KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13
Applied Mathematics
M2 7 Application of forces

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

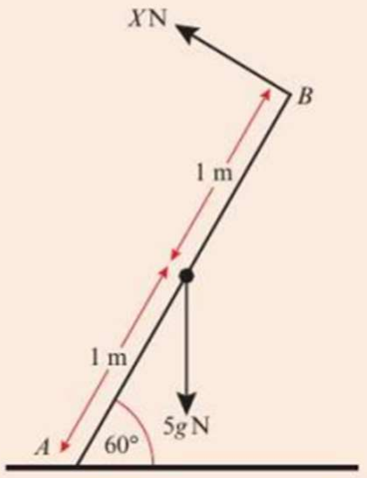
Contents

Prior knowledge check

Prior knowledge check

1 A particle of mass 2 kg sits on a rough plane that is inclined at 45° to the horizontal. A force of 10 N acts parallel to and up the plane. Given that the particle is on the point of moving, work out the coefficient of friction μ. ← Section 5.3

2 A uniform rod AB of length 2 m and mass 5 kg rests in equilibrium at an angle of 60° to a horizontal surface. The rod is pivoted at A and a force of magnitude X N acts perpendicular to the rod at B. Find the value of X.



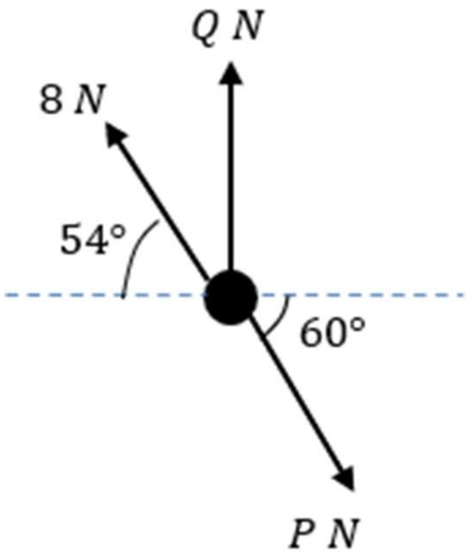
← Section 4.3

7.1) Static particles

Notes

Worked Example

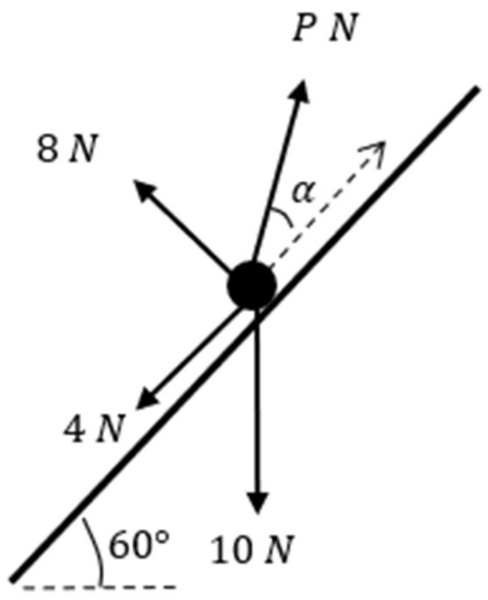
The diagram shows a particle in equilibrium under the forces shown. By resolving horizontally and vertically find the magnitudes of the forces P and Q.



T.130 7A: Qs 1-2, P.56 7.1: Qs 1-3

Worked Example

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force P and the size of the angle α.



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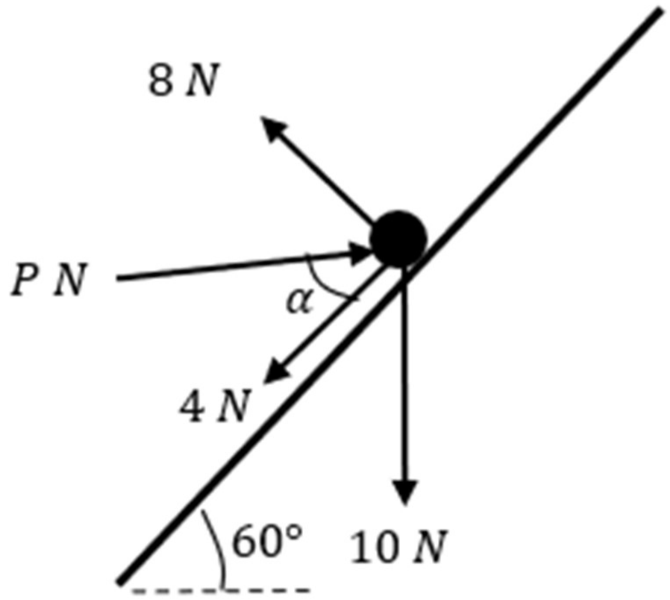


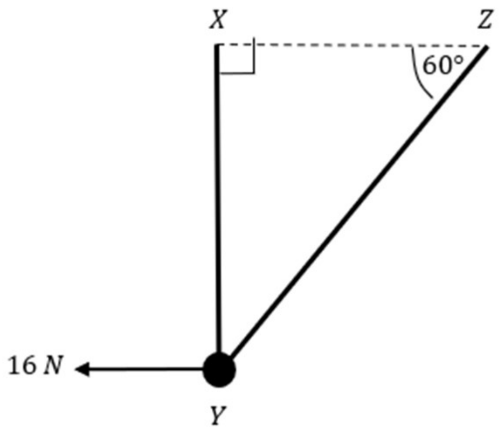
T.130 7A: Q 3, P.56 7.1: Qs 4+

7.2) Modelling with statics

Notes

Worked Example

A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points, X and Y, on the same horizontal level.
The bead is held in equilibrium by a horizontal force of magnitude 16 N acting parallel to ZX.
The bead Y is vertically below X and $\angle XZY = 60°$ as shown in the diagram.
Find the tension in the string and the weight of the bead.



Worked Example

A mass of 6kg rests on the surface of a smooth plane which is inclined at an angle of 30° to the horizontal.
The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane.
The cable carries a mass of 2kg freely suspended at the other end.
The masses are modelled as particles, and the cable as a light inextensible string.
There is a force of P N acting horizontally on the 6kg mass and the system is in equilibrium.

Calculate:

(a) the magnitude of P

(b) the normal reaction between the mass and the plane

(c) State how you have used the assumption that the pulley is smooth in your calculations.

Worked Example

A particle of weight $4\ N$ is attached at C to the ends of two light inextensible strings AC and BC.
The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at $45°$ to the horizontal and the string BC is inclined at $15°$ to the horizontal. Find:

a) The tension in the string AC
b) The tension in the string BC

T.134 7B: Qs 8+, P.57 7.2: Qs 6+

7.3) Friction and static particles

Notes

Worked Example

A mass of 4 kg rests on a rough horizontal plane.
The mass may be modelled as a particle, and the coefficient of friction between the mass and plane is 0.25.
Find the magnitude of the maximum force P N which acts on this mass without causing it to move if:
a) The force P is horizontal
b) The force P acts at an angle of $30°$ above the horizontal

Worked Example

A box of mass 20kg rests in limiting equilibrium on a rough plane inclined at $10°$ above the horizontal.

(a) Find the coefficient of friction between the box and the plane.

A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium,

(b) find the maximum possible value of P.

T.139 7C: Qs 1-5, P. 58 7.3: Qs 1-2

Worked Example

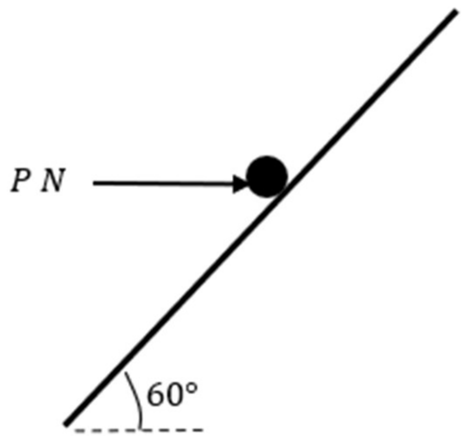
A parcel of weight $20\ N$ lies on a rough plane inclined at an angle of $60°$ to the horizontal.
A horizontal force of magnitude P Newtons acts on the parcel. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is $36\ N$. The coefficient of friction between the parcel and the plane is μ. Find:

a) The value of P

b) The value of μ

The horizontal force is removed.

c) Determine whether or not the parcel moves.



T.139 7C: Qs 6+, P. 58 7.3: Qs 3+

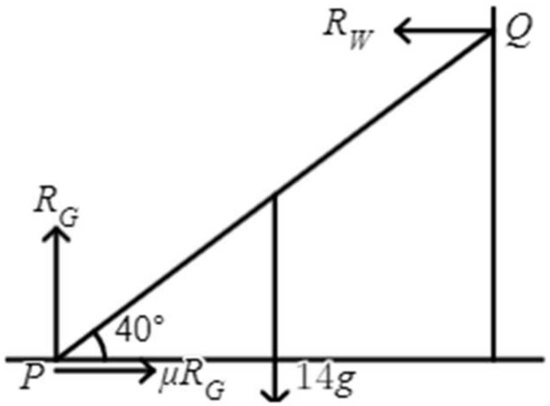
7.4) Static rigid bodies

Notes

659a: Determine the coefficient of friction of a ladder resting on a wall in limiting equilibrium.

A uniform ladder PQ of mass 14 kg and length 13 m rests against a smooth vertical wall with its lower end on rough horizontal ground.

The ladder is in limiting equilibrium at an angle of 40° to the horizontal.



Find the coefficient of friction between the ladder and the ground.

$\mu =$

Worked Example

A uniform rod AB of mass 20kg and length 5m rests with the end A on rough horizontal ground.
The rod rests against a smooth peg C where $AC = 4$ m.
The rod is in limiting equilibrium at an angle of $30°$ to the horizontal. Find:

(a) the magnitude of the reaction of C

(b) the coefficient of friction between the rod and the ground.

T.144 7D: Qs 1-2, P. 60 7.4: Qs 1-3

Worked Example

A ladder AB, of mass m and length $5a$, has one end A resting on rough horizontal ground.
The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = 2a$.
The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle.
The ladder rests in limiting equilibrium at an angle of $50°$ with the ground.
Find the coefficient of friction between the ladder and the ground.

T.144 7D: Qs 3+, P. 60 7.4: Qs 4+

7.5) Dynamics and inclined planes

Notes

Worked Example

A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$.
The coefficient of friction between the particle and the plane is 0.25.
The particle is released and slides down the plane. Find:

(a) the acceleration of the particle.
(b) the distance it slides in the first 4 seconds.

T.148 7E: Qs 1-3, P. 62 7.5: Qs 1,2

Worked Example

A box of mass $4\ kg$ is pushed up a rough plane by a horizontal force of magnitude $50\ N$.
The plane is inclined to the horizontal at an angle of $20°$.
Given that the coefficient of friction between the box and the plane is 0.1, find the acceleration of the box.

Worked Example

A particle of mass $0.3\ kg$ slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at $15°$ to the horizontal.
The particle passes through two points A and B, where $AB = 5\ m$.
The speed of P at A is $4\ ms^{-1}$.
It takes $7\ s$ to move from A to B. Find:

a) The speed of P at B
b) The acceleration of P
c) The coefficient of friction between P and the plane

T.148 7E: Qs 4+, P. 62 7.5: Qs 3+

7.6) Connected particles

Notes

Worked Example

Two particles P and Q of masses 4kg and 8kg respectively are connected by a light inextensible string.
The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane.
P rests on the inclined plane and Q hangs on the edge of the plane with the string vertical and taut.
The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{5}{12}$.
The coefficient of friction between P and the plane is 0.3. The system is released from rest.

(a) Find the acceleration of the system.

(b) Find the tension in the string.

Worked Example

One end of a light inextensible string is attached to a block A of mass 4kg. The block A is held at rest on a **smooth** fixed plane which is inclined to the horizontal at an angle of $45°$. The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 10kg. The system is released from rest. By modelling the blocks as particles and ignoring air resistance,

(a)(i) find the acceleration of block B

(ii) find the tension in the string.

(b) State how you have used the fact that the string is inextensible in your calculations.

(c) Calculate the magnitude of the force exerted on the pulley by the string.

Worked Example

A fixed rough plane is inclined at $45°$ to the horizontal.
A small smooth pulley P is fixed at the top of the plane.
Two particles A and B, of mass $3\ kg$ and $6\ kg$ respectively, are attached to the ends of a light inextensible string which passes over the pulley P.
The part of the string from A to P is parallel to the line of greatest slope of the plane and B hangs freely below P.
The coefficient of friction between A and the plane is $\frac{1}{\sqrt{2}}$.
Initially A is held at rest on the plane.
The particles are released from rest with the string taut and A moves up the plane.
Find the tension in the string immediately after the particles are released.

Extract from Formulae book

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Past Paper Questions

A2 SAMs

Moments

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

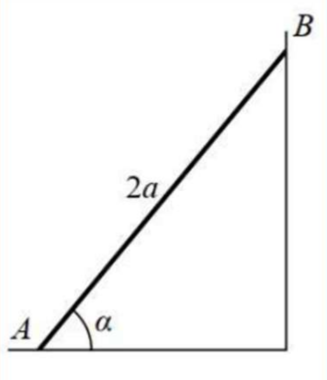


Figure 1

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

9(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1
	$S = 3W$ *	A1*
		(5)
(b)	$R = 8W$	B1
	$F = \frac{1}{4}R\ (= 2W)$	M1
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1
	$W \le P \le 5W$	A1
		(5)
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1
	also R increases (resolving vertically)	M1
	which increases max F available	M1
		(3)

Summary of Key Points

Summary of key points

1 A particle or rigid body is in static equilibrium if it is at rest and the resultant force acting on the particle is zero.

2 The maximum value of the frictional force $F_{MAX} = \mu R$ is reached when the body you are considering is on the point of moving. The body is then said to be in limiting equilibrium.

3 In general, the force of friction F is such that $F \leqslant \mu R$, and the direction of the frictional force is opposite to the direction in which the body would move if the frictional force were absent.

4 For a rigid body in static equilibrium:

- the body is stationary
- the resultant force in any direction is zero
- the resultant moment is zero.

T.154: mixed ex. , P. 65 : BSG