logarithmic fluency for a number of candidates. Others factorised 8 from $8+\frac{64}{x}$ but then struggled to apply the laws appropriately as they had not completed the process of writing the argument as a single fraction.

Misconceptions were seen such as expressing $\log\left(8+\frac{64}{x}\right)$ as $\log 8+\log\left(\frac{64}{x}\right)$ and sign errors often occurred.

Question 7

Overall, this was an accessible question on functions which allowed candidates to attempt all sections of it even when they found parts (a) or (b) particularly challenging. Part (c) had the lowest number of errors out of the entire question and was one of the most successfully answered parts over the entire paper.

In part (a), a large majority of candidates were able to score the B mark available, with nearly all attempting to give the range. Although the mark scheme was generously accepting notations such as $\mathrm{f}>3$ or $\text{range}>3$, some candidates lost the mark for unacceptable labels such as $x>3$ or $\mathrm{f}(x)\geq 3$.

In general, candidates correctly rearranged the formula and interchanged *x* and *y* successfully in part (b). A small number of candidates made some errors in applying the correct order of operations and this resulted in an incorrect expression for the inverse function. A common error was seen with candidates who incorrectly manipulated $y=3+\sqrt{x-2}$ to obtain $y^2=9+x-2$. The candidates who lost the A mark were often due to labelling the inverse as $y=$ instead of e.g. $\mathrm{f}^{-1}(x)=\ldots$. A small minority of candidates misread $\mathrm{f}^{-1}(x)$ as $\mathrm{f}'(x)$ and so they differentiated the function instead of finding the inverse of it. Candidates should be advised to read the question and the labels carefully to check which skill is required. A common error costing candidates the B mark was failing to record the domain of the inverse function. This mark was rarely scored across the entire cohort, despite 3 marks possibly indicating that more was required by candidates than just rearranging to find $\mathrm{f}^{-1}(x)$. Candidates who paid careful attention to how f and g were defined in the question would be able to attempt similarly for the inverse function.

Part (c) was well attempted by many candidates even when they could not attempt parts (a) or (b). Most candidates scored both marks available in this part. Finding $\mathrm{f}(6)$ first and then substituting this result into g was more common than substituting $x = 6$ into the composite function gf(x). This second approach caused some errors due to incorrect algebraic manipulation leading to various incorrect answers for the composite function.

A large number of candidates scored the first mark in part (d) as they correctly formed the equation $(a-3)(3+a)=15$ and proceeded to a quadratic in a. Several candidates failed to spot that $\sqrt{a^2} = a$, or that $a^2+2-2=a^2$. Some candidates that found $a = \pm 2\sqrt{6}$ did not reject the negative solution losing the last A mark. Some candidates possibly rejected the negative solution by seeing that the domain of f had to be greater than 2 and fortuitously scored the final mark. However, others may have considered that the $\left(-2\sqrt{6}\right)^2+2$ also satisfied this domain resulting in $\mathrm{f}\left(\left(-2\sqrt{6}\right)^2+2\right) > 0$, but that $\mathrm{g}\left(-2\sqrt{6}\right) < 0$ so this negative solution had to be rejected. Other errors included writing decimal answers only. On several occasions $a^2-9=15$ incorrectly became $a^2=6$.

A number of candidates were not able to obtain the quadratic equation needed (hence, did not score any marks) because they got confused by $3+\sqrt{a^2}$ and not realising that this was actually $3+a$. Others tried to manipulate the equation leading to a quartic equation and usually solved via a calculator (and successfully in a few cases), however those who had proceeded along this route typically were expected to apply algebraic division or the factor theorem to find the quadratic factor, having introduced other solutions from squaring and the complexity of their expressions meant it was rare for the method mark to be scored.

Question 8

Generally most candidates found this question accessible, working confidently in radians throughout, although inadvisable conversions to degrees were not uncommon with many candidates unaccountably preferring the unwieldy $\frac{\theta}{360}\times \pi r^2$ to the elegant $\frac{1}{2}r^2\theta$.

In part (a) a rigorous solution was expected; a very large number of candidates lost the second mark by failing to explicitly show the process of dividing the arc length by the angle (perhaps the mark most commonly lost on this question). Candidates need to appreciate that every step of working needs to be clearly demonstrated in order to gain full marks for "show that" questions and that their final result needs to be what the question asked them to show in the first place. As the question was worth 2 marks, candidates should not expect to just state $2.3r = 27.6$ and for this to be sufficient. Those who chose to convert 2.3 radians to degrees not only lost time but also invariably lost the accuracy mark. Others did show the division and did not label their answer so just an expression was seen.

Part (b) was typically answered well, with many candidates recognising the straight line giving a total of π radians and allowing them to solve the problem. Where candidates lost marks, this was often due to a lack of brackets, e.g. $\pi - 2.3 \div 2 = 0.421$, or an incorrect joined statement such as $\pi - 2.3 = \frac{0.842...}{2} = 0.421$. A small minority worked in degrees which needed much more work to secure both marks.

There was a wide range of marks for part (c), although many candidates produced highly competent solutions and gained full marks. Most candidates seemed familiar with the area of a sector formula and the vast majority scored both marks for this. Candidates also seemed fully aware of the area of a triangle formula, although they did not always use a correct method to work out the length 15.7. Some candidates seemed not to realise that they needed to use the information provided, i.e. the length of the front of the stage, in order to obtain all the lengths needed for their area calculations. There were also some more complex and longer methods to find the area of a triangle which were not generally successful, but the most common error was using 12 as opposed to 15.7. Some candidates spent time working out the length of *AB* but then realised that this was not necessary.

Question 9

This was a more demanding than expected question on geometric sequences with a number of candidates getting confused with the value of *k* and the subsequent values of *a* and *r* required for the question. There was still access to later parts as the quadratic in *k* was given such that part (b) and part (c) could be attempted without part (a).

Most candidates were able to score full marks in part (a) by correctly giving an equation in *k*, almost always $\frac{12-3k}{3k+4}=\frac{k+16}{12-3k}$ and proceeding to the given quadratic with no errors seen. A noticeable number of candidates failed to gain the final mark as they gave a -40 term in their quadratic. Given that this was a printed answer, it was disappointing that candidates did not identify their error and correct it. Some candidates wrote down a correct expression for *r* but failed to create an equation, whilst a few candidates formed an equation for the sum of three terms, but invariably failed to expand and simplify correctly. There were several valid but more complicated efforts such as expressing the third term as the first term multiplied by the square of the ratio. This was more likely to result in errors and there were a significant number who did not know where to start. The common mistake where they could not set up an equation in *r* was to look for a common difference as if it was an arithmetic sequence.

In part (b)(i) candidates were told that the given sequence converges. Candidates were required to find the value of *k* and to give a reason for their answer. Solving from calculator, or without working, was allowed and the majority of candidates were able to correctly solve the quadratic reaching the terms $k=20$ and $k=\frac{2}{3}$. However, many found giving a reason particularly challenging. Of those who did score both marks, listing values in the sequence was a common explanation, as was finding the value of *r* and using the fact that $|r|<1$. When attempting to reason via a calculation of the common ratio, there were often errors. A common response was to assume *k* was equal to *r* and then selecting $k=\frac{2}{3}$ as the final value. Some did not select a final value, keeping 20 and $\frac{2}{3}$. Some candidates who had chosen $k=20$ gave comments such as "because it is an integer/whole number" or "because it is a constant". A significant number chose $k=20$ giving a reason "because the sequence is convergent" which they had already been told and did not demonstrate an understanding of what the term means.

Part (b)(ii) proved to be very challenging for many candidates. It was quite often the case that candidates tried to use $r=\frac{2}{3}$ and just found the value $a=6$ when $k=\frac{2}{3}$. It was very frequent that those who had correctly identified $k=20$ in part (b)(i) did not show their working to find *a* and *r* and lost all marks in (b)(ii) as the sign error in substituting in $-\frac{3}{4}$ meant that it was unclear whether the correct value for *r* had been found, or if the correct sum to infinity formula was being used. Some candidates having found $\frac{256}{7}$ correctly then proceeded to give a rounded decimal as their answer which was condoned. Those who found $a = 6$ and $r = \frac{5}{3}$ were still able to score the B mark for a correct pair of values for *a* and *r*, even though they could not score the mark for using the sum to infinity formula with an invalid value for *r*.

Question 10

Candidates were typically able to make good progress in part (a) with the method of completing the square being familiar to many when dealing with the equation of the circle. Part (b) was focused on using the discriminant and solving a quadratic inequality, however, the manipulation proved to be a lot for some, resulting in a wide range of marks. Again, the access was good on this question such that candidates who could not do part (a) were not restricted from attempting part (b), although those with errors from part (a) often found that this caused issues in part (b) and resulted in losing many of the marks.

In general candidates obtained $(-3k,\ k)$ as the centre of the circle in part (a). Common errors included $(3k,\ -k)$ and $(-3kx,\ ky)$. If candidates were successful at completing the square, they could find the correct radius of the circle. When attempting to complete the square a common error was to subtract $3k^2$ instead of $(3k)^2$. Occasionally, candidates would find $r^2 = 10k^2 - 7$ but did not proceed to take the square root. Those who collected terms on the right-hand side of the equation in order to form the square of the radius made fewer sign errors than those who did not. Others tried to manipulate the equation mentally, so writing down a wrong radius with no evidence of its development (and thus no method mark). One error was to write $\sqrt{10k^2 - 7} = \sqrt{10}k - \sqrt{7}$. Sometimes there were sign slips such as writing the -7 as +7 when rearranging the equation.

In part (b), whilst a number of candidates had little idea how to proceed, the vast majority did realise what was required and attempted to produce an equation in *x* by substitution. Many lost their way in the algebraic complexity, or did not know what to do with the equation (sometimes trying to solve it to find a value for *k*). A significant number of candidates did produce a three-term quadratic for *x*, with coefficients in terms of *k*.

Centres should note that the scheme required the quadratic in *x* to be of the form specified, and that it be set out in a way which identified the coefficients (possibly implied by later use in the discriminant). So, for example, "$(2k-4)x$" was required rather than "$2kx-4x$" as this provided evidence of recognising that the coefficients were the key to solving this problem. Quite a few candidates had a constant term which was not of the required form which meant that no marks could be scored.

In general candidates who found critical values for their discriminant by solving $b^2-4ac=0$ were more successful than those who expressed it as an inequality. Often the inequality was simply carried forward to a statement about *k*; those who identified the two critical values by solving an equation perhaps thought more carefully about where the region was. The "need" to write a single inequality was seemingly pressing for many, so $7+\sqrt{85}<k<7-\sqrt{85}$ was seen. A few candidates did not use exact values of *k* and lost the final accuracy mark. Some candidates used 'and' with their two regions, however, most recognised that this needed to be 'or', either stating this explicitly or just using a comma between their two answers. A number of capable candidates brought some interesting approaches to bear on the problem, adopting a variety of techniques from Further Maths specifications.

Question 11

This question proved to be quite challenging for a number of candidates, however those who were able to manipulate logarithms using the rules correctly, along with calculating the gradient, were often very successful. It is worth noting that there were a number of candidates who did not pay attention to the labelling on the *y*-axis and therefore were unable to score any marks on this question. As with many modelling questions, candidates currently lack proficiency at relating the mathematics to the context of the question.

In part (a), most candidates were able to find the initial value correctly, although several lost the accuracy mark by forgetting to include units. The location of the unit was condoned e.g. 1000£ was seen on many occasions. A common reason for candidates not achieving any marks was to assume $V=3$ or $V=\log_{10}3$, instead of $\log_{10}V=3$. A few candidates used natural logs instead of $\log_{10}$, resulting in $V=e^3$ instead of $V=10^3$. Some candidates just stated the initial value was 3, not fully appreciating what information the linear graph was representing.

Part (b) was discriminating, requiring some understanding of the relationship between $V=ab^t$ and the linear graph. Those candidates who were most successful with this part of the question, made the link between part (a) and the value of *a* without having to do additional calculations. Many candidates were able to find the gradient of the graph correctly and pick up the first method mark. However, finding a value for *b* was difficult for many candidates. Those candidates who did not make the link with part (a) often had to recalculate *a* or attempted to use simultaneous equations, often with little success. There were a lot of very convoluted incorrect solutions in this question which showed a poor grasp of logarithms and the associated rules; $t\log ab$ was commonly seen; others failed to correctly relate $\log V = \log a + t\log b$ to $y = mx + c$ despite previous questions on the topic, confusing which were the variables and which were the constants. There were several cases of candidates mixing up the *x*, *y* coordinates in their substitution. It was encouraging to see that most candidates tried to give the full equation if they found values for *a* and *b*, but there was still a good number that did not and thus, lost the final mark. A few candidates were unable to score the final mark for leaving their answer as $\log_{10} V = 3 - 0.021t$. Candidates were able to gain the final mark of part (b), if the complete equation was seen in part (c), although this was rare to see.

In part (c), candidates who used $t = 24$ generally scored the method mark even if their model was incorrect but of the correct form. The most common form used was a continuation of their answer to part (b), usually $V = ab^t$ where *a* was positive. The majority of candidates were able to make a valid comparison accompanied by a sufficient explanation to earn the final accuracy mark. Some calculated the percentage error to justify their thinking which was acceptable. A minority thought the small difference was too great for the model to be reliable.

A few candidates used the method of substituting $t = 24$ into an equation of the form $\log_{10} V = p + qt$ to find a value for $\log_{10} V$. The discerning candidates either compared their $\log_{10} V$ with $\log_{10} 320$, or they proceeded to find *V* and compared this with 320. This method was slightly more complicated, and the success rate of this method was mixed. The units in this question caught a considerable number of candidates out, when checking the suitability of the model and substituting in 2 (years) rather than 24 (months), highlighting the importance of reading the question carefully. This led to a conclusion that the model was unreliable due to the vastly different amount that was yielded. Candidates did not consider the fact that this could mean that there was an error in their own calculation.

Question 12

This differentiation from first principles question was similar to the one that appeared in 2018 and candidates demonstrated a much greater confidence with the way in which they should approach this topic. The limits were provided in the question and the accessibility was evident as it was very rare to see a completely blank script, which is not always true for a differentiating from first principles question. Most candidates were able to obtain the first 3 marks of this question, with very few sign errors seen when using the addition formula. Most correctly started with the fraction for the gradient of the chord $\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$ as given in the formula booklet. Occasionally all a candidate would attempt was the addition formula in terms of *A* and *B*, but more often than not it was in the correct format and correct.

Scoring the last two marks was not as common, but still achieved more often than not. There were a variety of layouts used for the candidates to justify the replacement of terms with 0 and 1. If candidates lost marks here, it was often because they failed to separate their terms in *h* correctly. Some candidates introduced extra *h*'s, forming expressions like $\cos xh \times \sin hh$. At other times, candidates simply tried to cancel terms incorrectly or stopped. Notation was generally good with limiting arguments correct. The main mistakes that resulted in lost marks were either failing to isolate the required expressions and jumping to the answer or trying to replace each of the 3 terms separately, failing to spot the common factor of sin *x* in two of them. There were also several candidates incorrectly splitting up the product of each term resulting in an extra *h* in the denominators. Incorrect notation resulted in the loss of the final mark, in particular the limit notation was often seen right through to the end, in the final answer, even after the limits had been used.

A small number of candidates proceeded by using the small angle formula which was generally done well; where errors were seen it was generally not cancelling correctly. One surprising thing to note was the number of candidates who felt it acceptable to not clearly identify the numerator of the fraction by using very small fraction lines, resulting in their solution looking like only one term was being divided by *h*. Many candidates failed to write $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$ at the end, despite this being what they had been asked to show. If $\mathrm{f}'(x)$ had not been acceptable as an answer, then many candidates would have lost the final A mark. Only a very small handful of candidates used small angle approximations.

Question 13

A significant number of candidates found this question very challenging, with a fair proportion of blank answers, and another significant number working with various substitutions in (a) that were quickly abandoned. Those who were able to work with both the quadratic and trigonometric models confidently were able to score highly and demonstrated a good understanding of how mathematics can be used to model real life contextual problems.

In part (a), there were two common approaches that led to successfully scoring full marks. One was applying the knowledge that $b(t-20)^2 = 0$ when $H = 60$ which therefore meant $a = 60$. This then enabled the candidate to substitute $H = 2,\ t = 0$ and find a complete equation. It was clear when candidates were able to apply this knowledge, and knowing this gave them a huge advantage in this question. Noticing that *b* was positive was pivotal when taking this approach. The other approach that led to successful answers involved multiplying out the brackets, and differentiating with respect to*t*. This enabled the candidate to find that $t = 20$ when $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$, thus overcoming a shortfall in knowledge over when geometrically the maximum occurs in a negative quadratic. This second approach led to success less often, and candidates often ended up with a page of various quadratic expressions and equations that looked costly in terms of time and did not yield marks.

An overwhelming majority of candidates who scored full marks in (a) successfully found H to be $2\,\text{m}$ in part (b). Some candidates still managed to find this value with either an incorrect answer in (a) or no answer at all, by perhaps noticing that due to the symmetry of the quadratic, the height after 40 seconds was the same as at the start, which they are told was
$2\,\text{m}$ in the question.

In part (c), a significant number of candidates had no real idea how to start this. Many attempts involved fruitlessly expanding $\cos(9t+\alpha)$ but most candidates who successfully found α and β did not do this. The candidates who realised that when $H=60$, $\cos(9t+\alpha)=1$ which meant $\beta=31$ had the most success. Candidates who did this typically then substituted $t=20$ into $\cos(9t+\alpha)=1$ to find a value of α. Candidates who found one value typically found the other. There were a number of candidates who found the two values that did not 'find a complete equation' as the question required. However, there were a pleasing number of fully correct solutions, and almost all candidates who found α and β continued to give the full model.

In part (d), the most popular correct answers involved an identification of the cyclic nature of the trigonometric function, or realising the first model would become very negative. A significant number of incorrect answers involved candidates saying the original model would keep increasing over time, which showed a lack of understanding of the negative quadratic model. Others made reference to the alternative model being continuous which was too vague. Some candidates managed to score this mark without scoring any other mark on this question which was, again, pleasing to see candidates attempting later parts of questions even if they have made little progress on earlier parts.

Question 14

Candidates were required to prove the statement: $(n+1)^3-n^3$ is odd for all natural numbers, n. This was similar to some of the proof questions that have been on previous papers, so it was pleasing to see the majority of candidates were able to make more progress on this type of question this time.

A large number of candidates tried to start correctly by substituting $n=2k$ and $n=2k+1$ (or $n=2k-1$) for even and odd numbers; this was the most likely method to achieve full marks. Candidates would often produced the correct expressions which were written in the form 2(......)+1 and then conclude appropriately. The final mark was often lost in many of the complete attempts as at least one element of the proof was either missing or had errors. Some candidates only proved one case (odd or even only) and some candidates did not include an overall conclusion, which was necessary for the final mark.

Most common errors seen in expansions were from $(2k)^3 = 2k^3$ and this often led to candidates achieving a cubic expression rather than a quadratic. Some used $2n$ and $2n+1$ which was penalised by losing the final mark. Errors in expanding and simplifying the algebra were common: expanding triple brackets was the most common approach, although the binomial expansion was also seen, on occasion.

The other common approach was from an attempt of algebra with logic. It was common to see candidates expand and simplify the expression to achieve a three-term quadratic but only a few went on to factorise their quadratic as required, and even fewer explained the logic for why $n(n+1)$ was even correctly.

A significant number of candidates tried to expand and simplify the given expression and usually achieved $3n^2+3n+1$ scoring the first 2 marks, but very few got beyond this without making any logical arguments at all or did not factorise the expression. e.g. $3n(n+1)+1$ which gained 3 marks. Success was also seen when the given expression was expanded and simplified to $3n^2+3n+1$ and then candidates substituted $n = 2k$ and $n = 2k+1$ to much the same result as the main method seen in the mark scheme.

A number of candidates tried proof by contradiction, but these almost all fell into being marked either from the main scheme or by an expanding and factorising attempt. Whilst the question did require candidates to use algebra, credit was given for those candidates who did just try to apply logic with a maximum score of 2 marks. It was pleasing, however, to see that most candidates had appreciated the demand of the question and attempted some manipulation. A few candidates tried the Further Maths Method "proof by induction", but most could not complete the proof correctly.

Question 15

This question brought together a variety of topics and was appropriately placed at the end of the paper. This was very good at discriminating between candidates at the higher end, but it was also pleasing to see a good number of weaker candidates who were still able to attempt later parts such as part (c) and part (d) to a pleasing degree of success.

In part (a) a significant proportion of candidates understood that they needed to use the product rule on $7x\mathrm{e}^x$ and most candidates who did, applied it correctly. A common error was just to write $7\mathrm{e}^x$ or $7x\mathrm{e}^x$. Most candidates were unable to achieve the B1 mark, with most not applying the chain rule and so missing terms or making errors with powers. Candidates usually used their expressions correctly with the quotient rule, but few candidates scored the final method mark due to not having the correct expressions. Most candidates chose to use the quotient rule method, but those who used product rule instead, and initially rewrote the denominator as $\left(e^{3x}-2\right)^{-\frac{1}{2}}$, were generally successful in achieving the first four marks. Some, however, omitted brackets in their expression which lost the final two marks. The final A1 mark was achieved by very few candidates as most did not complete the rearrangement process. Some candidates adjusted their answer by using the given answer in part (b) to work out what the values of *A* and *B* should be. Whilst candidates should not typically use later information in earlier parts, this was condoned, as in some cases it would have been too difficult to determine whether this had been done so the benefit of the doubt was given. To achieve the correct form in part (a) from correct differentiation deserved full credit, regardless of the route to the final form required. It should be noted, however, that there were a few who integrated instead of differentiating.

In part (b) very few candidates attempted a solution having not completed a solution to part (a). Some determined candidates attempted a solution using *A* and *B* and so were able to achieve the method mark. Some candidates attempted a solution by using the given solution to work out what the values of *A* and *B* should be and then attempted a solution with those values. A minority of candidates who attempted a solution did not show all the steps required for a show that question.

In part (c), the majority of candidates scored the mark for a correct diagram, although some started their initial line higher up than the *x*-axis, which was condoned on this occasion. Several candidates used a cobweb diagram, and for others there were often lines drawn to the left of $x=1$. Only a handful of candidates put arrows on their lines, but their omission was, again, condoned.

Part (d), for many candidates who had not scored in or attempted earlier parts, was an opportunity to score full marks relatively easily. This was very well done. Occasionally, rounding x_2 to 1.50 rather than 1.502 was seen. However, those who wrote down an unrounded answer which rounded to 1.502 first were able to get the A mark. Candidates who showed the value embedded in the formula and then did not achieve 1.502 were able to get the M mark, too. A minority of candidates skipped 1.502 to go to the next term, 1.873 but this still was able to score the method mark as this was still evidence that the iterative formula had been used. The correct answer $\beta=1.968$ was widely achieved for part (b)(ii), although 1.967 was a common incorrect answer. The final mark was also lost for a small number of candidates who failed to round to 3 decimal places.

Part (e) was the final part on the paper and for many this resulted in no marks. Most of the candidates who attempted it found a suitable interval but substituted the values in the iterative formula instead of a valid equation set equal to zero. Many candidates wrote the required conclusion with no supporting evidence in an attempt to gain a mark. Some candidates, who used a valid function correctly, failed to score the final mark for not commenting that their function was continuous. A significant proportion of candidates attempted an iterative process with a different iterative formula, including attempting the Newton-Raphson method. The question required used of a suitable interval and so these methods scored no marks.

Overall, whilst this question was demanding, there were some very impressive solutions demonstrating an excellent grasp of calculus, strong algebraic skills, and the ability to provide a rigorous argument to prove given answers and results.

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