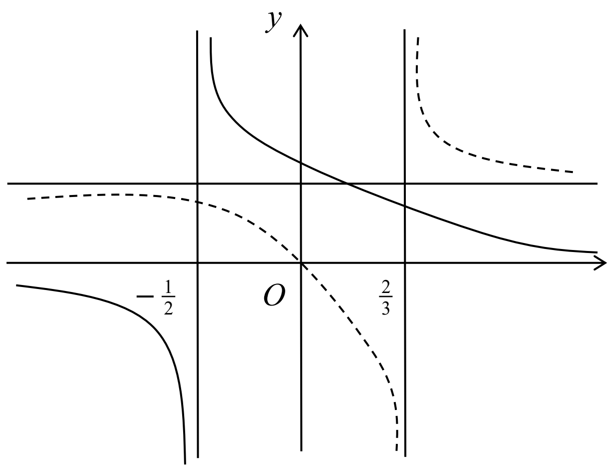
Further Pure Mathematics 1 Practice Paper 5 – mark schemes and answers

Origin of questions:

1. **P4 June 2003, Qn 2**

2.	Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$	B1, B1
	Establishing there are no further critical values	
	Obtaining $2x^2 - 2x + 2$ or equivalent	M1
	$\Delta = 4 - 16 < 0$	A1
	Using exactly two critical values to obtain inequalities	M1
	$-\frac{1}{2} < x < \frac{2}{3}$	A1
		(6 marks)
Graphical alt.	Identifying $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ as vertical asymptotes	B1, B1
	Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes.	M1
	Two correctly drawn curves with no intersections	A1
	As above	M1, A1
		

2. **P5 June 2004, Qn 3**

3.	**(a)**	As $4 = 9(1 - e^2)$, $\therefore e^2 = \frac{5}{9}$ Uses *ae* to obtain that the foci are at $(\pm\sqrt{5}, 0)$	M1, A1 M1 A1 (4)
	(b)	$PS + PS' = e(PM + PM')$ M1 for single statement e.g, PS = ePM $= e \times \frac{2a}{e}$ M1 needs complete method $= 2a = 6$	M1 M1 A1 (3)

3. **P6 June 2004, Qn 2**

(a) $f'(x) = \sec^2 x$ $\quad f''(x) = 2\sec x(\sec x \tan x)$ (or equiv.) M1 A1

$f'''(x) = 2\sec^2 x(\sec^2 x) + 2\tan x(2\sec^2 x \tan x)$ (or equiv.) A1

(3)

$(2\sec^2 x + 6\sec^2 x \tan^2 x)$

$(2\sec^4 x + 4\sec^2 x \tan^2 x)$, $(6\sec^4 x - 4\sec^2 x)$, $(2 + 8\tan^2 x + 6\tan^4 x)$

(b) $\tan\frac{\pi}{4} = 1$ or $\sec\frac{\pi}{4} = \sqrt{2}$ (1, 2, 4, 16) B1

$$\tan x = f\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)f'\left(\frac{\pi}{4}\right) + \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 f''\left(\frac{\pi}{4}\right) + \frac{1}{6}\left(x - \frac{\pi}{4}\right)^3 f'''\left(\frac{\pi}{4}\right)$$ M1

$$= 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3$$ (Allow equiv. fractions) A1(cso)

(3)

(c) $x = \frac{3\pi}{10}$, so use $\left(\frac{3\pi}{10} - \frac{\pi}{4}\right)$ $\quad \left(= \frac{\pi}{20}\right)$ M1

$$\tan\frac{3\pi}{10} \approx 1 + \frac{\pi}{10} + \left(2 \times \frac{\pi^2}{400}\right) + \left(\frac{8}{3} \times \frac{\pi^3}{8000}\right) = 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000}$$ (*) A1(cso)

(2)

(8 marks)

4. **FP1 textbook p159, Qn 8**

$$\frac{1}{n^{2n-1}}$$

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5. **FP3 June 2010, Qn 1**

Question Number	Scheme	Marks
1.	$\pm\frac{a}{e}=8, \quad \pm ae=2$	B1, B1
	$\frac{a}{e}\times ae=a^2=16$	
	$a=4$	B1
	$b^2=a^2(1-e^2)=a^2-a^2e^2$	
	$\Rightarrow b^2=16-4=12$	M1
	$\Rightarrow b=\sqrt{12}=2\sqrt{3}$	A1 (5)
		5

6. P6 June 2003, Qn 7

(a) Normal to plane is $(-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$ B1

Equation of plane: $\mathbf{r} \cdot (-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$ M1

$\Rightarrow \quad -x + 5y + 3z = -1 + 10 - 3 = 6$ or equivalent (*) M1 A1

(4)

[If vector equation of plane is by-passed, then B1 M2 A1]

(b) $\dfrac{1}{\sqrt{35}}$ B1

$|\, 6 - (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \cdot (-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) \,|$ M1 A1

or $|\, \overrightarrow{PQ} \cdot (-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) \,| = |\, 3\mathbf{k} \cdot (-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) \,|$

Distance = $\dfrac{9}{\sqrt{35}}$ or a.w.r.t 1.52 A1

(4)

(c) Direction of one line in plane = $(-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$ M1

Direction of another line in plane = $(3\mathbf{i} - 2\mathbf{k}) - (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1

$\Rightarrow \mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) + s(-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) + t(2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$
M1 A1 (4)

or $(3\mathbf{i} - 2\mathbf{k}) + s(-\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) + t(2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$

[12]

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7. **FP3 June 2010, Qn 8**

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = 4\sec t \tan t \quad \frac{dy}{dt} = 2\sec^2 t$	B1 (both)
	$\frac{dy}{dx} = \frac{2\sec^2 t}{4\sec t \tan t} \quad \left(= \frac{1}{2\sin t}\right)$	M1
	$y - 2\tan t = \frac{1}{2\sin t}(x - 4\sec t)$	M1 A1
	$2y\sin t - \frac{4\sin^2 t}{\cos t} = x - \frac{4}{\cos t}$	
	$2y\sin t = x - \frac{4 - 4\sin^2 t}{\cos t} = x - 4\cos t$ ✻	A1 (5)
(b)	Gradient of l_2 is $-2\sin t$	M1
	$y = -2x\sin t \quad (2)$	A1
	$2(-2x\sin t)\sin t = x - 4\cos t \Rightarrow x = \frac{4\cos t}{1 + 4\sin^2 t} \quad (1)$	M1 A1
	$y = \frac{-8\sin t\cos t}{1 + 4\sin^2 t}$	M1 A1
	$(x^2 + y^2)^2 = \left(\frac{16\cos^2 t}{(1+4\sin^2 t)^2} + \frac{64\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2}\right)^2$	
	$= \frac{256\cos^4 t}{(1+4\sin^2 t)^4}(1+4\sin^2 t)^2 = \frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	M1
	$16x^2 - 4y^2 = \frac{256\cos^2 t}{(1+4\sin^2 t)^2} - \frac{256\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2} = \frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	A1 (8) **13**

8. **FP1 textbook, p176 Qn 3**

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9. **FP1 textbook, p128 Qn 17**

17 a $6\tan\theta + 12\sin\theta + \cos\theta = 1$

$\Rightarrow \frac{12t}{1 - t^2} + \frac{24t}{1 + t^2} + \frac{1 - t^2}{1 + t^2} = 1 \Rightarrow t^4 - 6t^3 - t^2 + 18t = 0$

$\Rightarrow t(t - 2)(t^2 - 4t - 9) = 0$

b $\theta = 0, 2.21, 2.79, 4.26, 6.28$

10. FP3 June 2012, Qn 3

3. (a)	$\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{AC} \times \overrightarrow{BC} = 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$	B1, B1 M1 A1 (4)
(b)	Area of triangle $ABC = \frac{1}{2}\lvert 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}\rvert = \frac{1}{2}\sqrt{1225} = 17.5$	M1 A1 (2)
(c)	Equation of plane is $10x - 15y + 30z = -20$ or $2x - 3y + 6z = -4$ So **r.** (2**i** – 3**j** + 6**k**) = -4 or correct multiple	M1 A1 (2) **(8 marks)**