KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13
Pure Mathematics
P2 3 Sequences and Series Booklet

HGS Maths

Dr Frost Course

Name: ______________________________________

Class: ______________________________________

Contents

Prior knowledge check

Prior knowledge check

1 Write down the next three terms of each sequence.

a 2, 7, 12, 17 **b** 11, 8, 5, 2

c −15, −9, −3, 3 **d** 3, 6, 12, 24

e $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ **f** $-\frac{1}{16}, \frac{1}{4}, -1, 4$

← GCSE Mathematics

2 Solve, giving your answers to 3 s.f.:

a $2^x = 50$

b $0.2^x = 0.0035$

c $4 \times 3^x = 78\,732$

← Year 1, Section 14.6

3.1) Arithmetic sequences

Types of sequences:

common difference d

+3 +3 +3

$2, 5, 8, 11, 14, \ldots$

This is a:

Arithmetic Sequence

✎ An arithmetic sequence is one which has a common difference between terms.

common ratio r

$\times 2$ $\times 2$ $\times 2$

$3, 6, 12, 24, 48, \ldots$

Geometric Sequence
(We will explore these later in the chapter)

$1, 1, 2, 3, 5, 8, \ldots$

This is the **Fibonacci Sequence**. The terms follow a **recurrence relation** because each term can be generated using the previous ones. We will encounter recurrence relations later in the chapter.

THE SUM:

We use a to denote the **first term**. d is the **difference** between terms, and n is the **position** of the term we're interested in. Therefore:

1st Term	2nd Term	3rd Term	$\cdots$	nth term
a	$a+d$	$a+2d$	$\cdots$	$a+(n-1)d$

✎ **nth term of arithmetic sequence:**

$$u_n = a + (n-1)d$$

Notations

u_n — The n^{th} **term**. So u_3 would refer to the 3rd term.

n — The **position** of the term in the sequence.

$n = 3$ $\qquad$ $u_3 = 8$

$$2, 5, 8, 11, 14, \ldots$$

Notes

Worked Example

The nth term of an arithmetic sequence is
$u_n = 35 - 3n$.

a) Write down the first 3 terms of the sequence.

b) Find the first term in the sequence that is negative.

Worked Example

Find the nth term of each arithmetic sequence.

a) $-6, 2, 10, 18, 26, \ldots$

b) $788, 785, 782, 779, 886, \ldots$

T.61 3A: Qs 1-3, P.17 3.1: Qs1-3

Worked Example

Is 100 in the sequence:

$$-3, 4, 11, 18, \ldots ?$$

Is 10 in the sequence:

$$127\ 118, 109, 100, \ldots ?$$

Worked Example

The first five terms of each sequence are shown. Find two numbers which are in both sequences.

$$3, 10, 17, 24, 31, \ldots$$

$$-4, -1, 2, 5, 8, \ldots$$

Worked Example

Find the n^{th} term of the sequence

$$\frac{1}{3}, \frac{4}{5}, \frac{7}{7}, \frac{10}{9}, \ldots$$

Worked Example

573h: **Determine the value of a given term in an arithmetic sequence given the position and value of two other terms in the sequence.**

The 5th term of an arithmetic sequence is 37 and the 9th term is 61.

Find the **first term** and the **common difference.**

First Term $=$ ☐

Common Difference $=$ ☐

Worked Example

A sequence is generated by the formula
$u_n = an + b$ where a and b are constants to be found.
Given that $u_5 = 17$ and $u_9 = 33$, find the values of the constants a and b.

Worked Example

For which values of x would the expression $-2, 4x^2$ and $17x$ form the first three terms of an arithmetic sequence?

Worked Example

An arithmetic sequence has first term k^2 and common difference k, where $k < 0$. The third term of the sequence is 24. Find the value of k

T.61 3A: Qs 4+, P.17 3.1: Qs 4+

3.2) Arithmetic series

A **series** is a **sum** of terms in a sequence.
You will encounter 'series' in many places in A Level:

Arithmetic Series (this chapter!)

Sum of terms in an arithmetic sequence.

$$2 + 5 + 8 + 11$$

Binomial Series (Later in Year 2)

You did Binomial expansions in Year 1. But when the power is negative or fractional, we end up with an infinite series.

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{6}x^3 - \cdots$$

Taylor Series (Further Maths)

Expressing a function as an infinite series, consisting of polynomial terms.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Extra Notes: A 'series' usually refers to an infinite sum of terms in a sequence. If we were just summing some finite number of them, we call this a partial sum of the series.

e.g. The '*Harmonic Series*' is $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$, which is infinitely many terms. But a we could get a partial sum, e.g. $H_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$

However, in this syllabus, the term 'series' is used to mean either a finite or infinite addition of terms.

Terminology: A '*power series*' is an infinite polynomial with increasing powers of x. There is also a chapter on power series in the Further Stats module.

Arithmetic Series

n^{th} term

$$u_n = a + (n-1)d$$

✏ **Sum of first n terms**

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Alternative Formula

$$a + (a+d) + \cdots + L$$

Suppose last term was L.
We saw earlier that each opposite pair of terms (first and last, second and second last, etc.) added to the same total, in this case $a + L$.

There are $\frac{n}{2}$ pairs, therefore:

✏

$$S_n = \frac{n}{2}(a + L)$$

Worked Example

574b: Find the sum of the first n terms (the nth partial sum) of an arithmetic series.

Find the sum of the first 34 terms of the series that starts with:

$13 + 23 + 33 + 43 + \ldots$

Worked Example

574c: Find the sum of the first n terms of an arithmetic series used to model a given context.

Isaac started work 12 years ago. In year 1 Isaac's annual salary was £18100. Isaac's annual salary increased by £1200 each year, so that her annual salary in year 2 was £19300, in year 3 it was £20500 and so on, forming an arithmetic sequence.

Calculate the total amount that Isaac has earned in the 12 years.

Worked Example

Find the sum of the first 50 terms the sequence which begins:

$$p, 3p, 5p, 7p, 9p, \ldots$$

T.64 3B: Qs 1-5, P.18 3.2: Qs 1-3

Worked Example

574h: **Find the least number of terms for an arithmetic series to exceed a given value.**

Find the least number of terms for the sum of
$3 + 14 + 25 + 36 + \ldots$ to exceed 4820.

Worked Example

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £$(P + 900)$
Salary increases by £(T) each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £P
Salary increases by £$2T$ each year, forming an arithmetic sequence.

For the 10-year period, the total earned is the same for both salary schemes.

a) Find the value of T

b) For this value of T, the salary in Year 10 under Scheme 1 is £25890. Find the value of P

Worked Example

Prove that the sum of the first 200 natural numbers is 20100

Worked Example
Prove that the sum of the first n even numbers is $n^2 + n$

Worked Example

574m: Determine the value of a given term in an arithmetic series using the value of another term and a sum of terms.

The 17th term of an arithmetic sequence is -162.

The **sum** of the first 37 terms is -6512.

Find the value of the 25th term.

Worked Example

An arithmetic series is given by $(k+1)+(2k+5)+(3k+9)+\cdots+217$
Given that the sum of the series is 2250, find the value of k

Worked Example

The common difference of an arithmetic sequence is 4. The sum of the first 60 terms of this sequence is 7380. Find the first term.

Worked Example

The common difference of an arithmetic sequence is 4. The sum of the first 60 terms of this sequence is -240. Find the first term.

Worked Example

The first term of an arithmetic sequence is 3. The sum of the first 50 terms is 2600. Work out the common difference of the sequence.

Worked Example

The eighth term of an arithmetic sequence is 11. The fifth term of the same arithmetic sequence is 2.
Find the sum of the first 50 terms of this arithmetic sequence.

T.64 3B: Qs 6+, P.18 3.2: Qs 4+

3.3) Geometric sequences

common difference d

+3 +3 +3

2, 5, 8, 11, 14, ...

This is a:

Arithmetic Sequence

common ratio r

$\times 2$ $\times 2$ $\times 2$

3, 6, 12, 24, 48, ...

Geometric Sequence

✎ A geometric sequence is one in which there is a **common ratio** between terms.

Notes

Short exercise

Identify the common ratio r:

1. $1, 2, 4, 8, 16, 32, \ldots$
2. $27, 18, 12, 8, \ldots$
3. $10, 5, 2.5, 1.25, \ldots$
4. $5, -5, 5, -5, 5, -5, \ldots$
5. $x, -2x^2, 4x^3$
6. $1, p, p^2, p^3, \ldots$
7. $4, -1, 0.25, -0.0625, \ldots$

Worked Example

575a: **Determine a term of a geometric sequence given the first term and common ratio.**

A geometric progression has first term -2 and common ratio -4.

Find the value of the 6th term.

575b: **Determine a term of a geometric sequence used to model a given context.**

Beth has planned her physics revision to gradually increase by 40 percent each session. Her first session is 5 minutes long.

Find the length of session 8.
Give your answer to the nearest minute.

Worked Example

575c: **Determine the first term and common ratio of a geometric sequence given two terms in the sequence.**

The 6th term of a geometric sequence is 128 and the 11th term is 4096.

Find the **first term** and the **common ratio**.

Worked Example

575d: Determine the value of a term in a geometric sequence given two terms in the sequence.

The 5th term of a geometric progression is 112 and the 10th term is -3584.

Find the value of the 8th term.

Worked Example

The numbers $2, x$ and $x + 12$ form the first three terms of a positive geometric sequence. Find:

a) The value of x.

b) The 20^{th} term in the sequence.

T.69 3C: Qs 1-6, P.120 3.3: Qs 1-3

Worked Example

What is the first term in the geometric progression $2, 6, 18, 54, \ldots$ to exceed 1 million?

Worked Example

The second, third and fourth term of a geometric sequence are the following:

$$x, \qquad x+4, \qquad 10x-2$$

Given the common ratio is positive, find the common ratio and the first term of the sequence

Worked Example

The second, third and fourth term of a geometric sequence are the following:

$$x, \qquad x-8, \qquad 10x-2$$

Given the common ratio is negative, find the common ratio and the first term of the sequence

Worked Example

The first three terms of a geometric sequence are:

$$16, 144, 1296$$

Determine whether 944784 is in the sequence

T.69 3C: Qs 7+, P.20 3.3: Qs 4+

3.4) Geometric series

Sum of the first n terms of a geometric **series**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Can we prove this?

Worked Example

578b: Find the sum of the first n terms (the nth partial sum) of a geometric series.

Find the sum of the first 12 terms of the series that starts with:

$$6 - 15 + \frac{75}{2} - \frac{375}{4} + \dots$$

Give your answer correct to 1 decimal place where applicable.

Geometric series: $S_n = \frac{a(1-r^n)}{1-r}$

Worked Example

578e: **Determine the least number of terms for a geometric series to exceed or equal a given value.**

Find the least number of terms for the sum of $6 + 39 + 253.5 + 1647.75 + \ldots$ to exceed 54400.

Geometric series: $S_n = \frac{a(1-r^n)}{1-r}$

T.72 3D: Qs 1-6, P.21 3.4: Qs 1-4

Worked Example

A geometric series has first term a and common ratio r.
The sum of the first two terms of the series is 9.
The sum of the first four terms of the series is 45.
Find the two possible geometric sequences.

T.72 3D: Qs 7+, P.21 3.4: Qs 5+

3.5) Sum to infinity

Divergent vs Convergent:

$$1 + 2 + 4 + 8 + 16 + \ldots$$

This is **divergent** – the sum of the values tends towards infinity.

$$1 - 2 + 3 - 4 + 5 - 6 + \ldots$$

This is **divergent** – the running total alternates either side of 0, but gradually gets further away from 0.

$$1 + 0.5 + 0.25 + 0.125 + \ldots$$

This is **convergent** – the sum of the values tends towards a fixed value, in this case 2.

Definitely NOT in the A Level syllabus, and just for fun...

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

This is **divergent** . This is known as the Harmonic Series

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

This is **convergent** . This is known as the Basel Problem, and the value is $\pi^2/6$.

Sum to infinity

$1 + 0.5 + 0.25 + 0.125 + \ldots$ ← Why did this infinite sum converge (to 2)...

$1 + 2 + 4 + 8 + 16 + \ldots$ ← ...but this diverge to infinity?

- The infinite series will converge provided that $-1 < r < 1$ (which can be written as $|r| < 1$), because the terms will get smaller.

- Provided that $|r| < 1$, what happens to r^n as $n \to \infty$?
  For example $\left(\frac{1}{2}\right)^{100000}$ is very close to 0.
  We can see that as $n \to \infty, r^n \to 0$.

✎ A geometric series is convergent if $|r| < 1$.

- How therefore can we use the $S_n = \frac{a(1-r^n)}{1-r}$ formula to find the sum to infinity, i.e. S_∞?

✎ For a <u>convergent</u> geometric series,

$$S_\infty = \frac{a}{1-r}$$

Notes

Quickfire Examples

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$	$\boldsymbol{a} =$	$\boldsymbol{r} =$	$\boldsymbol{S_\infty} =$
$27, -9, 3, -1, \dots$	$\boldsymbol{a} =$	$\boldsymbol{r} =$	$\boldsymbol{S_\infty} =$
$p, p^2, p^3, p^4, \dots$ $where\ \ -1 < p < 1$	$\boldsymbol{a} =$	$\boldsymbol{r} =$	$\boldsymbol{S_\infty} =$
$p, 1, \frac{1}{p}, \frac{1}{p^2}, \dots$	$\boldsymbol{a} =$	$\boldsymbol{r} =$	$\boldsymbol{S_\infty} =$

Worked Example

578f: **Determine the sum to infinity of a geometric series.**

Find the sum to infinity of the series
$14 + 2.8 + 0.56 + 0.112 + \ldots$

Give your answer correct to 1 decimal place where applicable.

Worked Example

Find the sum to infinity of the series:

$a)\ p + p^3 + p^5 + p^7 + p^9 + \cdots$

$b)\ k + \frac{1}{k} + \frac{1}{k^3} + \frac{1}{k^5} + \frac{1}{k^7} + \cdots$

Worked Example

578g: **Determine the first term or common ratio of a series given its sum to infinity.**

A geometric series has $a = 15$ and $S_\infty = 30$

Find the common ratio of the series.

Geometric series: $S_\infty = \frac{a}{1-r}$ *for* $|r| < 1$

T.75 3E: Qs 1-5, P.22 3.5: Qs 1-5

Worked Example

The third term of a geometric series is 1.5 and the eighth term is 0.046875.

a) Show that this series is convergent.

b) Find the sum to infinity of this series.

Worked Example

For a geometric series with first term a and common ratio r, $S_4 = 12.75$ and $S_\infty = 12.8$.
a) Find the possible values of r.
b) Given that the terms in the series alternate between positive and negative values, find the value of a

T.75 3E: Qs 6+, P.22 3.5: Qs 6+

3.6) Sigma notation

What does each bit of this expression mean?

$$\sum_{r=1}^{5}(2r+1)$$

The Greek letter, capital sigma, means 'sum'.

The numbers top and bottom tells us what r varies between. It goes up by 1 each time.

We work out this expression for each value of r (between 1 and 5), and add them together.

$$\underset{r=1}{= 3} \quad \underset{r=2}{+5} \quad \underset{r=3}{+7} \quad \underset{r=4}{+9} \quad \underset{r=5}{+11} \quad = 35$$

If the expression being summed (in this case $2r + 1$) is **linear**, we get an **arithmetic series**. We can therefore apply our usual approach of establishing a, d and n before applying the S_n formula.

"Use of Technology" :
The Classwiz has a Σ button.

Try and use it to find:

$$\sum_{k=5}^{12} 2 \times 3^k$$

Notation Exercise

	First few terms?	Values of a, n, d or r?	Final result?
$\sum_{n=1}^{7} 3n$			
$\sum_{k=5}^{15} (10 - 2k)$			
$\sum_{k=1}^{12} 5 \times 3^{k-1}$			
$\sum_{k=5}^{12} 5 \times 3^{k-1}$			

Notes

Worked Example

Write in sigma notation:

$8 + 13 + 18 + 23 + 28 + 33 + 38 + 43 + 48 + 53$

$-11 + -13 + -15 + -17 + -19 + -21 + -23$

T.78 3F: Qs 1-3, P.23 3.6: Qs 1

Worked Example

Evaluate:

$$\sum_{n=9}^{30}(2 + 7n)$$

Worked Example

Given that

$$\sum_{r=1}^{k} 3 \times 2^r = 12282$$

Find the value of k

Worked Example

A convergent geometric series is given by

$$1 + 2x + 4x^2 + 8x^3$$

a) Find the range of possible values of x

b) Given that

$$\sum_{r=1}^{\infty} (2x)^{r-1} = 2$$

find the value of x

T.78 3F: Qs 4+, P.23 3.6: Qs 2+

3.7) Recurrence relations

$$u_n = 2n^2 + 3$$

This is an example of a position-to-term sequence, because each term is based on the position n.

$$u_{n+1} = 2u_n + 4$$
$$u_1 = 3$$

We need the first term because the recurrence relation alone is not enough to know what number the sequence starts at.

But a term might be defined based on previous terms.
If u_n refers to the current term, u_{n+1} refers to the next term.
So the example in words says "the next term is twice the previous term + 4"

This is known as a term-to-term sequence, or more formally as a **recurrence relation**, as the sequence 'recursively' refers to itself.

Increasing, decreasing and periodic sequences

A sequence is **strictly increasing** if the terms are always increasing, i.e. $u_{n+1} > u_n$ for all $n \in \mathbb{N}$.
e.g. $1, 2, 4, 8, 16, \ldots$

Textbook Error: It uses the term 'increasing' when it means 'strictly increasing'.

Similarly a sequence is **strictly decreasing** if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$

A sequence is **periodic** if the terms repeat in a cycle. The **order** k of a sequence is **how often it repeats**, i.e. $u_{n+k} = u_n$ for all n.
e.g. 2, 3, 0, 2, 3, 0, 2, 3, 0, 2, … is periodic and has order 3.

Notes

Worked Example

Find the first five terms of the sequence:

$$u_{n+1} = 2u_n + 3, u_1 = 3$$

Worked Example

582b: **Determine the order of a periodic sequence.**

A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2$$

$$a_{n+1} = \frac{4a_n - 4}{3a_n - 2}$$

State the order of this periodic sequence.

T.80 3G: Qs 1-3, P.25 3.7: Qs 1-3

Worked Example

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 4$$

$$u_{n+1} = pu_n + q, n \geq 1$$

Given that $u_2 = 5$ and $u_3 = 7$, find the values of p and q

Worked Example

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$

$$a_{n+1} = (a_n)^2 - ka_n, n \geq 1$$

where k is a constant.
Given that $a_3 = 1$, find the value of:

$$\sum_{r=1}^{100} a_n$$

Worked Example

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2$$

$$a_{n+1} = (a_n)^2 - 2, n \geq 2$$

where $p > 0$

a) Find a_3

b) Given that $a_2 = 2$, find the value of p

c) Find the sum of the first 100 terms

d) Find a_{199}

Worked Example

For each sequence:

i) State whether the sequence is increasing, decreasing or periodic.

ii) If the sequence is periodic, write down its order.

a) $u_{n+1} = u_n - 3, \; u_1 = 7$

b) $u_{n+1} = (u_n)^3, \quad u_1 = 2$

c) $u_{n+1} = \cos(45n°)$

T.80 3G: Qs 4+, P.25 3.7: Qs 4+

3.8) Modelling with series

Notes

Worked Example

Bruce starts a new company. In year 1 his profits will be £10 000. He predicts his profits to increase by £5000 each year, so that his profits in year 2 are modelled to be £15 000, in year 3, £20 000 and so on. He predicts this will continue until he reaches annual profits of £50 000. He then models his annual profits to remain at £50 000.

a) Calculate the profits for Bruce's business in the first 20 years.

b) State one reason why this may not be a suitable model.

c) Bruce's financial advisor says the yearly profits are likely to increase by 2.5% per annum. Using this model, calculate the profits for Bruce's business in the first 20 years.

T.80 3H: all Qs

Worked Example

A company predicts a yearly profit of £210 000 in the year 2031. The company predicts that the yearly profit will rise each year by 4%.

a) Find the predicted profit in the year 2035
b) Find the first year in which the yearly predicted profit exceeds £300 000
c) Find the total predicted profit for the years 2031 to 2042 inclusive, giving your answer to the nearest pound.

T.84 3I: all Qs, P.26 3.8 all Qs

Extract from Formulae book

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

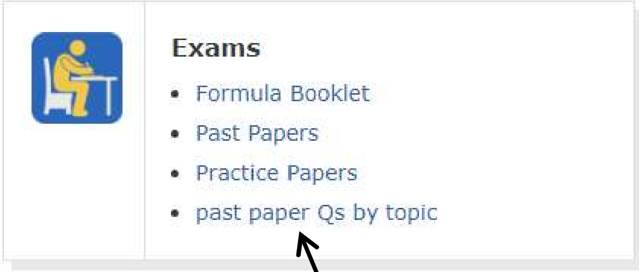
10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

10	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	1.2
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	1.2
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
			(4 marks)

Summary of Key Points

Summary of key points

1. In an **arithmetic sequence**, the difference between consecutive terms is constant.

2. The formula for the nth term of an arithmetic sequence is $u_n = a + (n - 1)d$, where a is the first term and d is the common difference.

3. An arithmetic series is the sum of the terms of an arithmetic sequence.

   The sum of the first n terms of an arithmetic series is given by $S_n = \frac{n}{2}(2a + (n - 1)d)$, where where a is the first term and d is the common difference.

   You can also write this formula as $S_n = \frac{n}{2}(a + l)$, where l is the last term.

4. A **geometric sequence** has a **common ratio** between consecutive terms.

5. The formula for the nth term of a geometric sequence is $u_n = ar^{n-1}$, where a is the first term and r is the common ratio.

6. The sum of the first n terms of a geometric series is given by

   $S_n = \frac{a(1 - r^n)}{1 - r}, r \neq 1$ or $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$

   where a is the first term and r is the common ratio.

7. A geometric series is convergent if and only if $|r| < 1$, where r is the common ratio.

   The **sum to infinity** of a convergent geometric series is given by $S_\infty = \frac{a}{1 - r}$

8. The Greek capital letter 'sigma' is used to signify a sum. You write it as $\sum$. You write limits on the top and bottom to show which terms you are summing.

9. A recurrence relation of the form $u_{n+1} = f(u_n)$ defines each term of a sequence as a function of the previous term.

10. A sequence is **increasing** if $u_{n+1} > u_n$ for all $n \in \mathbb{N}$.

    A sequence is **decreasing** if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$.

    A sequence is **periodic** if the terms repeat in a cycle. For a periodic sequence there is an integer k such that $u_{n+k} = u_n$ for all $n \in \mathbb{N}$. The value k is called the **order** of the sequence.

T.86 mixed ex, P.28 BSG