KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 13
Applied Mathematics
S1 6 Binomial Distribution

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

Prior knowledge check

1 Three coins are flipped. Calculate the probability that:

a all the coins land on tails

b all the coins land on heads

c exactly one of the coins lands on tails

d at least two coins land on heads.

← Chapter 5

2 Two fair dice are rolled. Calculate the probability that the sum of the scores on the dice is:

a five

b even

c odd

d a multiple of 3

e a prime number.

← Chapter 5

6.2) The binomial distribution

✐ We can model a random variable X with a **binomial distribution** $B(n, p)$ if

- there are a fixed number of trials, n,
- there are two possible outcomes: 'success' and 'failure',
- there is a fixed probability of success, p
- the trials are independent of each other

If $X \sim B(n, p)$ then:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Notation for using an 'off-the-shelf' distribution:

$X \sim B(n, p)$

"The random variable X…"

"has the distribution…"

"a binomial distribution…"

"with parameters n and p."

In our '6 out of 8 lefties' example

'Success' was **being a leftie**

$n = \mathbf{8}$

$p = \mathbf{0.1}$

$x = \mathbf{6}$

Notes

How to Identify When to Use a Binomial Distribution:

Back in 2010 Dr Frost was on holiday in Hawaii and visited the family of a friend. They noticed at the dinner table that out of the 8 people there, 6 were left-handed (including Dr Frost). One of them commented, "The chances of that must be very low".
"CHALLENGE ACCEPTED".

There are a **fixed number of 'trials'**. A trial represents each individual outcome, in this case each individual person's 'handedness'.

We are interested in **how many trials were successful**, i.e. how many left-handed people.

Each trial has **only two outcomes** (hence the 'bi'), 'success' or 'failure'.
Here 'success' is 'left-handed', i.e. the thing we're counting.

Notes

Quick Fire Qs

1. The probability that any given student is late to school is 0.05. I'm interested in knowing out of the 300 students attending the school, at least 10 will be late today.

No | Yes

2. I throw a fair dice 10 times. I want to know the probability I get 3 sixes, 4 fives and 2 ones.

No | Yes

3. Customers enter my shop at an average rate of 1 a minute. I want to know the probability that in the 10 minutes, I get no customers.

No | Yes

4. An Olympic archer hits the bullseye of a target with 30% probability. Out of 8 allowed shots, what's the probability that 4 hit the bullseye?

No | Yes

Fill in the gaps

a The probability of throwing a Heads on a biased coin is 0.7. What's the probability in 10 throws that I see 4 Heads?

Let X be...

$n =$

$p =$

$X \sim B(\quad)$

$x =$

$P(X = \square) = \binom{\square}{\square} \square^{\square} \square^{\square}$

b It rains on any given day with probability 0.35. What's the probability it rains for 5 days next week?

Let X be...

$n =$

$p =$

$X \sim B(\quad)$

$x =$

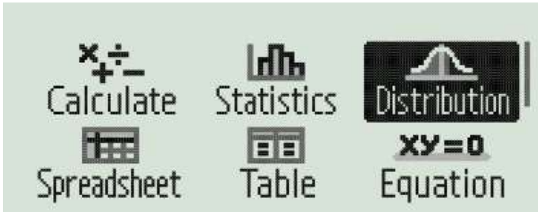
$P(X = \square) = \binom{\square}{\square} \square^{\square} \square^{\square}$

Using your calculator

The probability of throwing a Heads on a biased coin is 0.7. What's the probability in 10 throws that I see 4 Heads?

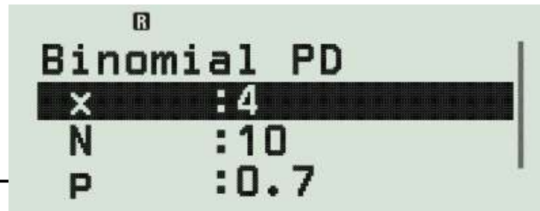
These are instructions for the
Casio fx-570/991CW

1 Use the arrows and OK to select Distribution.

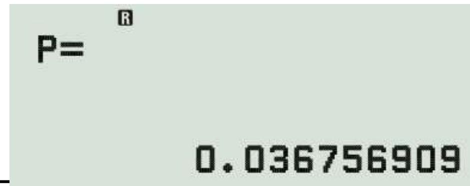


2 Choose Binomial PD (=Probability Distribution). We will use CD later. Choose Variable (List allows you to get multiple probabilities at once).

3 Enter your values, pressing EXE after each value and EXE again at the end.



4 Read off the value. Note that if you return to Calculate mode, this probability will be stored as ANS.



Assumptions with Binomial Distributions

We can model a random variable X with a **binomial distribution** $B(n, p)$ if

- there are a fixed number of trials, n,
- there are two possible outcomes: 'success' and 'failure',
- there is a fixed probability of success, p
- the trials are independent of each other

Worked Example

639c: **Calculate binomial probabilities in the form $P(X = x)$ (no context), and appreciate the relationship of this formula with the binomial expansion of $(p+q)^n$**

The random variable X has the distribution $X \sim B(20, 0.1)$

Find $P(X = 5)$.

T.90 6B: Qs 1-2, P.40 6.2: Qs 1-3

Worked Example

The probability of a lightbulb being faulty is 0.12. A random sample of 34 lightbulbs is taken from the production line.

a) Define a suitable distribution to model the number of faulty lightbulbs in this sample.

b) Find the probability that the sample contains fewer than 3 faulty lightbulbs.

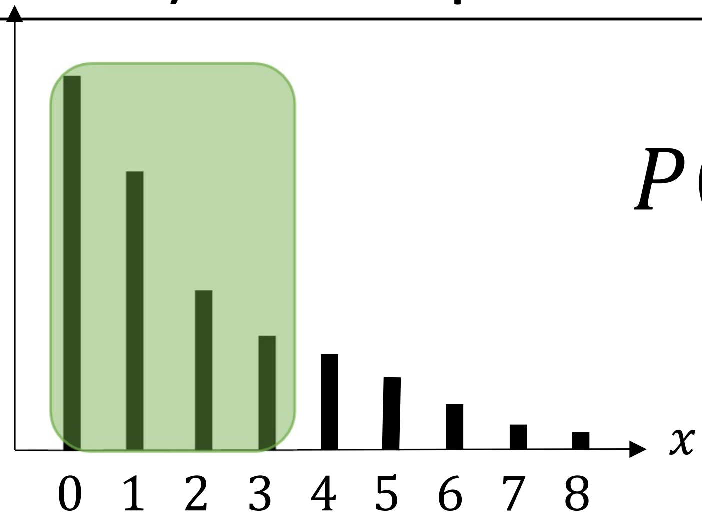
Worked Example

A company claims that a third of the lightbulbs sent to them are faulty.
To test this claim the number of faulty lightbulbs in a random sample of 100 is recorded.
Give two reasons why a binomial distribution may be a suitable model for the number of faulty lightbulbs in the sample.

T.90 6B: Qs 3+, P.40 6.2: Qs 4+

6.3) Cumulative probabilities

$X \sim B(8, 0.1)$



$P(X \leq 3)$

We might want to calculate the running total of the probability **up to** a certain number of successes.
$P(X \leq k)$ is known as a **cumulative probability function** (CF on calculators).

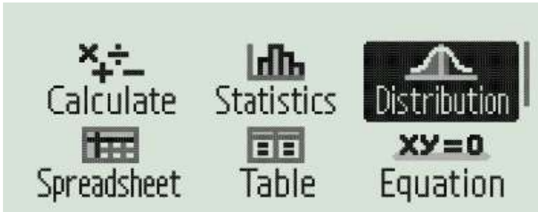
There is no easy way to calculate this directly other than adding the probabilities of each outcome:

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$
$$= 0.9^8 + \binom{8}{1} 0.1^1 0.9^7 + \binom{8}{2} 0.1^2 0.9^6 + \binom{8}{3} 0.1^3 0.9^5$$

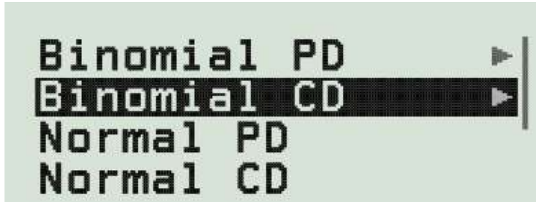
Calculating Cumulative Probabilities

These are instructions for the
Casio fx-570/991CW

1 Use the arrows and OK to select Distribution.



2 Choose Binomial CD (=Cumulative Distribution). Choose Variable (List allows you to get multiple probabilities at once).



3 Enter your values, pressing EXE after each value and EXE again at the end.

4 Read off the value.

Notes

Worked Example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp,

a) $P(X = 3)$
b) $P(X \leq 5)$
c) $P(X < 5)$
d) $P(X \geq 7)$
e) $P(X > 7)$
f) $P(4 < X < 9)$
g) $P(4 \leq X \leq 9)$
h) $P(4 \leq X < 9)$
i) $P(4 < X \leq 9)$

Worked Example

639f: **Calculate cumulative binomial probabilities (no context).**

A random variable X has the distribution $X \sim \mathrm{B}\left(15, \frac{5}{7}\right)$

Find $\mathrm{P}(X < 7)$.

T.93 6C: Qs 1-4, P.42 6.3: Qs 1-3

Worked Example

639g: **Calculate cumulative binomial probabilities in context.**

A game is designed to be a game of chance so that the probability of winning each game is 0.29

Thomas and his friends play the game 35 times.

Find the probability that Thomas wins less than 16 games.

Worked Example

639i: Use a binomial probability to calculate another binomial probability.

The probability that a screw is damaged is 0.11

There are 29 screws in a pack.

a. Find the probability that at least 5 screws are damaged in a pack.

A company buys 44 packs of 29 screws.

b. Find the probability that there will be at least 5 damaged screws in 6 of the 44 packs.

a.

b.

Worked Example

639k: **Determine the number of trials given the probability of at least one success.**

The probability that Hugo will buy an item online in any week is 0.29.

The probability that Hugo buy an item online at least once in a period of n weeks is greater than 0.95.

Find the smallest possible value of n.

Worked Example

The random variable $X \sim B(40, 0.3)$. Find:

a) The largest value of p such that $P(X \leq p) < 0.05$

b) The largest value of r such that $P(X < r) < 0.1$

c) The smallest value of s such that $P(X \geq s) < 0.15$

d) The smallest value of t such that $P(X > t) < 0.2$

Worked Example

The random variable $X \sim B(40, 0.3)$. Find the largest value of p such that $P(X \leq p) < 0.05$

Worked Example

The random variable $X \sim B(40, 0.3)$. Find the largest value of r such that $P(X < r) < 0.1$

Worked Example

The random variable $X \sim B(40, 0.3)$. Find the smallest value of s such that $P(X \geq s) < 0.15$

Worked Example

The random variable $X \sim B(40, 0.3)$. Find the smallest value of t such that $P(X > t) < 0.2$

Worked Example

Each day a person plays 10 games of chess. The probability that they win each game is 0.7.
They consider it a successful day if they win at least 8 games.
Calculate the probability that in a seven-day week, they have at least five successful days.

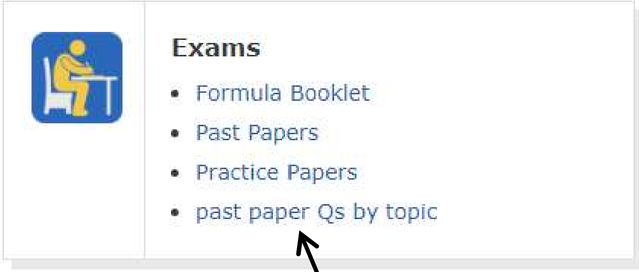
T.93 6C: Qs 5+, P.42 6.3: Qs 4+

[EdExcel Statistics 2 January 2009]

5. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.
(2)

(b) Find the probability that there are at least 2 defective components in the box.
(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

5	(a)	X represents the number of defective components.		
		$P(X=1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1	(2)
	(b)	$P(X \geq 2) = 1 - P(X \leq 1)$	M1	
		$= 1 - (q)^{10} - (a)$	M1/A1	
		$= 0.0043$	A1	(3)

Summary of Key Points

Summary of key points

1 A **probability distribution** fully describes the probability of any outcome in the sample space.

2 The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.

3 You can model X with a **binomial distribution**, **B(n, p)**, if:

- there are a fixed number of trials, n
- there are two possible outcomes (success or failure)
- there is a fixed probabillity of success, p
- the trials are independent of each other.

4 If a random variable X has the binomial distribution B(n, p) then its probability mass function is given by

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

T.94 mixed ex., P.43 BSG