KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13
Applied Mathematics
M2 6 Projectiles

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

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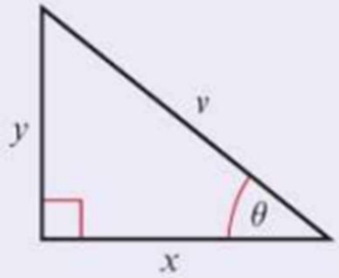
1 A small ball is projected vertically upwards from a point P with speed $15\ \text{m s}^{-1}$. The ball is modelled as a particle moving freely under gravity. Find:

a the maximum height of the ball

b the time taken for the ball to return to P. ← **Year 1, Chapter 9**

2 Write expressions for x and y in terms of v and θ. ← **GCSE Mathematics**



3 **a** Given $\sin\theta = \frac{5}{13}$ find

i $\cos\theta$ **ii** $\tan\theta$

b Given $\tan\theta = \frac{8}{15}$ find

i $\sin\theta$ **ii** $\cos\theta$ ← **Pure Year 1, Chapter 10**

6.1) Horizontal projection

In **vertical** direction, acceleration downwards is g ms^{-2}.

Use suvat equations as before.

In **horizontal** direction, acceleration is 0 ms^{-2}.

Constant velocity, so can use bog standard $speed = \frac{distance}{time}$

Notes

Worked Example

654a: Determine the horizontal displacement of a particle projected horizontally.

A ball is projected horizontally at 10.8 m s^{-1} from a point h m above the ground.

The ball hits the ground after 3.8 seconds.

Find the horizontal distance travelled by the ball before it reaches the ground.
Give your answer correct to 2 significant figures.

Worked Example

654b: Determine the vertical displacement of a particle projected horizontally.

A ball is projected horizontally at 40.3 m s^{-1} from a point h m above the ground.

The ball hits the ground after 1.6 seconds.

Find the value of h.
Give your answer correct to 2 significant figures.

Worked Example

654c: Determine the time taken for a particle to reach the ground after being projected horizontally.

A ball is projected horizontally at 32.9 m s^{-1} from a point 91.6 metres above the ground.

Find the time taken by the particle to reach the ground.
Give your answer correct to 2 significant figures.

Worked Example

654d: Determine the initial velocity of a particle projected horizontally.

A ball is projected horizontally at a velocity of U m s^{-1} from a point 110.5 m above the ground.

The particle hits the plane at a point which is at a horizontal distance of 86.1 m away from the starting point.

Find the initial velocity of the particle.
Give your answer correct to 2 significant figures.

Worked Example

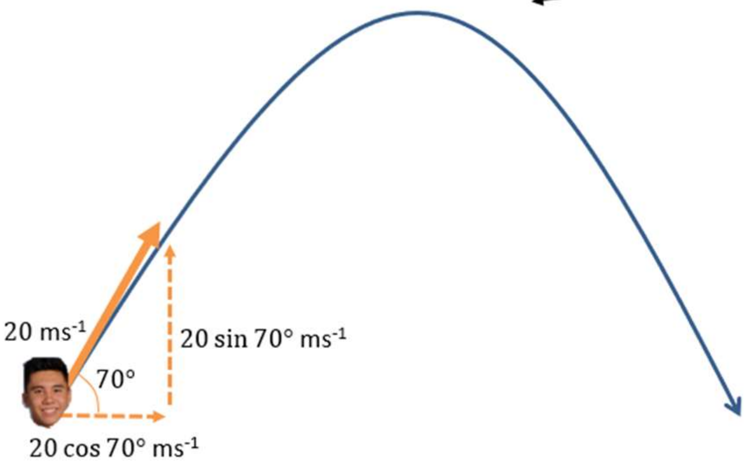
A particle of mass $10\ kg$ is projected along a horizontal rough surface with a velocity of $20\ ms^{-1}$.
After travelling a distance of $40\ m$ the ball leaves the rough surface as a projectile and lands on the ground which is $3\ m$ vertically below. Given that the total time taken for the ball to travel from the initial point of projection to the point when it lands is 4.0 seconds, find:

a) The time for which the particle is in contact with the surface
b) The coefficient of friction between the particle and the surface
c) The horizontal distance travelled from the point of projection to the point where the particle hits the ground

T.110 : 6A Qs all, P. 49 : 6.1 Qs all

6.2) Horizontal and vertical components

Just as **we split forces into its horizontal and vertical components**, in order to consider forces in the horizontal and vertical directions respectively, we can do **exactly the same with velocity**!



When the object is at its highest point:
The vertical velocity is 0.

We know that the scalar form of velocity is **speed**, and thus we just find the **magnitude** of the velocity vector:

$$\begin{pmatrix} 12 \\ 5 \end{pmatrix} ms^{-1}$$

$$\Rightarrow \sqrt{\mathbf{12^2 + 5^2}}$$

$$= \mathbf{13\ ms^{-1}}$$

Notes

Worked Example

A particle is projected from a point on a horizontal plane with an initial velocity of $39\ ms^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

a) Find the horizontal and vertical components of the initial velocity

b) Express the initial velocity as a vector in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$

T.112 : 6B Qs 1-4, P. 50 : 6.2 Qs 1,2,4,5,6

Worked Example

A particle is projected with velocity $\boldsymbol{U} = (2\boldsymbol{i} + 7\boldsymbol{j})\ ms^{-1}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the unit vectors in the horizontal and vertical directions respectively.
Find the initial speed of the particle and its angle of projection.

Worked Example

A particle is projected with velocity $\boldsymbol{U} = (5k\boldsymbol{i} + 2k\boldsymbol{j})\ ms^{-1}$.

a) Find the angle of projection

Given that the initial speed is $5\sqrt{29}\ ms^{-1}$

b) Find the possible values of k

T.112 : 6B Qs 5+, P. 50 : 6.2 Qs 3,7

6.3) Projection at any angle

Notes

Worked Example

654e: Determine the maximum height of a particle projected at an angle.

Find the maximum height reached by a ball thrown from a height of 2.1 m with speed 38.3 m s^{-1} at an angle of 41° to the horizontal.

Worked Example

654f: Determine the angle of projection of a particle given the maximum height it has reached.

A ball is struck so that its initial speed is 34.4 m s^{-1} at angle α above the horizontal.

The maximum height reached by the ball is 13 m.

Find the value of α.

654j: Determine the initial velocity of a particle projected at an angle.

A ball is kicked so that, initially, it moves with speed U m s $^{-1}$, at 65° above the horizontal.

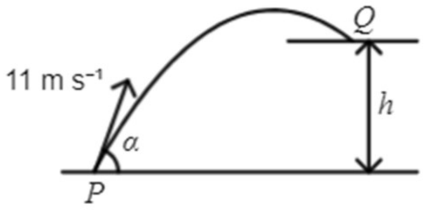
The ball hits the ground for the first time 3.5 seconds later.

Find the value of U.

Worked Example

654m: Determine the height of a particle projected at an unknown angle at a specific instant.

A small ball is projected with speed 11 m s^{-1} from a point P on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground.



The ball moves freely under gravity until it hits the platform at the point Q, as shown. The speed of the ball immediately before it hits the platform at Q is 9 m s^{-1}.

Find the value of h.

T.117 : 6C Qs 1-5, P. 51 : 6.3 Qs 1-3

Worked Example

A particle is projected from a point O with speed V ms^{-1} and at an angle of elevation of θ, where $\tan\theta = \frac{4}{3}$.
The point O is 42.5m above a horizontal plane. The particle strikes the plane at a point A, 5 s after it is projected.

(a) Find V

(b) Find the distance between O and A.

Worked Example

A ball is struck by a racket at a point which is $2\ m$ above horizontal ground. Immediately after the ball is truck, the ball has velocity $(10\boldsymbol{i} + 16\boldsymbol{j})ms^{-1}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors horizontally and vertically respectively.
After being struck, the ball travels freely under gravity until it strikes the ground.
Find:

a) The greatest height above the ground reached by the ball
b) The speed of the ball as it reaches the ground
c) The angle the velocity of the ball makes with the ground as the ball reaches B

Worked Example

A particle is projected from a point on level ground with speed $U\ ms^{-1}$ and an angle of elevation of α.
The maximum height reached by the particle is $15.3061\ m$ (4 dp) above the ground and the particle hits the ground $35.3480\ m$ (4 dp) from its point of projection.
Find the value of α and U

T.117 : 6C Qs 6+, P. 51 : 6.3 Qs 4+

6.4) Projectile motion formulae

Exam Note: You may be asked to derive these. But don't attempt to memorise them or actually use them to solve exam problems – instead use the techniques used earlier in the chapter.

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \frac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \frac{U \sin}{g}$
- Range on horizontal plane $= \frac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory: $y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$
  where y is vertical height of particle and x horizontal distance.

Notes

Worked Example

A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$

A particle is projected from a point O on a horizontal plane, with speed $14\ ms^{-1}$ at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 16m from O and at a height of 4m above the plane.

(b) Find the two possible values of α, giving your answers to the nearest degree.

T.123 : 6D Qs all, P. 52 : 6.4 Qs all

10.

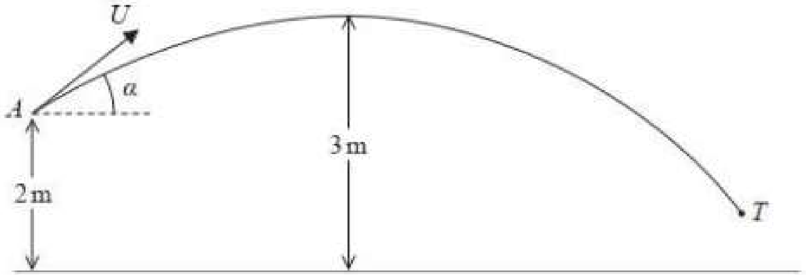


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is $2\,\text{m}$ above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is $3\,\text{m}$ above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2\alpha}$.

(2)

The point T is at a horizontal distance of $20\,\text{m}$ from A and is at a height of $0.75\,\text{m}$ above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

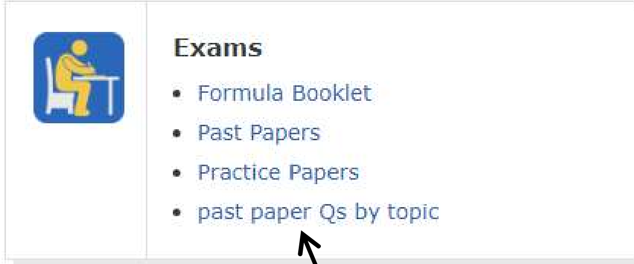
(9)

(c) State one limitation of the model that could affect your answer to part (b).

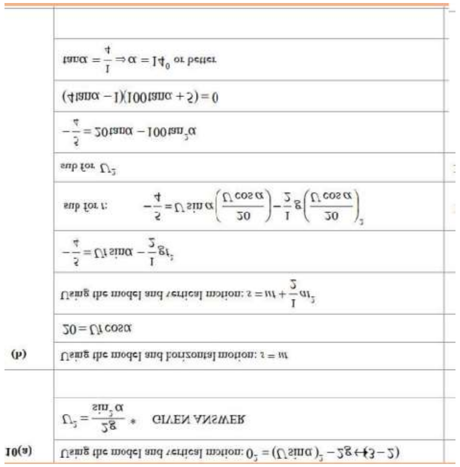
(1)

(d) Find the time taken for the ball to travel from A to T.

(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



Summary of Key Points

Summary of key points

1 The **horizontal** motion of a projectile is modelled as having **constant velocity** ($a = 0$).
You can use the formula $s = vt$.

2 The **vertical** motion of a projectile is modelled as having **constant acceleration** due to gravity ($a = g$).

3 When a particle is projected with initial velocity U, at an angle α above the horizontal:
- The **horizontal component** of the initial velocity is $U\cos\alpha$
- The **vertical component** of the initial velocity is $U\sin\alpha$

4 A projectile reaches its point of greatest height when the vertical component of its velocity is equal to 0.

5 For a particle which is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal, and that moves freely under gravity:
- Time of flight $= \frac{2U\sin\alpha}{g}$
- Time to reach greatest height $= \frac{U\sin\alpha}{g}$
- Range on horizontal plane $= \frac{U^2\sin 2\alpha}{g}$
- Equation of trajectory: $y = x\tan\alpha - gx^2\frac{(1 + \tan^2\alpha)}{2U^2}$

where y is the vertical height of the particle, x is the horizontal distance from the point of projection, and g is the acceleration due to gravity.

T.125 : mixed ex, P. 54 : BSG