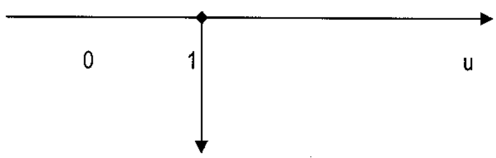
Further Pure Mathematics 2 Practice Paper 5 – mark schemes and answers

Origin of questions:

1. P6 January 2006, Qn 8

Qn	Scheme	Marks	
8.	(a) Relating lines and angle (generous)		
	[*angle between* $\pm 2i$ *to P and* ± 2 *to P*]		
	Angle between correct lines is $\frac{\pi}{2}$		
	Circle		
	Selecting correct ("top half") semi-circle .		
	[If algebraic approach:	M1	
	Method for finding Cartesian equation — M1		
	Correct equation, any form, $\Rightarrow x(x+2) + y(y-2) = 0$ — A1	A1	
	Sketch: showing circle — M1	M1	
	Correct circle { centre (–1, 1)}, choosing only "top half" — A1]	A1	
			(4)
	(b) $\lvert z + 1 - i \rvert$ is radius; $= \sqrt{2}$	M1A1	
			(2)
	(c) $z = \dfrac{2(1+i) - 2\omega}{\omega}$ $\left(= \dfrac{2(1+i)}{\omega} - 2\right)$	M1	
	$\dfrac{z - 2i}{z + 2} = \dfrac{2(1+i) - 2(1+i)\omega}{2(1+i)}$ $(= 1 - \omega)$	M1A1	
	Arg $(1 - \omega) = \dfrac{\pi}{2}$ is line segment, passing through (1,0)	A1,A1	
		A1	
			(6)
		Total 12 marks	

2. P6 June 2003, Qn 3

Question number	Scheme	Marks
3.	(a) Deriving characteristic equation $(4-\lambda)(-9-\lambda)+30=0$	M1 A1
	$\Rightarrow \lambda^2+5\lambda-6=0 \Rightarrow (\lambda+6)(\lambda-1)=0 \Rightarrow \lambda=-6, \lambda=1$	M1 A1 (4)
	(b) Stating, implying or showing $\lambda=1$ associated with point invariant line.	B1
	$\Rightarrow \begin{pmatrix}4 & -5\\6 & -9\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$	
	Equation is $4x-5y=x \Rightarrow 3x-5y=0$ any equivalent form	M1 A1 (3) [7]

3. P6 June 2002, Qn 6(a)

Question Number	Scheme	Marks
6. (*a*)	For $n=1$ $2^5+5^2=57$, which is divisible by 3	M1, A1
	Assume true for $n=k$ $(k+1)$th term is $2^{3k+5}+5^{k+2}$	B1
	$(k+1)$th term $\pm$ kth term $=2^{3k+5}+5^{k+2}\pm 2^{3k+2}+5^{k+1}$	M1
	$=2^{3k+2}(2^3\pm 1)+5^{k+1}(5\pm 1)$	M1, A1
	$=6(2^{3k+2}+5^{k+1})+3.\ 2^{3k+2}$ or $=4(2^{3k+2}+5^{k+1})+3.\ 2^{3k+2}$	M1
	which is divisible by 3 $\Rightarrow$ $(k+1)$th term is divisible by 3	A1
	Thus by induction frue for all n cso	B1 **(9)**

4. Pearson FP2 textbook, p38, Qn 3

3 a $362 = 17 \times 21 + 5$, $21 = 4 \times 5 + 1$, $5 = 5 \times 1 + 0$
so $\gcd(362, 21) = 1$

b $x = 690$, $y = -40$

5. P5 June 2005, Qn 4

4.	(a)	$I_n = \frac{1}{2}x^n e^{2x} - \frac{n}{2}\int x^{n-1}e^{2x}\,dx, \quad I_n = \frac{1}{2}(x^n e^{2x} - nI_{n-1})$	(*)	M1 A1 A1 (3)
	(b)	$\int_0^1 x^2 e^{2x}\,dx = I_2 = \left[\frac{1}{2}x^2 e^{2x}\right]_0^1 - I_1 = \frac{1}{2}e^2 - I_1$	one correct stat	M1
		$I_1 = \left[\frac{1}{2}xe^{2x}\right]_0^1 - \frac{I_0}{2} = \frac{1}{2}e^2 - \frac{1}{2}I_0$	linking all three	M1 A1
		$I_0 = \int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1 = \frac{e^2}{2} - \frac{1}{2}$		
		$I_2 = \frac{e^2}{2} - \left(\frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{1}{4}\right)\right) = \frac{1}{4}\left(e^2 - 1\right)$	use of limits	M1 A1 (5)
				(8)

6. P5 June 2004, Qn 7

7. (a)	$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \qquad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	B1
	$s = \int\sqrt{(9a^2(c^4s^2 + s^4c^2))}\,d\theta$	M1
	$= 3a\int\sqrt{c^2s^2}\,d\theta$	M1
	$= 3a\int\cos\theta\sin\theta\,d\theta$	A1
	Total length = $4\times\frac{3a}{2}[\sin^2\theta]_0^{\frac{\pi}{2}}$	M1
	$=6a$	M1 A1 (7)
(b)	$A = 2\pi\int a\sin^3\theta\times 3a\cos\theta\sin\theta\,d\theta$	M1 A1
	$= 6\pi a^2\int\sin^4\theta\cos\theta\,d\theta$	
	$= \frac{6\pi a^2}{5}\left[\sin^5\theta\right]_0^{\frac{\pi}{2}}\times 2$	M1
	$= \frac{12\pi a^2}{5}$	M1 A1 (5)

7. Pearson FP2 textbook, p149, Qn 20

20 **a** $u_n = \cos\left(\frac{n\pi}{4}\right) + (\sqrt{2} - 1)\sin\left(\frac{n\pi}{4}\right)$

b cos and sin are periodic of period 2π, so period for u_n is $\frac{2\pi}{\frac{\pi}{4}} = 8$.

8. Pearson FP2 textbook, p82, Qn 8

8 **a** **i**

∘	2	4	8	10	14	16
2	4	8	16	2	10	14
4	8	16	14	4	2	10
8	16	14	10	8	4	2
10	2	4	8	10	14	16
14	10	2	4	14	16	8
16	14	10	2	16	8	4

ii Closure: All entries in the table are in G.
Identity: The row and column corresponding to 10 are the same as the column and row headings, so 10 is the identity.
Inverse: 10 and 8 are self-inverse; $2^{-1} = 14$, $4^{-1} = 16$
Associativity: Assumed
So G forms a group under ∘.

b $4^2 = 16$ and $4^3 = 10$, so 4 has order 3.

c 2; $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $10 = 2^6$, $14 = 2^5$, $16 = 2^4$
14; $2 = 14^5$, $4 = 14^4$, $8 = 14^3$, $10 = 14^6$, $14 = 14^1$, $16 = 14^2$