KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13
Applied Mathematics
S2 3 Normal Distribution

HGS Maths

Dr Frost Course

N ________________________________

Class: ________________________________

Contents

Prior knowledge check

Prior knowledge check

1 The probability that a one-month old Labrador puppy weighs under 2 kg is 0.735. Two puppies are chosen at random from different litters. Find:

a P(both weigh under 2 kg)

b P(exactly one weighs under 2 kg)

← Year 1, Chapter 1, Chapter 5

2 $X \sim B(20, 0.4)$. Find:

a $P(X = 6)$

b $P(X \geqslant 8)$

c $P(3 \leqslant X \leqslant 10)$ ← Year 1, Chapter 6

3 The probability that a plate made using a particular production process is faulty is given as 0.16. A sample of 20 plates is taken. Find:

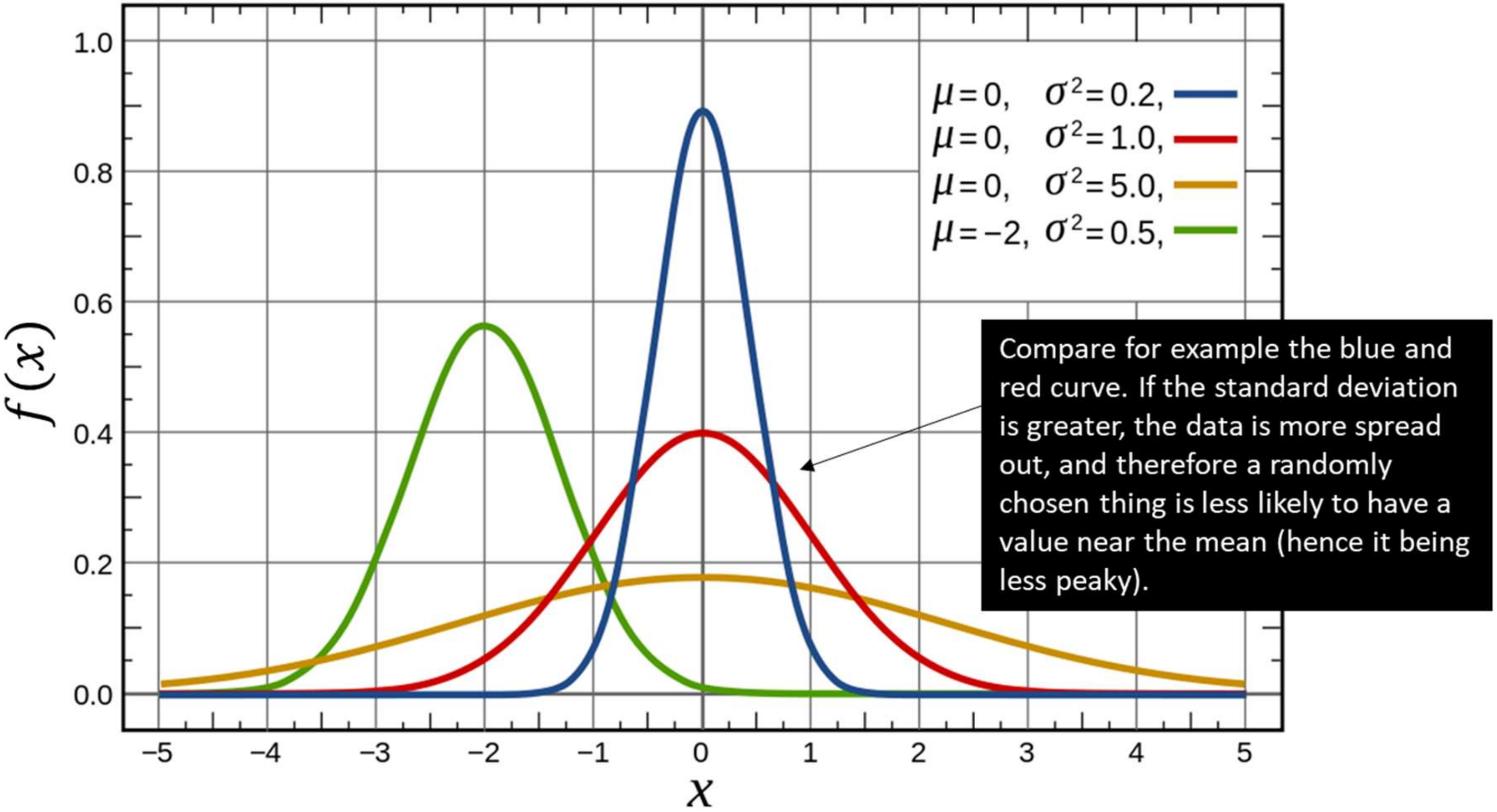
a the probability that exactly two plates are faulty

b the probability that no more than three plates are faulty.

← Year 1, Chapter 6

3.1) The normal distribution

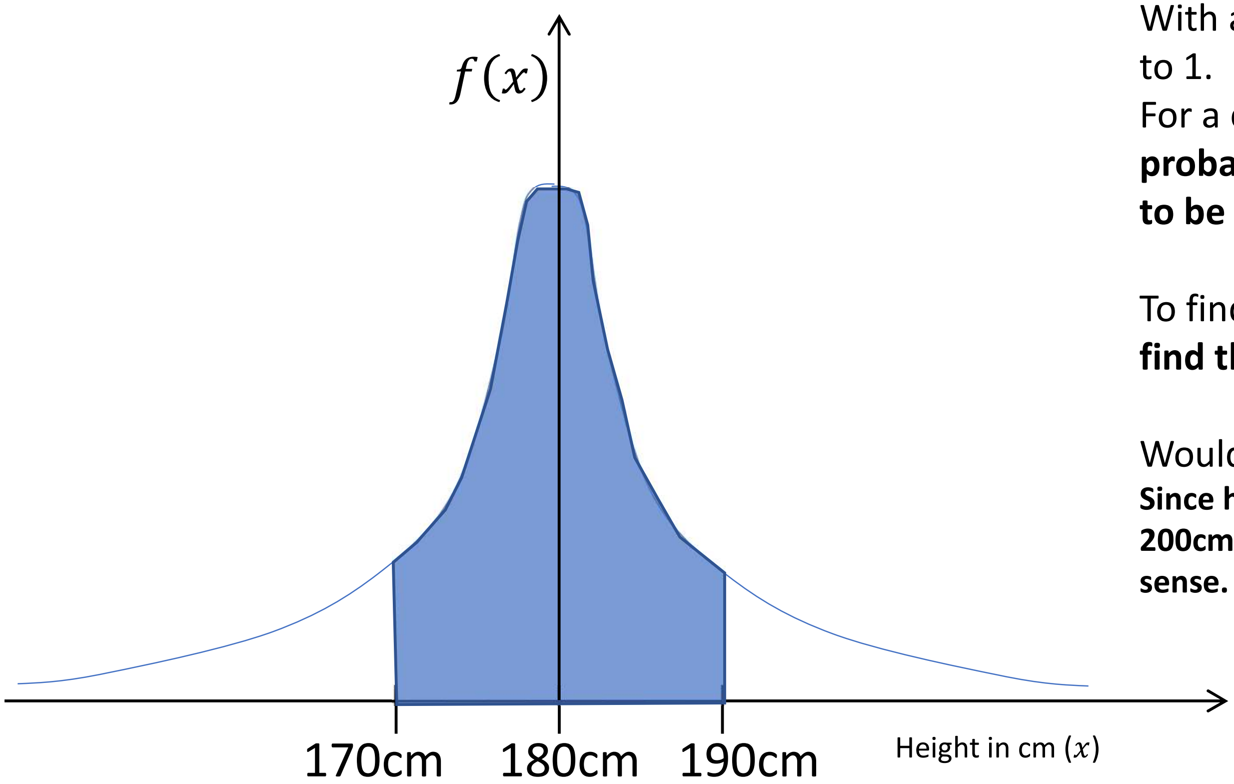
What does it look like?



We can set the mean μ and the standard deviation σ of the Normal Distribution. If a random variable X is normally distributed, then we write

$$\boldsymbol{X \sim N(\mu, \sigma^2)}$$

Notes



For a Normal Distribution to be used, the variable has to be **continuous**

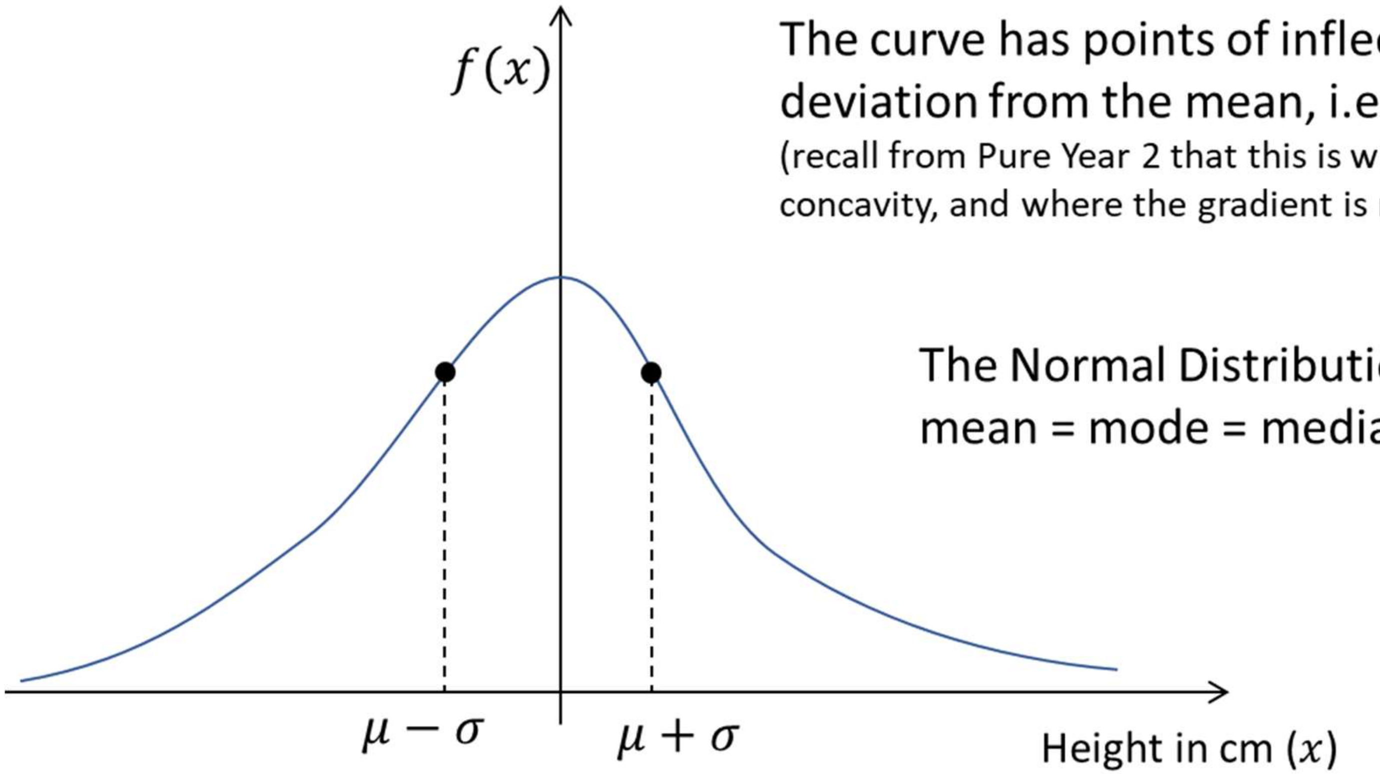
With a discrete variable, all the probabilities had to add up to 1.
For a continuous variable, similarly: **the area under the probability graph has to be 1.**

To find $P(170 < X < 190)$, we could:
find the area between these values.

Would we ever want to find $P(X = 200)$ say?
Since height is continuous, the probability someone is 'exactly' 200cm is infinitesimally small. So not a 'probability' in the normal sense.

Side Notes: You might therefore wonder what the y-axis actually is. It is **probability density**, i.e. "the probability per unit cm". This is analogous to frequency density with histograms, where the y-value is frequency density area under the graph gives frequency. We use $f(x)$ rather than $p(x)$, to indicate probability density.

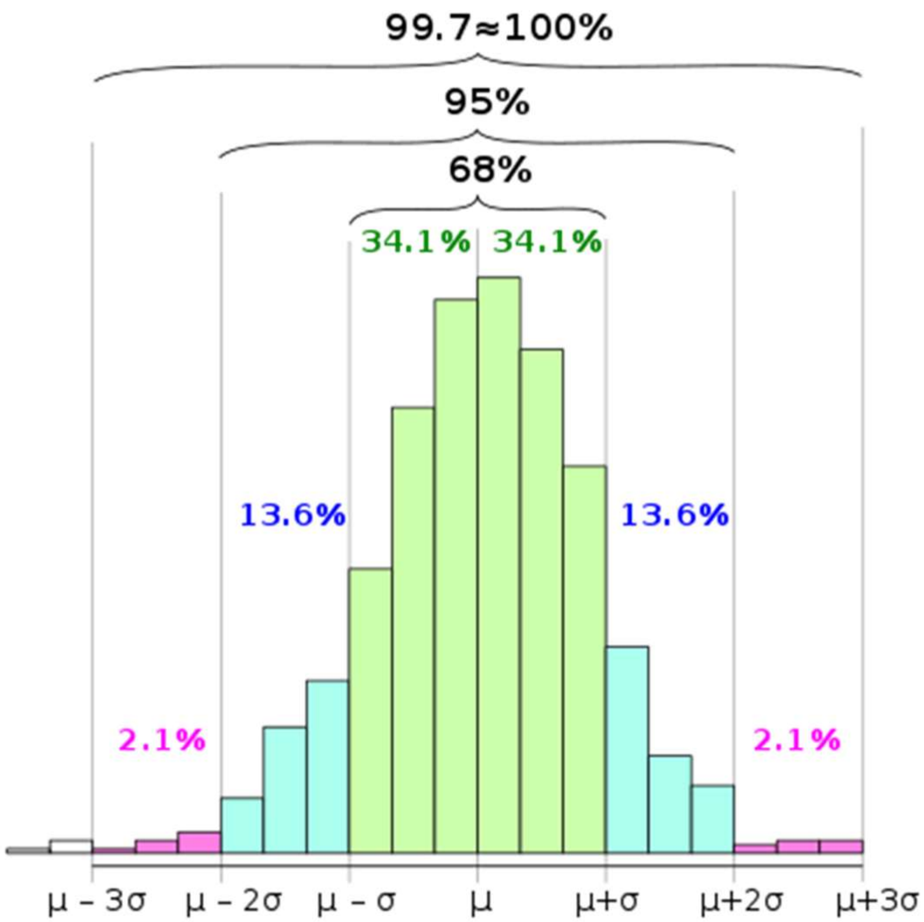
Further Notes



The curve has points of inflection one standard deviation from the mean, i.e. $\mu \pm \sigma$
(recall from Pure Year 2 that this is where the curve changes concavity, and where the gradient is not changing)

The Normal Distribution is symmetrical, i.e.
mean = mode = median

The 68-95-99.7 rule



The histogram above is for a quantity which is approximately normally distributed.
Source: Wikipedia

The 68-95-99.7 rule is a shorthand used to remember the percentage of data that is within 1, 2 and 3 standard deviations from the mean respectively.
<u>You need to memorise this!</u>

$\approx$ 68% of data is within one standard deviation of the mean.
$\approx$ 95% of data is within two standard deviations of the mean.
$\approx$ 99.7% of data is within three standard deviations of the mean.

For practical purposes we consider all data to lie within $\mu \pm 5\sigma$

Only one in 1.7 million values fall outside $\mu \pm 5\sigma$. CERN used a "5 sigma level of significance" to ensure the data suggesting existence of the Higgs Boson wasn't by chance: this is a 1 in 3.5 million chance (if we consider just one tail).

Notes

Worked Example

The diameters of a rivet produced by a particular machine, X mm, is modelled as $X \sim N(10, 0.3^2)$. Find:

a) $P(X < 10)$

b) $P(9.4 < X < 10.6)$

c) $P(9.1 < X < 10.9)$

d) $P(X \neq 9.7)$

Worked Example

The mass of a group of animals, M grams, is modelled as

$$M \sim N(\mu, 25)$$

If 84% of the animals have a mass less than 50 grams, find μ

Worked Example

The mass of a group of animals, M grams, is modelled as

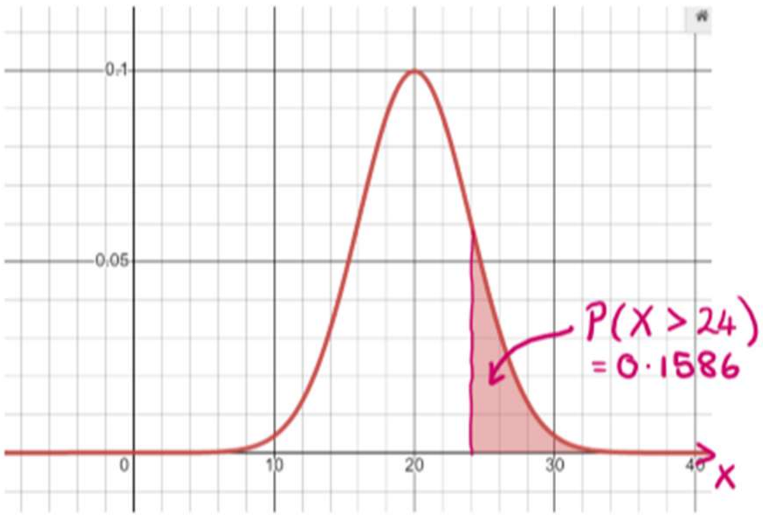
$$M \sim N(\mu, \sigma^2)$$

84% of the animals have a mass less than $70.9\ kg$ and 97.5% of the animals have a mass less than $76.3\ kg$.

Find the population mean and variance.

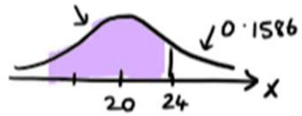
3.2) Finding probabilities for normal distributions

Can use: Normal Distribution Calculator – GeoGebra (https://www.geogebra.org/m/kbjKw7Cd)

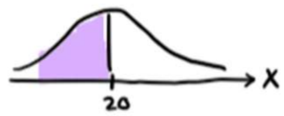


$P(X > 24) = P(X \geq 24)$
$= 0.1586$

$P(X < 24) = 1 - 0.1586$
$= 0.8414$



$P(X < 20) = 0.5$

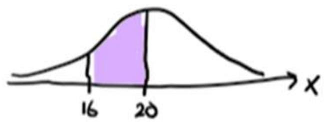


$P(X < 16) = 0.1586$

$P(16 \leq X \leq 24) = 1 - 2(0.1586)$
$= 0.6828$

$P(X = 20) = 0$

$P(16 < X < 20) = 0.3414$



Any others?

Notes

Worked Example *KS640f*

IQ is distributed using $X \sim N(100, 15^2)$. Find

(a) $P(X < 91)$

(b) $P(X \geq 107)$

(c) $P(80 < X < 90)$

(d) $P(X < 86 \text{ or } X > 112)$

Worked Example

IQ is distributed using $X \sim N(100, 15^2)$.
Adults scoring at least 131 on an IQ test are eligible to join Mensa.
Thirty adults take the test.
Find the probability that at least three of them are eligible to join.

3.3) The inverse normal distribution function

Notes

Worked Example *KS641a & d*

$$X \sim N(30, 4)$$

Find, correct to two decimal places, the values of a such that:

a. $P(X < a) = 0.7$

b. $P(X > a) = 0.45$

c. $P(24 < X < a) = 0.2$

Worked Example

The IQ of a population is distributed using

$$X \sim N(100, 15^2)$$

a) Determine the IQ corresponding to the top 30% of the population.

b) Determine the interquartile range of IQs.

3.4) The standard normal distribution

Z is the number of standard deviations above the mean.

If again we use IQ distributed as $X \sim N(100, 15^2)$ then: (in your head!)

IQ	Z
100	0
130	2
85	-1
165	4.333
62.5	-2.5

This formula makes sense if you think about the definition above. For an IQ of 130: $Z = \frac{130-100}{15} = 2$ as expected.

Z represents the coding:

$$Z = \frac{X - \mu}{\sigma}$$

and $Z \sim N(0, 1^2)$. Z is known as a standard normal distribution.

The 0 and 1 of $Z \sim N(0, 1^2)$ are the result of the coding. If we've subtracted μ from each value the mean of the normal distribution is now 0. If we've divided all the values by σ the standard deviation is now $\frac{\sigma}{\sigma} = 1$

$\Phi(z)$ - refers to the area/probability to the LEFT of z

Notes

Worked Example

Z is the number of standard deviations above the mean.

Assume $X \sim N(100, 15^2)$

Find z if

$$X = 100$$

$$X = 130$$

$$X = 62.5$$

Worked Example

The random variable $X \sim N(40, 5^2)$.
Write in terms of $\Phi(z)$ for some value of z.

(a) $P(X \leq 45)$

(b) $P(X > 43)$

Worked Example

If $X \sim N(100, 15^2)$, determine, in terms of Φ:

(a) $P(X > 70)$

(b) $P(88 < X < 122.5)$

Worked Example

The systolic blood pressure of an adult population, S mmHg, is modelled as a normal distribution with mean 721 and standard deviation 4.

A medical research wants to study adults with blood pressures higher than the 90th percentile.

Find the minimum blood pressure for an adult included in her study.

Worked Example

Determine:

$$P(Z > -1.7)$$

$$P(Z \leq -1.5)$$

Worked Example

Determine:

$$P(-1 < Z < 0)$$

$$P(-1.5 < Z < 0.5)$$

Worked Example

Determine a such that:

$$P(Z > a) = 0.3$$

$$P(Z < a) = 0.4$$

Worked Example

Determine a such that:

$$P(-a < Z < a) = 0.4$$

$$P(-a < Z < a) = 0.5$$

Worked Example

Use the percentage points table to find values of z which correspond to the 10% to 80% interpercentile range.

3.5) Finding μ and σ

Notes

Worked Example- *K643b*

$$X \sim N(\mu, 4^2)$$

Given that $P(X > 30) = 0.1$, find the value of μ.

Worked Example –*K343c*

A machine makes metal sheets with width, X cm, modelled as a normal distribution such that

$$X \sim N(70, \sigma^2)$$

(a) Given that $P(X < 64) = 0.02275$, find the value of σ.

(b) Find the 80th percentile of the widths.

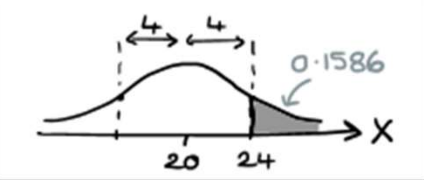
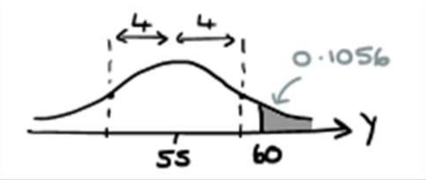
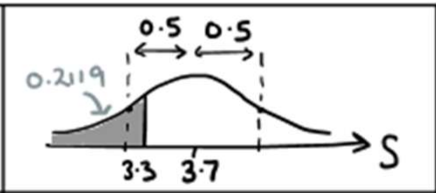
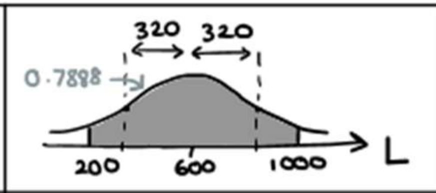
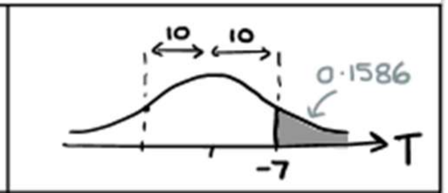
Worked Example–*K343d*

A random variable

$$X \sim N(\mu, \sigma^2)$$

Given that $P(X < 13) = 0.1964$ and $P(X > 51) = 0.01$, find the values of μ and σ

Activity

				
$X \sim N(20, 4^2)$	$Y \sim$ □			
$\mu = 20$	$\mu =$ □		$\mu = 600$	
$\sigma = 4$	$\sigma =$ □	$\sigma = 0.5$		
$P(X > 24) = 0.1586$	$P(Y > 60) = 0.1056$	$P(S >$ □ $) = 0.2119$		
$P(X < 16) = 0.1586$	$P(Y <$ □ $) = 0.1056$	$P(S < 3.3) = 0.2119$		
$P(X \leq 24) = 0.8414$	$P(Y < 60) =$ □	$P(S <$ □ $) =$ □		
$P(16 < X \leq 20) = 0.3413$	$P($ □ $< Y \leq$ □ $) =$ □	$P($ □ $) =$ □	$P(200 < L \leq 1000) = 0.7888$	

Activity

$M \sim N(4, 20^2)$		$D \sim N(100, 25)$		$Z \sim N(0,1)$
$\mu =$	$\mu = 24$		$\mu = 20$	
$\sigma =$	$\sigma = 0.8$		$\sigma = 4$	
$P(X > 24) = 0.1586$				$P(Z > 1) =$
		$P(D \leq 96) = 0.2119$		
	$P(G \leq 25) = 0.8944$			
			$P(\quad < R \leq \quad) = 0.7888$	

Worked Example

The time taken for a journey, X, has a normal distribution with mean 200 minutes and standard deviation d minutes.
Given that 30% of the journeys take longer than 230 minutes, find the standard deviation.

Worked Example

The time taken for a journey, X, is normally distributed with mean μ days and standard deviation σ days.
15% of journeys are shorter than 532 days.
2.5% are longer than 682 days.
Find the values between which the middle 95% of journeys lie.

Worked Example

The mass of an animal is found to be normally distributed with mean μ and standard deviation σ.
10% of the animals have a mass less than 9 kg. 5% of the animals have a mass greater than 60 kg.
8 animals are chosen at random.
Find the probability that at least two of them have a mass greater than 50 kg.

3.6) Approximating a binomial distribution

If we're going to use a normal distribution to approximate a Binomial distribution, it makes sense that we set the mean and standard deviation of the normal distribution to match that of the original binomial distribution:

$$\mu = np$$
$$\sigma = \sqrt{np(1-p)}$$

✎ If n is large and p close to 0.5, then the binomial distribution $X \sim B(n, p)$ can be approximated by the normal distribution $N(\mu, \sigma^2)$ where

$$\mu = np$$
$$\sigma = \sqrt{np(1-p)}$$

Quickfire Questions:

$X \sim B(10,0.2) \rightarrow Y \sim N(2, 1.6)$

$X \sim B(20,0.5) \rightarrow Y \sim N(10, 5)$

$X \sim B(6, 0.3) \rightarrow Y \sim N(1.8, 1.26)$

We tend to use the letter Y to represent the normal distribution approximation of the distribution X.

Why use a normal approximation?

- Tables for the binomial distribution only goes up to $n = 50$ and your calculator will reject large values of n.
- The formula for $P(X = x)$ makes use a factorials. Factorials of large numbers cannot be computed efficiently. Type in 65! for example; your calculator will hesitate! Now imagine how many factorials would be required if you wanted to find $P(X \leq 65)$. ☺

Notes

Worked Example

A biased coin has $P(tails) = 0.47$.
The coin is tossed 200 times and the number of tails is recorded.

a) Write a binomial model for X

b) Show that X can be approximated with a normal distribution $Y \sim N(\mu, \sigma^2)$ and find the values of μ and σ

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(X = 5)$$

$$P(X = 4)$$

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(X \leq 5)$$

$$P(X \leq 4)$$

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(X < 5)$$

$$P(X < 4)$$

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(X \geq 5)$$

$$P(X \geq 4)$$

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(X > 5)$$

$$P(X > 4)$$

Worked Example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(5 \leq X \leq 8)$$

$$P(4 < X < 7)$$

Worked Example *K644c*

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(5 < X \leq 8)$$

$$P(4 \leq X < 7)$$

Worked Example

For a particular type of flower bulbs, 44% will produce red flowers. A random sample of 160 bulbs is planted.

(a) Calculate the actual probability that there are exactly 70 red flowers.

(b) Use a normal approximation to find an estimate that there are exactly 70 red flowers.

(c) Hence determine the percentage error of the normal approximation for 70 red flowers.

3.7) Hypothesis testing with the normal distribution

0	1	2	3	4	5	6	7	8	9

Imagine we have 10 children, one of each age between 0 and 9.
This is our population. There is a **known population mean** of $\mu = 4.5$

					$\bar{x}$
Sample 1:	1	3	7	8	4.75
Sample 2:	6	2	0	9	4.25

...

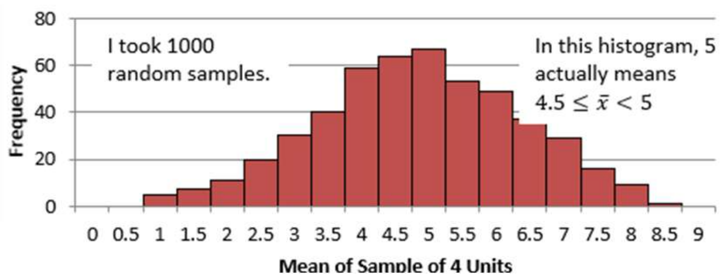
The mean of this sample is $\bar{x} = 4.75$. This sample mean $\bar{x}$ is close the true population mean μ, but is naturally going to vary as we consider different samples.

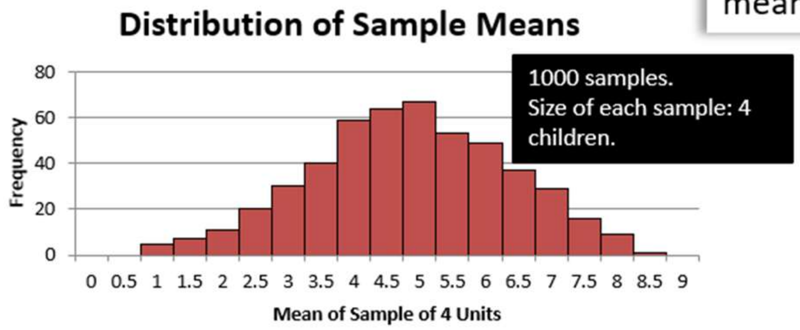
For a different sample of 4, we might obtain a different sample mean. What would happen if we took lots of different samples of 4, and found the mean $\bar{x}$ of each? How would these means be distributed?

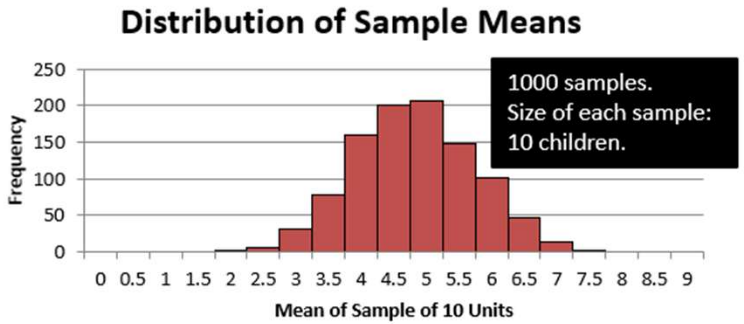
Suppose we took a sample of 4 children.

Sample mean $\bar{x}$	Tally
4.00	
4.25	|
4.50	
4.75	|
5.00	

Distribution of Sample Means $\bar{X}$







✏ For a random sample of size n taken from a random variable X, the sample mean $\bar{X}$ is normally distributed with $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

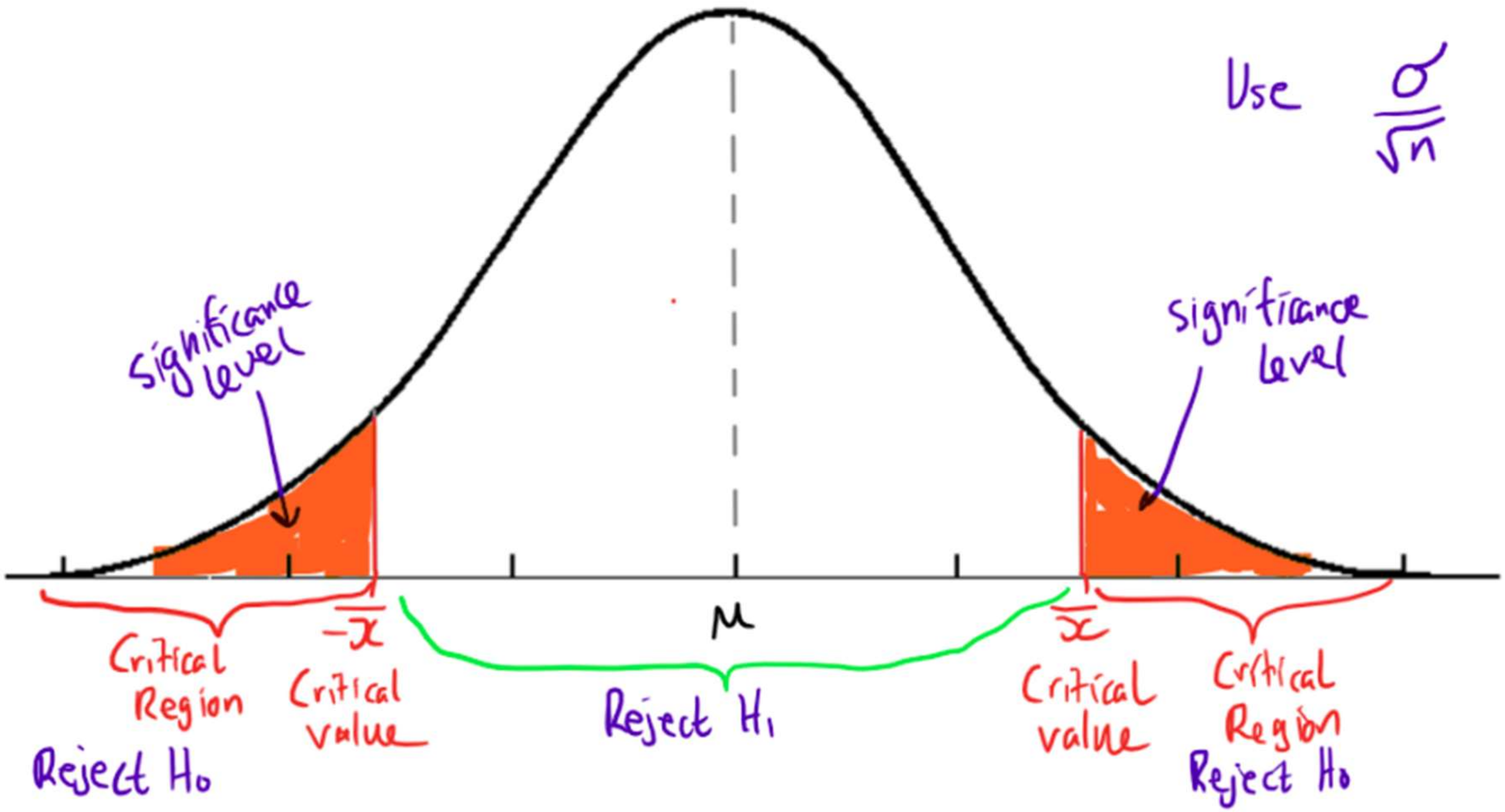
$\bar{X}$ is our distribution across different sample means as we consider different samples.

Question 1: What type of distribution is $\bar{X}$?
From the left it seems like it is approximately normally distributed!

Question 2: On average, what sample mean do we see? (i.e. the mean of the means!)
μ. The sample means $\bar{x}$ vary around the population mean μ, but on average is μ.

Question 3: Is the variance of $\bar{X}$ (i.e. how spread out the sample means are) the same as that of the variance of the population of children?
No! On the left, we can see that how spread out the sample means are depends on the sample size. If the sample size is small, the sample means are likely to vary quite a bit. But with a larger sample size, we expect the different $\bar{x}$ to be closer to the population mean μ.

Normal Distribution Hypothesis Testing – A Visual Approach



Writing Frames for test

VERSION 1: p-value

State hypotheses	$H_0: \mu = \ldots$ $H_1: \mu > \ldots.$ $\mu < \ldots$ $\mu \neq \ldots$
Write parameters of the sampling distribution	$\mu =$ *(Same as H_0)* *calculate:* $\frac{\sigma}{\sqrt{n}}$ *Write as* $\bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$
Calculate probability	$\mu <$: $P(X \leq \bar{x}) = p$ $\mu >$: $P(X \geq \bar{x}) = p$ $\mu \neq$: $P(X \leq \bar{x}) = p$ or $P(X \geq \bar{x}) = p$
Compare p to significance level	*accept Ho if* $p > sig\ level$ *accept Ho if* $p > \frac{sig\ level}{2}$
Accept/reject H_0	
Conclusion in context – *using wording from Q*	there is **insufficient/sufficient** evidence to accept/reject the claim (state it) at the___ % significance level

Writing Frames for test

VERSION 2: critical value

<table>
<tr><td>State hypotheses</td><td>H_0: $\mu = \ldots$ H_1: $\mu > \ldots.$ $\mu < \ldots$ $\mu \neq \ldots$</td></tr>
<tr><td>Write parameters of the sampling distribution</td><td>$\mu =$ (same as H_0)   calculate: $\frac{\sigma}{\sqrt{n}}$
Write as $\bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$</td></tr>
<tr><td>Find critical value x_c</td><td><table><tr><th>$\mu <$</th><th>$\mu <$</th><th>$\mu \neq$</th></tr><tr><td>Use inverse normal using
sig. level
as area</td><td>Use inverse normal using
[1- sig. level]
as area</td><td>Use inverse normal using
Half of sig. level
as area.
Then find the value the same distance from μ on the right</td></tr></table></td></tr>
<tr><td>Compare $\bar{x}$ to x_c</td><td></td></tr>
<tr><td>Accept/reject H_0</td><td><table><tr><th>$\mu <$</th><th>$\mu <$</th><th>$\mu \neq$</th></tr><tr><td>Accept if
$x_c < \bar{x}$</td><td>Accept if
$\bar{x} > x_c$</td><td>Accept if
$x_{c_1} < \bar{x} < x_{c_2}$</td></tr></table></td></tr>
<tr><td>Conclusion in context – using wording from Q</td><td>there is <u>insufficient/sufficient</u> evidence to accept/reject the claim (state it) at the___ % significance level</td></tr>
</table>

Worked Example

A random sample of size n is taken from a population X having a normal distribution with mean μ and variance σ^2. Conduct a hypothesis test on the population mean, given:

Sample size $n = 40$
Sample mean $\bar{x} = 49$
Population standard deviation $\sigma = 4$
5% significance level

$H_0\colon\ \mu = 50$
$H_1\colon\ \mu < 50$

Worked Example

A random sample of size n is taken from a population X having a normal distribution with mean μ and variance σ^2. Conduct a hypothesis test on the population mean, given:

Sample size $n = 30$
Sample mean $\bar{x} = 51$
Population standard deviation $\sigma = 4$
10% significance level

H_0: $\mu = 50$
H_1: $\mu > 50$

Worked Example

A random sample of size n is taken from a population X having a normal distribution with mean μ and variance σ^2. Conduct a hypothesis test on the population mean, given:

Sample size $n = 61$
Sample mean $\bar{x} = 51$
Population standard deviation $\sigma = 4$
10% significance level

H_0: $\mu = 50$
H_1: $\mu \neq 50$

Worked Example

A certain company sells fruit juice in cartons.
The amount of juice in a carton has a normal distribution with a standard deviation of 5ml.
The company claims that the mean amount of juice per carton, μ, is 40ml.
A trading inspector has received complaints that the company is overstating the mean amount of juice per carton and wishes to investigate this complaint.
The trading inspector takes a random sample of 36 cartons and finds that the mean amount of juice per carton is 38.9ml.

Using a 5% level of significance, and stating your hypotheses clearly, test whether or not there is evidence to uphold this complaint.

Worked Example

A machine products bolts of diameter D where D has a normal distribution with mean 0.580 cm and standard deviation 0.015 cm. The machine is serviced and after the service a random sample of 50 bolts from the next production run is taken to see if the mean diameter of the bolts has changed from 0.580 cm. The distribution of the diameters of bolts after the service is still normal with a standard deviation of 0.015 cm.

(a) Find, at the 1% level, the critical region for this test, stating your hypotheses clearly.

The mean diameter of the sample of 50 bolts is calculated to be 0.587 cm.

(b) Comment on this observation in light of the critical region.

3.x) Conditional probabilities

This is not in the textbook. But given the recent Chapter 2 on Conditional Probabilities and the fact that the type of question below occurred frequently in S1 papers, it seems worthwhile to cover!

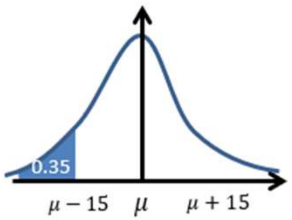
Edexcel S1 May 2014(R) Q4

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(*c*) find $P(X > \mu + 15 \mid X > \mu - 15)$. **(3)**



$$P(X > \mu + 15 \mid X > \mu - 15) = \frac{P(X > \mu + 15)}{P(X > \mu - 15)} = \frac{0.35}{0.65} = \frac{7}{13}$$

The intersection of "above $\mu + 15$" and "above $\mu - 15$" is just "above $\mu + 15$" because the stronger statements takes precedent. If I said "my age is above 20, and above 30", this is equivalent to saying just "my age is above 30".

Notes

Worked Example

The time taken, X minutes, for a flight has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 25) = 0.45$,

Find $P(X > \mu + 25 \mid X > \mu - 25)$

Your Turn

The time taken, X minutes, for a flight has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$,

Find $P(X > \mu + 15 \mid X > \mu - 15)$

$$\frac{7}{13}$$

Worked Example

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 20 hours and a standard deviation of 3 hours.

A person is about to go on a 2 hour journey.
Given that it is 25 hours since they last charged their phone, find the probability that their phone will not need charging before the journey is completed.

Your Turn

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

A person is about to go on a 6 hour journey.
Given that it is 127 hours since they last charged their phone, find the probability that their phone will not need charging before the journey is completed.

0.39 (2 sf)

Extract from Formulae book

Percentage Points of The Normal Distribution

The values z in the table are those which a random variable $Z - N(0, 1)$ exceeds with probabilit
is, $P(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

5. The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

(a) Find the probability that a randomly selected battery will last for longer than 16 hours. **(1)**

At the start of her exams Alice put 4 new batteries in her calculator.
She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

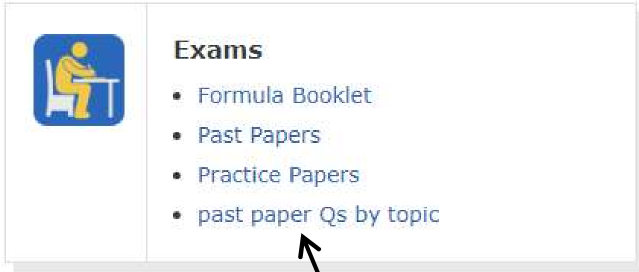
(b) Find the probability that her calculator will not stop working for Alice's remaining exams. **(5)**

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

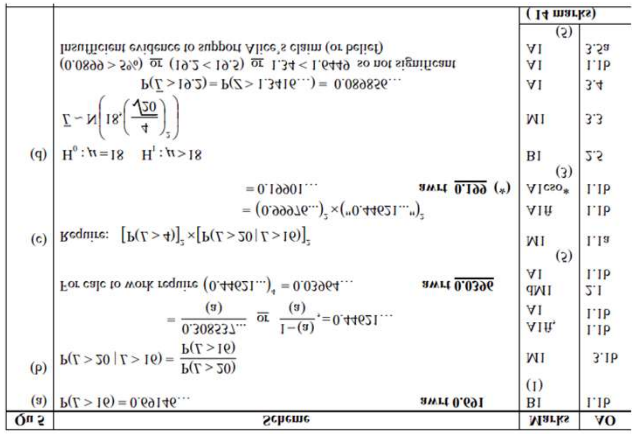
(c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures. **(3)**

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

(d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief. **(5)**



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



Qu 5	Scheme	Marks	AO
(a)	$P(L>16)=0.69146...$ awrt **0.691**	B1 (1)	1.1b
(b)	$P(L>20 \mid L>16)=\frac{P(L>20)}{P(L>16)}$	M1	3.1b
	$=\frac{0.308537...}{(a)}$ or $\frac{1-(a)}{(a)}$, $=0.44621...$	A1ft, A1	1.1b, 1.1b
	For calc to work require $(0.44621...)^4=0.03964...$ awrt **0.0396**	dM1, A1 (5)	2.1, 1.1b
(c)	Require: $[P(L>4)]^2\times[P(L>20 \mid L>16)]^2$	M1	1.1a
	$=(0.99976...)^2\times(\text{"}0.44621...\text{"})^2$	A1ft	1.1b
	$=0.19901...$ awrt **0.199** (*)	A1cso* (3)	1.1b
(d)	$H_0:\mu=18\quad H_1:\mu>18$	B1	2.5
	$\bar{L}\sim N\left(18,\left(\frac{4}{\sqrt{20}}\right)^2\right)$	M1	3.3
	$P(\bar{L}>19.2)=P(Z>1.3416...)=0.089856...$	A1	3.4
	$(0.0899>5\%)$ or $(19.2<19.5)$ or $1.34<1.6449$ so not significant	A1	1.1b
	Insufficient evidence to support Alice's claim (or belief)	A1 (5)	3.5a
		(14 marks)	

Summary of Key Points

Summary of key points

1. The area under a continuous probability distribution is equal to 1.

2. If X is a normally distributed random variable, you write $X \sim N(\mu, \sigma^2)$ where μ is the population mean and σ^2 is the population variance.

3. The normal distribution
   - has parameters μ, the population mean, and σ^2, the population variance
   - is symmetrical (mean = median = mode)
   - has a bell-shaped curve with asymptotes at each end
   - has total area under the curve equal to 1
   - has points of inflection at $\mu + \sigma$ and $\mu - \sigma$

4. The standard normal distribution has mean 0 and standard deviation 1.
   The standard normal variable is written as $Z \sim N(0, 1^2)$.

5. If n is large and p is close to 0.5, then the binomial distribution $X \sim B(n, p)$ can be approximated by the normal distribution $N(\mu, \sigma^2)$ where
   - $\mu = np$
   - $\sigma = \sqrt{np(1-p)}$

6. If you are using a normal approximation to a binomial distribution, you need to apply a **continuity correction** when calculating probabilities.

7. For a random sample of size n taken from a random variable $X \sim N(\mu, \sigma^2)$, the sample mean, $\overline{X}$, is normally distributed with $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

8. For the sample mean of a normally distributed random variable, $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ is a normally distributed random variable with $Z \sim N(0, 1)$.