



Year 12 Pure Mathematics P2 1.2-1.5 Algebraic Methods

HGS Maths







Name:

Class:

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1.2) Algebraic fractions

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1.4) Repeated factors

1.5) Algebraic division

Past Paper Practice Summary

Prior knowledge check

Prior knowledge check

- Factorise each polynomial: 1
 - **a** $x^2 6x + 5$ **b** $x^2 16$

 - **c** $9x^2 25$ \leftarrow Year 1, Section 1.3
- 2 Simplify fully the following algebraic fractions.

a $\frac{x^2 - 9}{x^2 + 9x + 18}$ **b** $\frac{2x^2 + 5x - 12}{6x^2 - 7x - 3}$ c $\frac{x^2 - x - 30}{-x^2 + 3x + 18}$ ← Year 1, Section 7.1 387d: Simplify algebraic fractions where factorisation of a simple quadratic on the numerator or denominator is required.

Simplify

$$\frac{x-3}{x^2-4x+3}$$

387e: Simplify algebraic fractions where factorisation of a simple quadratic on both the numerator and denominator is required.

$$\frac{x^2-7x+10}{x^2-6x+5}$$

1.2) Algebraic fractions		

Notes

$$\frac{2x^3 - 5x^2 - 3x}{2x - 6}$$

$$\frac{2x^3 + 5x^2 - 3x}{4x^2 - 1}$$

$$\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$$

$$\frac{2x^2 - 5x - 3}{3x^2 - 11x + 6}$$

Simplify: $\frac{3x^2 + 2x - 8}{6x^2 - 23x - 35} \times \frac{7x^2 - 29x - 30}{4x^2 + 5x - 6}$

$$\frac{x^2-x}{y^2} \div \frac{x^2+x-2}{y^5}$$

$$\frac{3x^2 - 10x - 8}{6x^2 + 37x - 35} \div \frac{x^2 - 3x - 4}{x^2 - 49}$$

Write as a single fraction:

$$3 - (x - 4) \div \frac{x^2 - 16}{x - 5}$$

	Worked Example
Write as a single fraction: $\frac{3}{5x+2} - \frac{2}{x-3}$	Worked Example

390a: Add or subtract algebraic fractions where non-quadratic factorising is required (changing one denominator).

Simplify

$$\frac{2}{x-1} - \frac{10}{x^2+3x-4}$$

Leave the denominator in factorised form when applicable.

390c: Add or subtract algebraic fractions where factorising is required (changing two denominators).

Simplify

$$\frac{5}{y^2+18y+80}+\frac{9}{y^2+8y-20}$$

Leave the denominator in factorised form when applicable.

390e: Solve an equation with fractions requiring prior factorisation of one of the fractions.

Solve the following equation:

$$rac{4}{2x^2+x-15}-rac{1}{2x-5}=3$$

Give your answer in exact form.

1.3) Partial fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x-2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$$

Notation reminder: \equiv means 'equivalent/identical to', and indicates that both sides are equal for <u>all</u> values of x.

Notes	

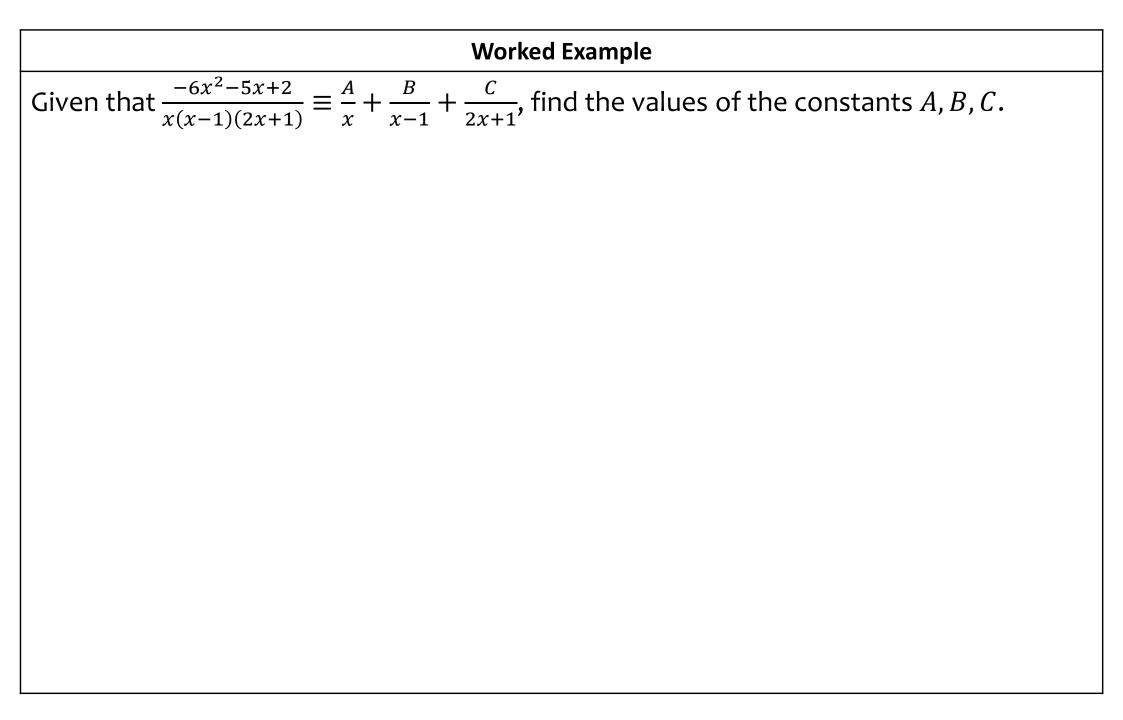
559a: Decompose a rational function into partial fractions: $\frac{ax+b}{(cx\pm d)(ex\pm f)}$

Express

$$\frac{5x-1}{(3x-2)(2x+1)}$$

in the form

$$\frac{A}{3x-2} + \frac{B}{2x+1}$$



Express as partial fractions: $\frac{6x^2 + 14x - 12}{x^3 - 4x}$

1.4) Repeated factors

Suppose we wished to express $\frac{2x+1}{(x+1)^2}$ as $\frac{A}{x+1} + \frac{B}{x+1}$. What's the problem?

Because the denominators are the same, we'd get $\frac{A+B}{x+1}$. There's no constant values of A and B we can choose such that $\frac{2x+1}{(x+1)^2} \equiv \frac{A+B}{x+1}$ because the denominators will still be different.

Notes	

Express as partial fractions:

$$\frac{4x}{(x-4)^2}$$

559d: Decompose a rational function into partial fractions: $\frac{ax\pm b}{(cx\pm d)(ex\pm f)^2}$

Express

$$rac{4z+5}{(z-1)(z-4)^2}$$

in the form

$$\frac{A}{z-1}+\frac{B}{z-4}+\frac{C}{(z-4)^2}$$

Express as partial fractions:

$$\frac{11x^2 - 22x + 9}{(x-1)^2(2x-1)}$$

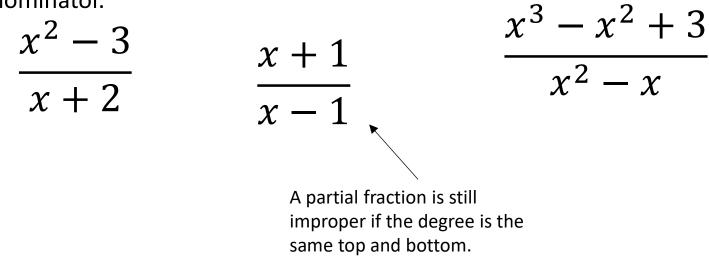
Worked Example Express as partial fractions: $\frac{5x^2 + 4x + 1}{x^3 + x^2}$

Express as partial fractions: $\frac{15x^2 - 5x + 2}{9x^3 - 6x^2 + x}$

1.5) Algebraic division

In Pure Year 1, we saw that the '**degree**' of a polynomial is the highest power, e.g. a quadratic has degree 2.

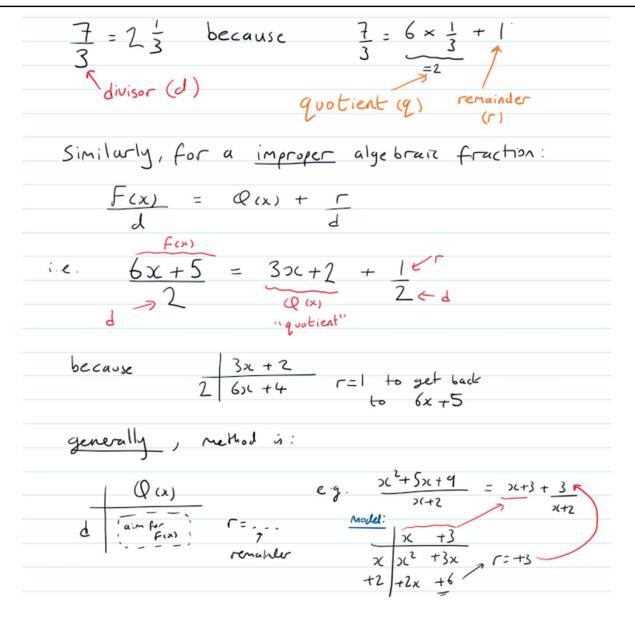
An algebraic fraction is **improper** if the degree of the numerator is **at least** the degree of the denominator.



Questions might take one of two forms:

- Do the division to express as a quotient and a remainder, e.g. $\frac{x+1}{x-1} \rightarrow 1 + \frac{2}{x-1}$
- Express as partial fractions, e.g. $\frac{x^2+x}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$

Reducing to Quotient and Remainder



Notes

497a: Determine a factor from algebraic division of a quadratic or cubic expression, given another factor

Given that

$$\frac{3x^2 - 22x + 35}{x - 5} = Ax + B$$

Use algebraic long division to work out the values of the constants ${\cal A}$ and ${\cal B}.$

Find the values of *A*, *B* and *C*:

$$\frac{x^2 + 9x - 5}{x + 3} = Ax + B + \frac{C}{x + 3}$$

Find the values of *A*, *B* and *C*:

$$\frac{x^2 - 5x + 9}{x - 2} = Ax + B + \frac{C}{x - 2}$$

Your Turn

Find the values of *A*, *B* and *C*:

$$\frac{x^2 + 5x - 9}{x - 3} = Ax + B + \frac{C}{x - 3}$$

A = 1, B = 8, C = 15

Worked Example				
Simplify:	$\frac{x^4 - 81}{x + 3}$			
	<i>λ</i> Τ 3			

559b: Decompose a rational function into partial fractions: $\frac{ax^2+bx+c}{(dx\pm e)(fx\pm g)}$

Express $rac{2x^2+4x+4}{(x-1)\,(x+4)}$

in the form $A+rac{B}{x-1}+rac{C}{x+4}$

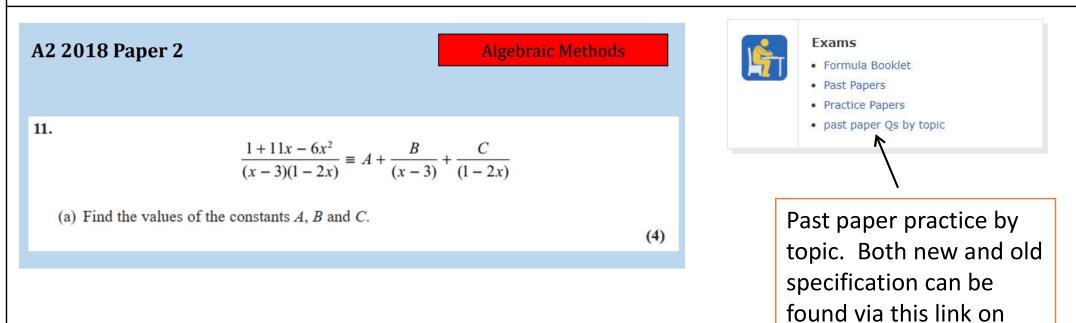
Split into partial fractions: $\frac{9x^2 + 20x - 10}{(x-2)(3x+1)}$

Worked Example			
Split into partial fractions:			
$9x^2 + 16$			
$\frac{9x^2 + 16}{9x^2 - 16}$			

Find the values of A, B, C and D: $\frac{x^3 - x^2 + 7}{x - 2} = Ax^2 + Bx + C + \frac{D}{x - 2}$

Find the values of *A*, *B*, *C*, *D* and *E*: $\frac{x^4 - x^3 - x + 10}{x^2 - 2x - 3} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 + 2x - 3}$

Past Paper Questions



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	(†)		
d1.1	IA	$B = 4 \text{ and } C = -2 \text{ which have been found using}$ $-10x + 10 \equiv B(1-2x) + C(x-3)$	
d1.1	IM	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	
d1.1	BI	ε = <i>F</i> .	
1.2	IM	$-10x + 10 \equiv B(1 - 2x) + C(x - 3) \Longrightarrow B = \dots, C = \dots$	
		$\frac{0I+x0I-}{(x2-I)(\varepsilon-x)} + \varepsilon \equiv \frac{^{x}x\partial - xII+I}{(x2-I)(\varepsilon-x)} \{\text{sovig noisivib gnoI}\}$	(8) (8)

Summary of Key Points

- **3** To multiply fractions, cancel any common factors, then multiply the numerators and multiply the denominators.
- 4 To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.
- 5 To add or subtract two fractions, find a common denominator.
- 6 A single fraction with two distinct linear factors in the denominator can be split into two separate fractions with linear denominators. This is called splitting it into partial fractions:

$$\frac{5}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$$

7 The method of partial fractions can also be used when there are more than two distinct linear factors in the denominator:

 $\frac{7}{(x-2)(x+6)(x+3)} = \frac{A}{x-2} + \frac{B}{x+6} + \frac{C}{x+3}$

8 A single fraction with a repeated linear factor in the denominator can be split into two or more separate fractions:

 $\frac{2x+9}{(x-5)(x+3)^2} = \frac{A}{x-5} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

- 9 An improper algebraic fraction is one whose numerator has a degree equal to or larger than the denominator. An improper fraction must be converted to a mixed fraction before you can express it in partial fractions.
- 10 You can either use:
 - algebraic division
 - or the relationship $F(x) = Q(x) \times \text{divisor} + \text{remainder}$

to convert an improper fraction into a mixed fraction.