Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Practice Paper 5

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/4A**

Further Mathematics
Advanced
Paper 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 75. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. In the Argand diagram the point P represents the complex number z.

Given that $\arg\left(\dfrac{z-2\mathrm{i}}{z+2}\right) = \dfrac{\pi}{2}$,

(a) sketch the locus of P,

(4)

(b) deduce the value of $|z + 1 - \mathrm{i}|$.

(2)

The transformation T from the z-plane to the w-plane is defined by

$$w = \frac{2(1+\mathrm{i})}{z+2}, \quad z \neq -2.$$

(c) Show that the locus of P in the z-plane is mapped to part of a straight line in the w-plane, and show this in an Argand diagram.

(6)

(Total for Question 1 is 12 marks)

2.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 6 & -9 \end{pmatrix}$$

(a) Find the eigenvalues of $\mathbf{M}$.

(4)

A transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix $\mathbf{M}$. There is a line through the origin for which every point on the line is mapped onto itself under T.

(b) Find a cartesian equation of this line.

(3)

(Total for Question 2 is 7 marks)

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3. For $n \in \mathbb{Z}^+$ prove that $2^{3n+2} + 5^{n+1}$ is divisible by 3.

(Total for Question 3 is 9 marks)

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4. (a) Use the Euclidean algorithm to show that 21 and 362 are relatively prime.

(3)

(b) Hence find a solution to the equation $21x + 362y = 10$, $y \in \mathbb{Z}$.

(5)

(Total for Question 4 is 8 marks)

5.

$$I_n = \int x^n e^{2x} \, dx, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 1$,

$$I_n = \frac{1}{2}(x^n e^{2x} - nI_{n-1}).$$

(3)

(*b*) Find, in terms of e, the exact value of

$$\int_0^1 x^2 e^{2x} \, dx.$$

(5)

(Total for Question 5 is 8 marks)

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6. **Figure 1**

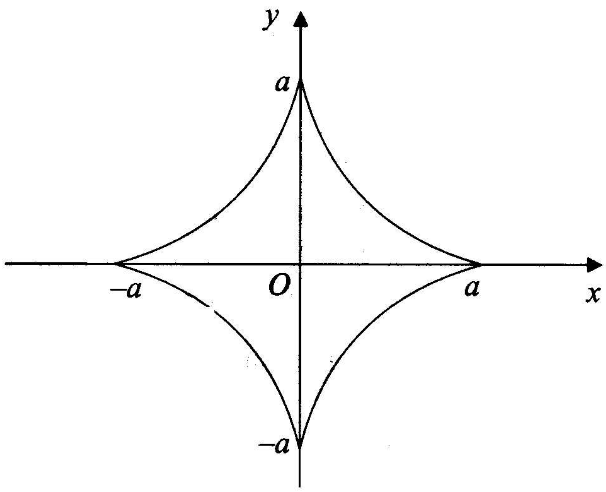

Figure 1 shows the curve with parametric equations

$$x = a\cos^3\theta, \quad y = a\sin^3\theta, \quad 0 \le \theta < 2\pi.$$

(*a*) Find the total length of this curve.

(7)

The curve is rotated through π radians about the x-axis.

(*b*) Find the area of the surface generated.

(5)

(Total for Question 6 is 12 marks)

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7. (a) Find a closed form for the sequence defined by the recurrence relation

$$u_n = \sqrt{2}(u_{n-1}) - u_{n-2}, \text{ with } u_0 = u_1 = 1.$$

(5)

(b) Hence show that the sequence is periodic and state its period.

(3)

(Total for Question 7 is 8 marks)

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8. The binary operator multiplication modulo 18, denoted by *, is defined on the set $G = \{2, 4, 8, 10, 14, 16\}$.

(a) (i) Copy and complete the Cayley table below.

*	**2**	**4**	**8**	**10**	**14**	**16**
2		8		2		
4	8	16	14	4	2	10
8		14		8		
10	2	4	8	10	14	16
14		2		14		
16		10		16		

(ii) Show that group (G, *) is a group. You may assume that the associative axiom is satisfied.

(6)

(b) Show that the element 4 has order 3.

(2)

(c) Find an element which generators (G, *) and write each element in terms of this generator.

(3)

(Total for Question 8 is 11 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 75 MARKS

Guide:

1. P6 January 2006, Qn 8

2. P6 June 2003, Qn 3

3. P6 June 2002, Qn 6(a)

4. Pearson FP2 textbook, p38, Qn 3

5. P5 June 2005, Qn 4

6. P5 June 2004, Qn 7

7. Pearson FP2 textbook, p149, Qn 20

8. Pearson FP2 textbook, p82, Qn 8