KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12
Pure Mathematics
P2 1.2-1.5 Algebraic Methods

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

Prior knowledge check

1 Factorise each polynomial:

a $x^2 - 6x + 5$ **b** $x^2 - 16$

c $9x^2 - 25$

← Year 1, Section 1.3

2 Simplify fully the following algebraic fractions.

a $\dfrac{x^2 - 9}{x^2 + 9x + 18}$ **b** $\dfrac{2x^2 + 5x - 12}{6x^2 - 7x - 3}$

c $\dfrac{x^2 - x - 30}{-x^2 + 3x + 18}$

← Year 1, Section 7.1

387d: **Simplify algebraic fractions where factorisation of a simple quadratic on the numerator or denominator is required.**

Simplify

$$\frac{x-3}{x^2-4x+3}$$

387e: **Simplify algebraic fractions where factorisation of a simple quadratic on both the numerator and denominator is required.**

Simplify

$$\frac{x^2-7x+10}{x^2-6x+5}$$

1.2) Algebraic fractions

Notes

Worked Example

Simplify:

$$\frac{2x^3 - 5x^2 - 3x}{2x - 6}$$

Worked Example

Simplify:

$$\frac{2x^3 + 5x^2 - 3x}{4x^2 - 1}$$

Worked Example

Simplify:

$$\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$$

Worked Example

Simplify:

$$\frac{2x^2 - 5x - 3}{3x^2 - 11x + 6}$$

Worked Example

Simplify:

$$\frac{3x^2 + 2x - 8}{6x^2 - 23x - 35} \times \frac{7x^2 - 29x - 30}{4x^2 + 5x - 6}$$

Worked Example

Simplify:

$$\frac{x^2 - x}{y^2} \div \frac{x^2 + x - 2}{y^5}$$

Worked Example

Simplify:

$$\frac{3x^2 - 10x - 8}{6x^2 + 37x - 35} \div \frac{x^2 - 3x - 4}{x^2 - 49}$$

Worked Example

Write as a single fraction:

$$3 - (x - 4) \div \frac{x^2 - 16}{x - 5}$$

Worked Example

Write as a single fraction:

$$\frac{3}{5x+2}-\frac{2}{x-3}$$

Worked Example

390a: Add or subtract algebraic fractions where non-quadratic factorising is required (changing one denominator).

Simplify

$$\frac{2}{x-1} - \frac{10}{x^2+3x-4}$$

Leave the denominator in factorised form when applicable.

Worked Example

390c: Add or subtract algebraic fractions where factorising is required (changing two denominators).

Simplify

$$\frac{5}{y^2+18y+80}+\frac{9}{y^2+8y-20}$$

Leave the denominator in factorised form when applicable.

Worked Example

390e: Solve an equation with fractions requiring prior factorisation of one of the fractions.

Solve the following equation:

$$\frac{4}{2x^2+x-15}-\frac{1}{2x-5}=3$$

Give your answer in exact form.

T.8: 1C Qs 3+ , P.2 : 1.2 Qs 3+

1.3) Partial fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x-2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$$

Notation reminder: ≡ means 'equivalent/identical to', and indicates that both sides are equal for **all** values of x.

Notes

Worked Example

559a: Decompose a rational function into partial fractions: $\frac{ax+b}{(cx\pm d)(ex\pm f)}$

Express

$$\frac{5x-1}{(3x-2)(2x+1)}$$

in the form

$$\frac{A}{3x-2}+\frac{B}{2x+1}$$

T.11: 1D Qs 1-3 , P.2 : 1.3 Qs 1,2

Worked Example

Given that $\frac{-6x^2-5x+2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$, find the values of the constants A, B, C.

Worked Example

Express as partial fractions:

$$\frac{6x^2 + 14x - 12}{x^3 - 4x}$$

T.11: 1D Qs 4+ , P.2 : 1.3 Qs 3+

1.4) Repeated factors

Suppose we wished to express $\frac{2x+1}{(x+1)^2}$ as $\frac{A}{x+1} + \frac{B}{x+1}$. What's the problem?

Because the denominators are the same, we'd get $\frac{A+B}{x+1}$. There's no constant values of A and B we can choose such that $\frac{2x+1}{(x+1)^2} \equiv \frac{A+B}{x+1}$ because the denominators will still be different.

Notes

Worked Example

Express as partial fractions:

$$\frac{4x}{(x-4)^2}$$

Worked Example

559d: Decompose a rational function into partial fractions: $\frac{ax \pm b}{(cx \pm d)(ex \pm f)^2}$

Express

$$\frac{4z+5}{(z-1)(z-4)^2}$$

in the form

$$\frac{A}{z-1}+\frac{B}{z-4}+\frac{C}{(z-4)^2}$$

Worked Example

Express as partial fractions:

$$\frac{11x^2 - 22x + 9}{(x-1)^2(2x-1)}$$

Worked Example

Express as partial fractions:

$$\frac{5x^2 + 4x + 1}{x^3 + x^2}$$

Worked Example

Express as partial fractions:

$$\frac{15x^2 - 5x + 2}{9x^3 - 6x^2 + x}$$

T.13: 1E all Qs , P.3 : 1.4 all Qs

1.5) Algebraic division

In Pure Year 1, we saw that the '**degree**' of a polynomial is the highest power, e.g. a quadratic has degree 2.

An algebraic fraction is **improper** if the degree of the numerator is **at least** the degree of the denominator.

$$\frac{x^2 - 3}{x + 2} \qquad \frac{x+1}{x-1} \qquad \frac{x^3 - x^2 + 3}{x^2 - x}$$

A partial fraction is still improper if the degree is the same top and bottom.

Questions might take one of two forms:

- Do the division to express as a quotient and a remainder, e.g. $\frac{x+1}{x-1} \rightarrow 1 + \frac{2}{x-1}$
- Express as partial fractions, e.g. $\frac{x^2+x}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$

Reducing to Quotient and Remainder

$\frac{7}{3} = 2\frac{1}{3}$ because $\frac{7}{3} = \underbrace{6 \times \frac{1}{3}}_{=2} + 1$

(3 ← divisor (d)); (2 ← quotient (q)); (1 ← remainder (r))

Similarly, for a improper algebraic fraction:

$$\frac{F(x)}{d} = Q(x) + \frac{r}{d}$$

i.e. $\frac{6x+5}{2} = 3x+2 + \frac{1}{2}$

($6x+5$ ← $F(x)$; 2 ← d; $3x+2$ ← $Q(x)$ "quotient"; 1 ← r; 2 ← d)

because

	$3x + 2$
2	$6x + 4$

$r = 1$ to get back to $6x+5$

generally, method is:

	$Q(x)$
d	aim for $F(x)$

$r = ...$ ← remainder

e.g. $\frac{x^2+5x+9}{x+2} = x+3 + \frac{3}{x+2}$

Model:

	x	$+3$
x	x^2	$+3x$
$+2$	$+2x$	$+6$

$r = +3$

Notes

Worked Example

497a: Determine a factor from algebraic division of a quadratic or cubic expression, given another factoı

Given that

$$\frac{3x^2 - 22x + 35}{x - 5} = Ax + B$$

Use algebraic long division to work out the values of the constants A and B.

Worked Example

Find the values of A, B and C:

$$\frac{x^2 + 9x - 5}{x + 3} = Ax + B + \frac{C}{x + 3}$$

Worked Example

Find the values of A, B and C:

$$\frac{x^2 - 5x + 9}{x - 2} = Ax + B + \frac{C}{x - 2}$$

Your Turn

Find the values of A, B and C:

$$\frac{x^2 + 5x - 9}{x - 3} = Ax + B + \frac{C}{x - 3}$$

$$A = 1, B = 8, C = 15$$

Worked Example

Simplify:

$$\frac{x^4 - 81}{x + 3}$$

T.16: 1F Qs all, P.4 : 1.5 Qs1-4

Worked Example

559b: Decompose a rational function into partial fractions: $\frac{ax^2+bx+c}{(dx\pm e)(fx\pm g)}$

Express $\dfrac{2x^2+4x+4}{(x-1)(x+4)}$

in the form $A+\dfrac{B}{x-1}+\dfrac{C}{x+4}$

Worked Example

Split into partial fractions:

$$\frac{9x^2 + 20x - 10}{(x - 2)(3x + 1)}$$

Worked Example

Split into partial fractions:

$$\frac{9x^2 + 16}{9x^2 - 16}$$

Worked Example

Find the values of A, B, C and D:

$$\frac{x^3 - x^2 + 7}{x - 2} = Ax^2 + Bx + C + \frac{D}{x - 2}$$

Worked Example

Find the values of A, B, C, D and E:

$$\frac{x^4 - x^3 - x + 10}{x^2 - 2x - 3} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 + 2x - 3}$$

A2 2018 Paper 2

Algebraic Methods

11.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

(4)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

(a) **Way 2**	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B = ..., C = ...$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = ...$ or $C = ...$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	

Summary of Key Points

3 To multiply fractions, cancel any common factors, then multiply the numerators and multiply the denominators.

4 To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

5 To add or subtract two fractions, find a common denominator.

6 A single fraction with two distinct linear factors in the denominator can be split into two separate fractions with linear denominators. This is called splitting it into **partial fractions**:

$$\frac{5}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$$

7 The method of partial fractions can also be used when there are more than two distinct linear factors in the denominator:

$$\frac{7}{(x-2)(x+6)(x+3)} = \frac{A}{x-2} + \frac{B}{x+6} + \frac{C}{x+3}$$

8 A single fraction with a repeated linear factor in the denominator can be split into two or more separate fractions:

$$\frac{2x+9}{(x-5)(x+3)^2} = \frac{A}{x-5} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

9 An improper algebraic fraction is one whose numerator has a degree equal to or larger than the denominator. An improper fraction must be converted to a mixed fraction before you can express it in partial fractions.

10 You can either use:

- algebraic division
- or the relationship $F(x) = Q(x) \times \text{divisor} + \text{remainder}$

to convert an improper fraction into a mixed fraction.