KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 13
Applied Mathematics
S1 7 Hypothesis testing binomial

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

Prior knowledge check

1 $X \sim \text{B}(20, 0.4)$. Calculate:

a $\text{P}(X = 5)$ **b** $\text{P}(X = 10)$

c $\text{P}(X \leqslant 2)$ **d** $\text{P}(X \geqslant 18)$ ← Chapter 6

2 Wanda rolls a fair dice eight times.

a Suggest a suitable model for the random variable X, the number of times the dice lands on five.

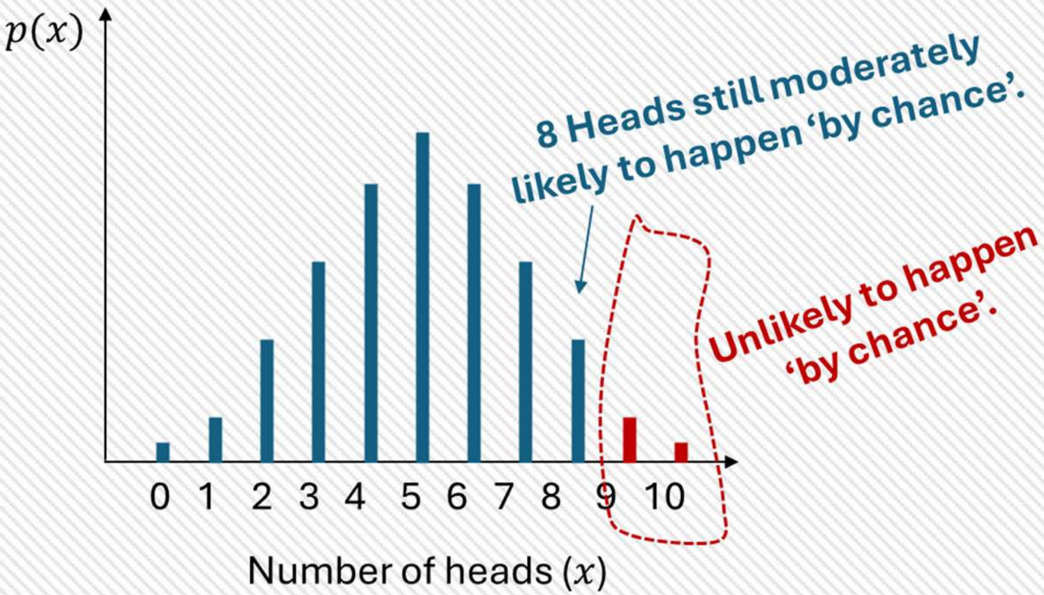
b Calculate:

i $\text{P}(X = 2)$ **ii** $\text{P}(X \geqslant 4)$ ← Chapter 6

7.1) Hypothesis testing

I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?

Our intuition is that the further away we are from the 'expected' number of heads (i.e. 5 heads out of 10), the more unlikely it is.



Notes

I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?

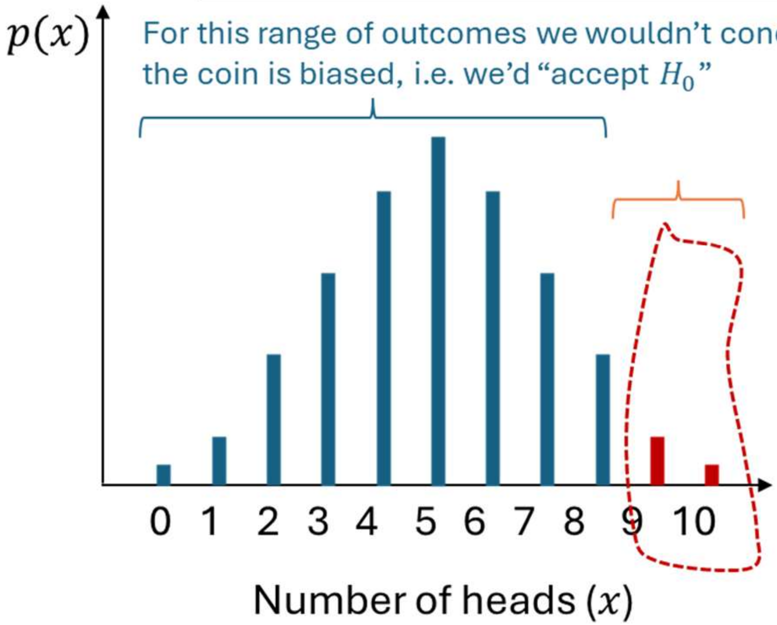
- A hypothesis is a statement made about the value of a **population parameter** that we wish to test by collecting evidence in the form of a sample.
- The **null hypothesis**, H_0 is the default position, i.e. that nothing has changed, unless proven otherwise.
- The **alternative hypothesis**, H_1, is that there has been some change in the population parameter.

In this context...

We're asking "is the coin biased". This is making a statement about the probability p of getting Heads (i.e. the p in $B(n, p)$)

The 'default position' is that the coin is fair, i.e. $p = 0.5$.

The 'alternative' position is that the coin is biased towards heads, i.e. p is more than 0.5.

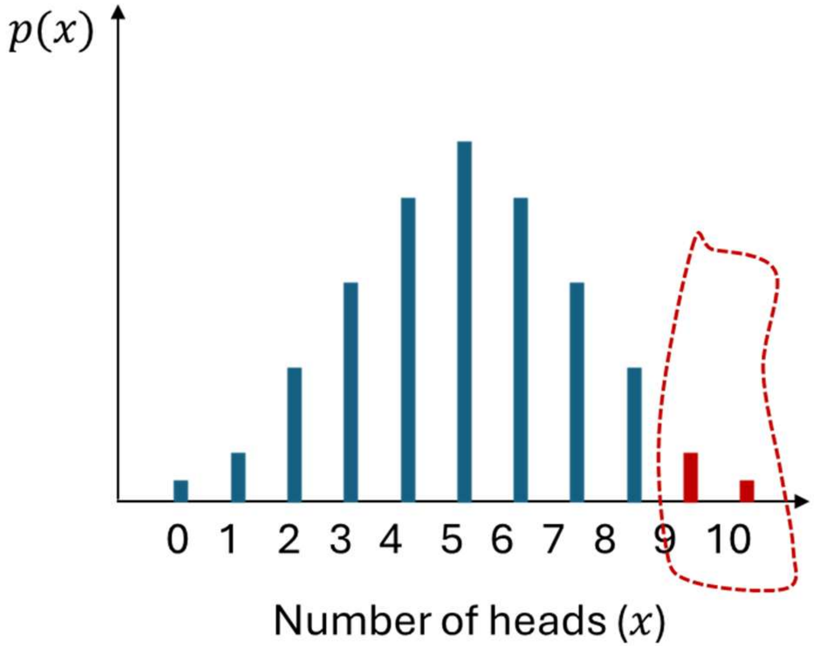


For this range of outcomes we'd conclude that this number of heads was too unlikely to happen by chance, and hence reject H_0 (i.e. that coin was fair) and accept H_1 (i.e. that coin was biased).

Notes

I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?

✎ In a hypothesis test, the evidence from the sample is a **test statistic**.



In this context...

The test statistic X is **what we observed**, in this case, X is the number of heads seen in 10 throws.

Note that the test statistic **is a distribution** (i.e. across the possible things we might observe).

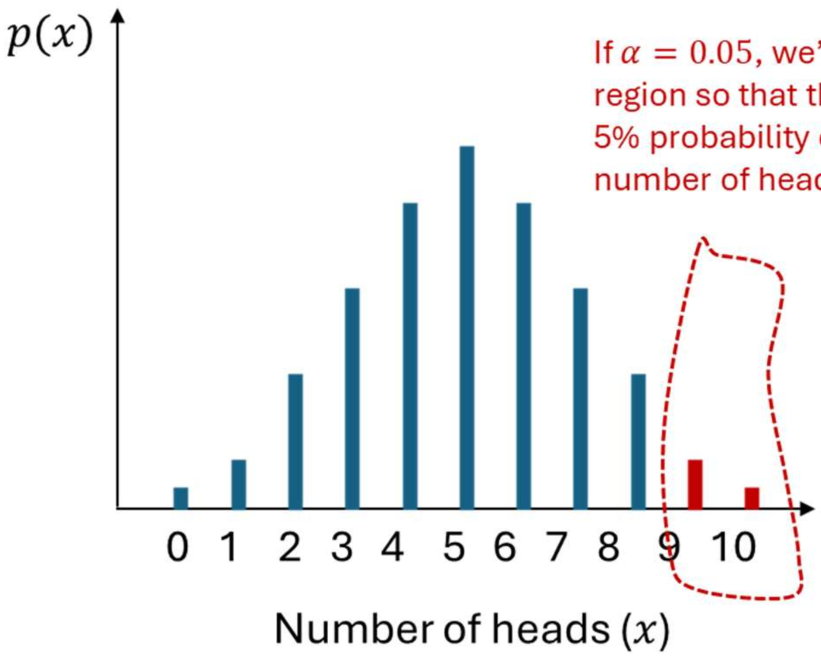
$$X \sim B(10, p)$$

noting that p is not known until we start making assumptions.

Notes

I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?

> ✎ The **level of significance α** is the maximum probability where we would reject the null hypothesis.
> This is usually 5% or 1%.



If $\alpha = 0.05$, we've set this region so that there's at most a 5% probability of seeing these number of heads 'by chance'.

In this context...

We said that if we saw a number of heads within ranges of outcomes that were sufficiently unlikely, then we'd rule out that the coin is fair and conclude it was in fact biased.

But how unlikely is 'sufficiently unlikely'? If $\alpha = 5\%$, then we'd find a region of outcomes where there's (at most) a 5% chance of one of these extreme values happening 'by chance' (i.e. if the coin was fair).

Notes

Hypothesis testing in a nutshell then is:

1. We have some hypothesis we wish to see if true (e.g. coin is biased towards heads), so…
2. We collect some sample data by throwing the coin (giving us our 'test statistic') and…
3. If that number of heads (or more) is sufficiently unlikely to have emerged 'just by chance', then we conclude that our (alternative) hypothesis is correct, i.e. the coin is biased.

Notes

Worked Example

Joan believes a six-sided dice is biased in favour of rolling a 4.
She rolls the dice 10 times and counts the number of times, X, it rolls a 4.
Define the test statistic and state your null and alternative hypotheses.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether the candidate is over-estimating his support.
The researcher asks 30 people whether they support the candidate or not. 2 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.

A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.

The researcher asks 30 people whether they support the candidate or not. 11 people say they do.

a) Write down a suitable test statistic.

b) Write down two suitable hypotheses.

c) Explain the condition under which the null hypothesis would be rejected.

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
A sample of 200 lightbulbs is tested, and 11 are found to be faulty.
The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Worked Example

Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$.
She rolls the dice 10 times and counts the number of times, X, it rolls a 4.
Define the test statistic and state your null and alternative hypotheses.

Worked Example

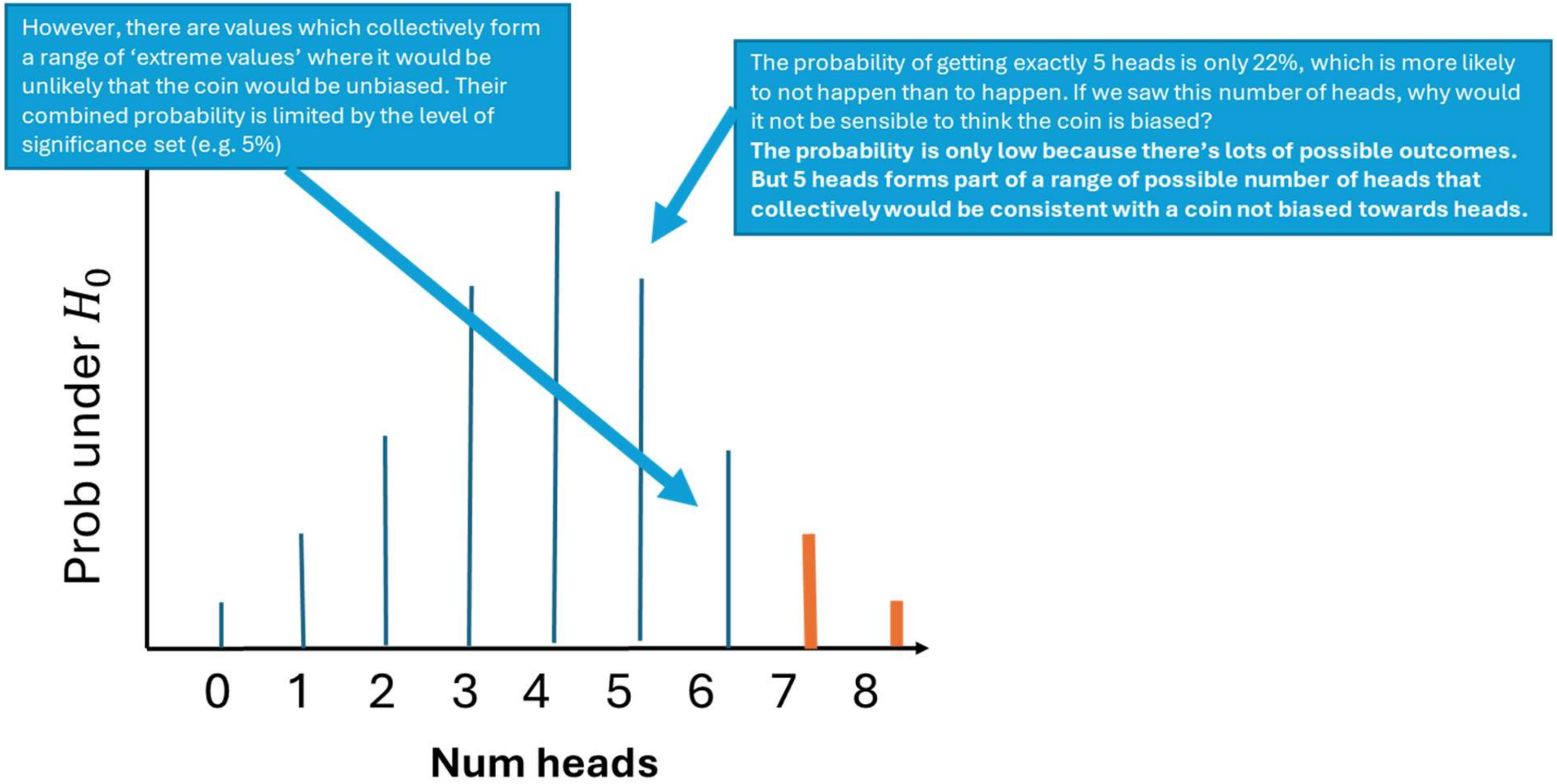
An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether this claim is true.
The researcher asks 30 people whether they support the candidate or not. 2 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

7.2) Finding critical values

John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X, it lands head uppermost. **What values would lead to John's hypothesis being rejected?**

As before, we're interested how likely a given outcome is likely to happen 'just by chance' under the null hypothesis (i.e. when the coin is not biased).



Notes

John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X, it lands head uppermost. **What values would lead to John's hypothesis being rejected**, if the significance level was 5%?

What's the probability that we would see **6 heads**, or an **even more extreme value**? Is this sufficiently unlikely to support John's claim that the coin is biased?

$$\boldsymbol{P(X \geq 6) = 1 - P(X \leq 5)}$$
$$\boldsymbol{= 0.1445}$$

Insufficient evidence to reject null hypothesis (since $\boldsymbol{0.1445 > 0.05}$).

What's the probability that we would see **7 heads**, or an **even more extreme value**?

$$\boldsymbol{P(X \geq 7) = 1 - P(X \leq 6)}$$
$$\boldsymbol{= 0.0352}$$

Since 0.0352 < 0.05, this is very unlikely, so we reject the null hypothesis and accept the alternative hypothesis that the coin is biased.

C.D.F. Binomial table: $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961

Worked Example

John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X, it lands head uppermost. **What values would lead to John's hypothesis being rejected**, if the significance level was 5%?

The **critical region** is the range of values of the test statistic that would lead to you rejecting H_0

If level of significance 5%, critical region?

We saw that 95% is exceeded when $X = 6$. This means $P(X \geq 7) = 1 - P(X \leq 6)$
$= 0.0352 < 5\%$
$\therefore 7 \leq X \leq 8$

Tip: Use the first value AFTER the one in the table that exceeds 95%.

The value(s) on the boundary of the critical region are called **critical value(s)**.

Critical value: 7

C.D.F. Binomial table:
$p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961
8	1

Your turn

Joan believes a six-sided dice is biased in favour of rolling a 4.
She rolls the dice 10 times and counts the number of times, X, it rolls a 4.

a) Using a 5% significance level, find the critical region for this test.

b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Quickfire Critical Regions

Determine the critical region when we throw a coin where we're trying to establish if there's the specified bias, given the specified number of throws, when the level of significance is 5%.

Coin thrown 5 times. Trying to establish if biased towards heads.

$p = 0.5, n = 5$

x	$P(X \leq x)$
0	0.0312
1	0.1875
2	0.5000
3	0.8125
4	0.9688

Coin thrown 10 times. Trying to establish if biased towards heads.

$p = 0.5, n = 10$

x	$P(X \leq x)$
0	0.0010
1	0.0107
2	0.0547
…	…
7	0.9453
8	0.9893
9	0.9990

Coin thrown 10 times. Trying to establish if biased towards tails.

$p = 0.5, n = 10$

x	$P(X \leq x)$
0	0.0010
1	0.0107
2	0.0547
…	…
7	0.9453
8	0.9893
9	0.9990

Reminder: At the positive tail, use the value AFTER the first that exceeds 95% (100 - 5).

At the negative tail, we just use the first value that goes under the significance level.

Worked Example

549e: **Determine the critical region for a one-tailed hypothesis test using a binomial model.**

The probability that a customer buys a tin of baked beans in a supermarket is 0.33. It is claimed that the probability has decreased. The supermarket records the shopping baskets of 25 different people.

$H_0 : p = 0.33$ and $H_1 : p < 0.33$

Using a 5% level of significance, find the critical region for the test.

$X \leq$ ☐

Actual Significance Level

John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X, it lands head uppermost. **What values would lead to John's hypothesis being rejected**, if the significance level was 5%?

We saw earlier that the critical region was $X \geq 7$, i.e. the region in which John would reject the null hypothesis (and conclude the coin was biased).

We ensured that $P(X \geq 7)$ was less than the significance level of 5%.
But what actually is $P(X \geq 7)$?

$$\boldsymbol{P(X \geq 7) = 1 - P(X \leq 6) = 0.0352}$$

This is known as the actual significance level, i.e. the probability that we're in the critical region. We expected this to be less than, but close to, 5%.

The **actual significance level** is the actual probability of being in the critical region.

C.D.F. Binomial table: $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961
8	1

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region for this test.
b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
A sample of 200 lightbulbs is tested. The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.

a) Find the critical region for this test.
b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Two-tailed test

Suppose I threw a coin 8 times and was now interested in how may heads would suggest it was a **biased coin** (i.e. either way!). How do we work out the critical values now, with 5% significance?

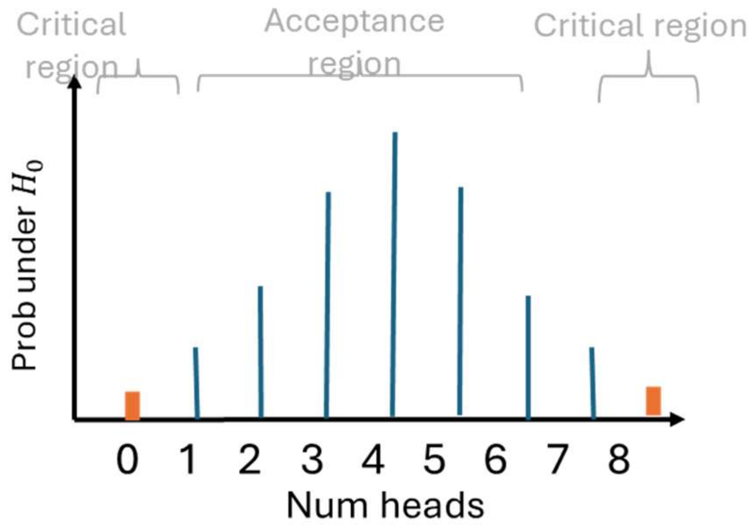
We split the 5% so there's 2.5% at either tail, then proceed as normal:

Critical region at positive tail:
Look at closest value above 0.975 (then go one above):

$$\boldsymbol{X = 8}$$

Critical region at negative tail:
Look at closest value below 0.025.

$$\boldsymbol{X = 0}$$



C.D.F. Binomial table: $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
…	…
6	0.9648
7	0.9961
8	1

Worked Example

549g: Determine the critical region for a two-tailed hypothesis test using a binomial model.

The probability that a mechanical component fails is 0.7. The company thinks the reliability of the component has changed. The company tests a sample of 27 components.

$H_0 : p = 0.7$ and $H_1 : p \neq 0.7$

Using a 5% level of significance, find the critical region for the test.

$X \leq$ ____

$X \geq$ ____

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether this claim is true.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region(s) for this test.
b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether this claim is true.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region(s) for this test.

b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
The manufacturing process is changed.
A sample of 200 lightbulbs is tested.
The manager wishes to test at the 2% significance level whether or not there has been a change in the proportion of faulty lightbulbs.

a) Find the critical region(s) for this test.
b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

Worked Example

A random variable has distribution $B(40, p)$.
A single observation is used to test $H_0: p = 0.1$
against $H_1: p \neq 0.1$.
Using a 1% level of significance, find the critical region for this test. The probability in each tail should be as close as possible to 0.005

7.3) One-tailed tests

Doing a full one-tailed hypothesis test

We've done various bits of a hypothesis test, and haven't actually properly conducted one yet. Let's do an example!

> John tosses a coin 8 times and it comes up heads 6 times. He claims the coin is **biased towards heads**. With a significance level of 5%, test his claim.

X is number of heads.
p is probability of heads.
$X \sim B(8, p)$

STEP 1: Define test statistic X (stating its distribution), and the parameter p.

$H_0: p = 0.5$
$H_1: p > 0.5$

STEP 2: Write null and alternative hypotheses.

Assume H_0 is true, $X \sim B(8, 0.5)$

$$P(X \geq 6) = 1 - P(X \leq 5)$$
$$= 1 - 0.8555$$
$$= 0.1445$$

STEP 3: Determine probability of observed test statistic (or 'more extreme'), assuming null hypothesis.
i.e. Determine probability we'd see this outcome just by chance.

14.45% > 5%, so insufficient evidence to reject H_0.
Coin is not biased.

STEP 4: Two-part conclusion:
1. Do we reject H_0 or not?
2. Put <u>in context of original problem</u>.

C.D.F. Binomial table: $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961

NEW TO A LEVEL 2017: The probability of 'the observed value or more extreme' is known as the p-value.

Alternative method using critical regions

We can also find the critical region and see if the test statistic lies within it.

John tosses a coin 8 times and it comes up heads 6 times. He claims the coin is **biased towards heads**. With a significance level of 5%, test his claim.

X is number of heads.
p is probability of heads.
$X \sim B(8, p)$

STEP 1: Define test statistic X (stating its distribution), and the parameter p.

$H_0: p = 0.5$
$H_1: p > 0.5$

STEP 2: Write null and alternative hypotheses.

$P(X \geq 7) = 1 - 0.9648 = 0.0352$
Critical region is $X \geq 7$

STEP 3 (Alternative): Determine critical region.

6 is not in critical region, so do not reject H_0.
Coin is not biased.

STEP 4: Two-part conclusion:
1. Do we reject H_0 or not?
2. Put in context of original problem.

C.D.F. Binomial table: $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
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7	0.9961

More on p-values

(Note that this is not covered in the Pearson textbook, but **is** in the specification)

Sheila wants to know if a coin is biased towards heads and throws it a large number of times, counting the number of heads. The p-value is less than 0.03. Conduct a hypothesis test at the 5% significance level.

Let $\boldsymbol{p}$ be the probability of heads.
$\boldsymbol{H_0: p = 0.5}$
$\boldsymbol{H_1: p > 0.5}$

$\boldsymbol{0.03 < 0.05}$ **so reject** $\boldsymbol{H_0}$**.**
Sufficient evidence to suggest the coin is biased.

Ordinarily we'd calculate the probability of seeing the observed number of heads 'or more extreme'. But this has already been done for us (i.e. the p-value), so we just need to compare this against the threshold.

Worked Example

549f: Conduct a one-tailed hypothesis test on a binomial model.

A random variable has distribution $X \sim B(20, p)$. A single observation $x = 14$ is taken from this distribution.

Using a 10% level of significance, test $H_0 : p = 0.49$ against $H_1 : p > 0.49$

Assuming H_0 true, $P(X \geq 14) =$

________ (3 decimal places)

which is ________ than 0.1

therefore we ________ H_0

Writing frame

VERSION 1: p-value

State hypotheses	H_0: $p = \ldots$ H_1: $p > \ldots.$ p< … p≠ …
Write parameters of the sampling distribution	X = number of …. p = probability of …. *Write as* $X \sim B(n, p)$
Calculate probability	**$p <$**: Just use Binomial CD with x given **$p >$**: use $P(X \geq x) = 1 - P(X \leq x - 1)$ **$p \neq$**: Do both of $P(X \leq x)$ and $P(X \geq x)$
Compare p to significance level	*accept Ho if* $p > sig\ level$ *accept Ho if* $p > \frac{sig\ level}{2}$
Accept/reject H_0	
Conclusion in context – *using wording from Q*	there is **insufficient/sufficient** evidence to accept/reject the claim (state it) at the__ % significance level

VERSION 2: critical value

State hypotheses	H_0: $p = \ldots$ H_1: $p > \ldots.$ p< … p≠ …
Write parameters of the sampling distribution	X = number of …. p = probability of …. *Write as* $X \sim B(n, p)$
Calculate probability	Go to Binomial CD LIST (enter either all possible x, or just some from top and/or bottom end)
Comparing	**$p <$**: Look for first x where p value below sig. level **$p >$**: - Look for first value where p greater than (1-sig level) - Then add 1 **$p \neq$**: Do both of these
Accept/reject H_0	**$p <$**: Accept if $x > x_c$ **$p >$**: Accept if $x < x_c$ **$p \neq$**: Accept if $x_{c_1} < x < x_{c_2}$
Conclusion in context – *using wording from Q*	there is **insufficient/sufficient** evidence to accept/reject the claim (state it) at the__ % significance level

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether the candidate is over-estimating his support.
The researcher asks 30 people whether they support the candidate or not. 6 people say they support the candidate.
Carry out a hypothesis test for the researcher.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether the candidate is over-estimating his support.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region for this test.
b) 6 people say they support the candidate. Comment on this observation in light of the critical region.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.
The researcher asks 30 people whether they support the candidate or not. 14 people say they support the candidate.
Carry out a hypothesis test for the researcher.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region for this test.

b) 14 people say they support the candidate. Comment on this observation in light of the critical region.

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
The manufacturing process is changed.
A sample of 200 lightbulbs is tested.
8 lightbulbs are found to be faulty.
The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.
Carry out this hypothesis test.

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
The manufacturing process is changed.
The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.
A sample of 200 lightbulbs is tested.

a) Find the critical region for this test.
b) 8 lightbulbs are found to be faulty. Comment on this observation in light of the critical region.

Worked Example

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful more than 99.8% of the time.
They test the benefits of the drug on 4500 patients.
The test is successful in 4498 cases.
Is there enough evidence, at the 1% significance level, to support the medical team's claim?

Worked Example

A medical team are testing the negative side effects of a new drug.
They claim that the drug gives negative side effects less than 0.2% of the time.
They test the drug on 4500 patients.
The drug has negative side effects in 5 patients.
Is there enough evidence, at the 1% significance level, to support the medical team's claim?

Worked Example

A medical team are testing the negative side effects of a new drug.
They claim that the drug gives negative side effects less than 0.2% of the time.
They test the drug on 4500 patients.

a) Find the critical region for this test at the 2% significance level.

b) The drug has negative side effects in 5 cases. Comment on this observation in light of the critical region.

7.4) Two-tailed tests

Notes

Worked Example

549i: Conduct a two-tailed hypothesis test on a binomial model.

A random variable has distribution $X \sim B(26, p)$. A single observation $x = 4$ is taken from this distribution.

Using a 5% level of significance, test $H_0 : p = 0.23$ against $H_1 : p \neq 0.23$

Assuming H_0 true, $P(X \leq 4) =$

(3 decimal places)

which is than 0.025

therefore we H_0

Writing frame

VERSION 1: p-value

State hypotheses	H_0: $p = \ldots$ H_1: $p > \ldots.$ p< … p≠ …
Write parameters of the sampling distribution	X = number of …. p = probability of …. *Write as* $X \sim B(n, p)$
Calculate probability	**$p <$**: Just use Binomial CD with x given **$p >$**: use $P(X \geq x) = 1 - P(X \leq x-1)$ **$p \neq$**: Do both of $P(X \leq x)$ and $P(X \geq x)$
Compare p to significance level	*accept Ho if* $p > sig\ level$ *accept Ho if* $p > \frac{sig\ level}{2}$
Accept/reject H_0	
Conclusion in context – *using wording from Q*	there is **<u>insufficient/sufficient</u>** evidence to accept/reject the claim (state it) at the___ % significance level

VERSION 2: critical value

State hypotheses	H_0: $p = \ldots$ H_1: $p > \ldots.$ p< … p≠ …
Write parameters of the sampling distribution	X = number of …. p = probability of …. *Write as* $X \sim B(n, p)$
Calculate probability	Go to Binomial CD LIST (enter either all possible x, or just some from top and/or bottom end)
Comparing	**$p <$**: Look for first x where p value below sig. level **$p >$**: - Look for first value where p greater than (1-sig level) - Then add 1 **$p \neq$**: Do both of these
Accept/reject H_0	**$p <$**: Accept if $x > x_c$ **$p >$**: Accept if $x < x_c$ **$p \neq$**: Accept if $x_{c_1} < x < x_{c_2}$
Conclusion in context – *using wording from Q*	there is **<u>insufficient/sufficient</u>** evidence to accept/reject the claim (state it) at the___ % significance level

Worked Example

Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$.
She rolls the dice 10 times and rolls a 4 five times.
Using a 5% significance level, test her belief.

Worked Example

Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$.

She rolls the dice 10 times.

a) Find the critical region(s) for this test at the 5% significance level.

b) A 4 is rolled five times. Comment on this observation in light of the critical region.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
The researcher asks 30 people whether they support the candidate or not. 1 person says they support the candidate.
Test, at the 1% significance level, whether the candidate's claim is true.

Worked Example

An election candidate believes he has the support of 30% of the residents in a particular town.
The researcher asks 30 people whether they support the candidate or not.

a) Find the critical region(s) for a test of the candidate's claim at the 1% significance level.
b) 16 people say they support the candidate. Comment on this observation in light of the critical region.

Worked Example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
The manufacturing process is changed.
A sample of 200 lightbulbs is tested.
7 lightbulbs are found to be faulty.
Test, at the 2% significance level, whether or not there has been a change in the proportion of faulty lightbulbs.

Worked Example

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful 99.8% of the time.
They test the benefits of the drug on 4500 patients.
The test is successful in 4498 cases.
Is the medical team's claim supported at the 1% significance level?

3. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Qu	Scheme	Marks	AO
3 (a)	Let N = the number of games Naasir wins $N \sim B(15, \frac{1}{3})$	M1	3.3
(i)	$P(N = 2) = 0.059946...$ **awrt 0.0599**	A1	1.1b
(ii)	$P(N > 5) = 1 - P(N \leqslant 5) = 0.38162...$ **awrt 0.382**	A1	1.1b
		(3)	
(b)	$H_0 : p = \frac{1}{3}$ $H_1 : p > \frac{1}{3}$	B1	2.5
	Let X = the number of games Naasir wins $X \sim B(32, \frac{1}{3})$	M1	3.3
	$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 0.03765$ (< 0.05)	A1	3.4
	[Significant result so reject H_0 (the null model) and conclude:] There is evidence to support Naasir's claim (o.e.)	A1	3.5a
		(4)	
		(7 marks)	

Summary of Key Points

Large data set

You will need access to the large data set and spreadsheet software to answer these questions.

1 The proportion of days with a recorded daily mean temperature greater than 15 °C in Leuchars between May 1987 and October 1987 was found to be 0.163 (3 s.f.).

A meteorologist wants to use a randomly chosen sample of 10 days to determine whether the probability of observing a daily mean temperature greater than 15 °C has increased significantly between 1987 and 2015.

- **a** Using a significance level of 5%, determine the critical region for this test.
- **b** Select a sample of 10 days from the 2015 data for Leuchars, and count the number of days with a mean temperature of greater than 15 °C.
- **c** Use your observation and your critical region to make a conclusion.

2 From the large data set, in Beijing in 1987, 23% of the days from May to October had a daily mean air temperature greater than 25 °C. Using a sample of size 10 from the data for daily mean air temperature in Beijing in 2015, test, at the 5% significance level, whether the proportion of days with a mean air temperature greater than 25 °C increased between 1987 and 2015.

Summary of key points

1 The null hypothesis, H_0, is the hypothesis that you assume to be correct.

2 The alternative hypothesis, H_1, tells us about the parameter if your assumption is shown to be wrong.

3 Hypothesis tests with alternative hypotheses in the form H_1: $p < \ldots$ and H_1: $p > \ldots$ are called one-tailed tests.

4 Hypothesis tests with an alternative hypothesis in the form H_1: $p \neq \ldots$ are called two-tailed tests.

5 A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.

6 The critical value is the first value to fall inside of the critical region.

7 The actual significance level of a hypothesis test is the probability of incorrectly rejecting the null hypothesis.

8 For a two-tailed test the critical region is split at either end of the distribution.

9 For a two-tailed test, halve the significance level at each end you are testing.