Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

A level Further Mathematics
Core Pure Mathematics

Practice Paper 5

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **8** questions in this question paper. The total mark for this paper is **74**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The complex number z is given by

$$z = \frac{p + 2\mathrm{i}}{3 + p\mathrm{i}}$$

where p is an integer.

(a) Express z in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

(b) Given that $\arg(z) = \theta$, where $\tan \theta = 1$ find the possible values of p.

(5)

(Total 9 marks)

[Mark scheme for Question 1]

[Examiner Comments]

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2. $$\mathrm{f}(x) = 2x^3 - 6x^2 - 7x - 4.$$

(a) Show that $\mathrm{f}(4) = 0$.

(1)

(b) Use algebra to solve $\mathrm{f}(x) = 0$ completely.

(4)

(Total 5 marks)

[Mark scheme for Question 2]

[Examiner Comments]

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3. The curve C has polar equation

$$r = 1 + 2\cos\theta, \qquad 0 \le \theta \le \frac{\pi}{2}.$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(Total 7 marks)

[Mark scheme for Question 3]

[Examiner Comments]

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4. Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = \frac{\ln x}{x}, \quad x > 0,$$

giving your answer in the form $y = \mathrm{f}(x)$.

(Total 8 marks)

[Mark scheme for Question 4]

[Examiner Comments]

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5. The line l passes through the point $P(2, 1, 3)$ and is perpendicular to the plane Π whose vector equation is

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = 3$$

Find

(a) a vector equation of the line l,

(2)

(b) the position vector of the point where l meets Π.

(4)

(c) Hence find the perpendicular distance of P from Π.

(2)

(Total 8 marks)

[Mark scheme for Question 5]

[Examiner Comments]

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6. $$z = -8 + (8\sqrt{3})\text{i}$$

(a) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^3,

(2)

(c) find the values of w such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

(5)

(Total 10 marks)

Mark scheme for Question 6

Examiner Comments

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7. (i) Find, without using a calculator,

$$\int_3^5 \frac{1}{\sqrt{15 + 2x - x^2}}\,\text{d}x$$

giving your answer as a multiple of π.

(5)

(ii) (a) Show that

$$5\cosh x - 4\sinh x = \frac{\text{e}^{2x} + 9}{2\text{e}^x}.$$

(3)

(b) Hence, using the substitution $u = \text{e}^x$ or otherwise, find

$$\int \frac{1}{5\cosh x - 4\sinh x}\,\text{d}x.$$

(4)

(Total 12 marks)

Mark scheme for Question 7

Examiner Comments

8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \qquad t \geq 0,$$

describes the motion of a particle along the x-axis.

(a) Find the general solution of this differential equation.

(8)

(b) Find the particular solution of this differential equation for which, at $t = 0$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$.

(5)

On the graph of the particular solution defined in part (*b*), the first turning point for $t > 30$ is the point A.

(c) Find approximate values for the coordinates of A.

(2)

(Total 15 marks)

[Mark scheme for Question 8](#)

[Examiner Comments](#)

TOTAL FOR PAPER: 74 MARKS

Further Core Pure Mathematics – Practice Paper 05 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$z = \dfrac{p+2\mathrm{i}}{3+p\mathrm{i}}.\dfrac{3-p\mathrm{i}}{3-p\mathrm{i}}$	M1
	$= \dfrac{3p-p^2\mathrm{i}+6\mathrm{i}+2p}{9+p^2}$	M1
	$= \dfrac{5p}{p^2+9}, \quad + \dfrac{6-p^2}{p^2+9}\mathrm{i}$	A1A1
		(4)
(b)	$\arg(z) = \arctan\left(\dfrac{\frac{6-p^2}{p^2+9}}{\frac{5p}{p^2+9}}\right)$	M1
	$\dfrac{6-p^2}{5p} = 1$	M1
	$p^2 + 5p - 6 = 0$	A1
	$(p+6)(p-1) = 0 \Rightarrow x =$	M1
	$p = 1, p = -6$	A1
		(5)
		(9 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	$\text{f}(x) = 2x^3 - 6x^2 - 7x - 4$	
	$\text{f}(4) = \underline{128 - 96 - 28 - 4 = 0}$	**B1**
		(1)
(b)	$\text{f}(4) = 0 \Rightarrow (x-4)$ is a factor.	
	$\text{f}(x) = (x-4)(\,2x^2 + 2x^2 + 1)$	**M1A1**
	So, $x = \dfrac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$ $(2)\left(x^2 + x + \frac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \frac{1}{2}\right)^2 \pm k \pm \frac{1}{2}\right) k \neq 0 \Rightarrow x =$	**M1**
	$\Rightarrow x = 4,\ \dfrac{-2 \pm 2\text{i}}{4}$	**A1**
		(4)
		(5 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3	$y = r\sin\theta = \sin\theta + 2\sin\theta\cos\theta$	B1
	$\dfrac{dy}{d\theta} = \cos\theta + 2\cos 2\theta$	M1
	$4\cos^2\theta + \cos\theta - 2 = 0$	A1oe
	$\cos\theta = \dfrac{-1 \pm \sqrt{1+32}}{8}$	M1 A1
	$OP = r = 1 + \dfrac{-1+\sqrt{1+32}}{4} = \dfrac{3+\sqrt{33}}{4}$	M1 A1
		(7)
		(7 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4	$\frac{\mathrm{d}y}{\mathrm{d}x}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Integrating factor $\mathrm{e}^{\int\frac{5}{x}}$	M1
	$e^{\int\frac{5}{x}}=e^{5\ln x}=x^5$	A1
	$\int x^3\ln x dx=\frac{x^4\ln x}{4}-\int\frac{x^3}{4}dx$	M1M1 A1
	$=\frac{x^4\ln x}{4}-\frac{x^4}{16}\ (+C)$	A1
	$x^5y=\frac{x^4\ln x}{4}-\frac{x^4}{16}+C$ $y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$	M1A1
		(8)
	(8 marks)	

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	$P(2,\,1,\,3)$ and $\mathbf{r}.(\mathbf{i}-2\mathbf{j}-\mathbf{k})=3$	
	l is parallel to $\mathbf{i}-2\mathbf{j}-\mathbf{k}$	M1
	$"\mathbf{r}"=2\mathbf{i}+\mathbf{j}+3\mathbf{k}+t(\mathbf{i}-2\mathbf{j}-\mathbf{k})$ or $\left(\mathbf{r}-(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\right)\times\left(\mathbf{i}-2\mathbf{j}-\mathbf{k}\right)=\mathbf{0}$ or $\mathbf{r}\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})=(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times\left(\mathbf{i}-2\mathbf{j}-\mathbf{k}\right)$ or $\mathbf{r}\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})=5\mathbf{i}+5\mathbf{j}-5\mathbf{k}$	A1
		(2)
(b)	$\begin{pmatrix}2+t\\1-2t\\3-t\end{pmatrix}\begin{pmatrix}1\\-2\\-1\end{pmatrix}=3$	M1
	$2+t-2(1-2t)-(3-t)=3$	A1
	$2+t-2+4t-3+t=3\Rightarrow t=\ldots\ldots$	dM1
	$t=1\Rightarrow l$ meets Π at $\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$	A1
		(4)
(c)	$PQ=\left\|3\mathbf{i}-\mathbf{j}+2\mathbf{k}-(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\right\|$	
	$=\left\|\mathbf{i}-2\mathbf{j}-\mathbf{k}\right\|=\sqrt{1^2+2^2+1^2}$	M1
	$=\sqrt{6}$	A1
		(2)
		(8 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	Modulus = 16	B1
	Argument = $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$	M1A1
		(3)
(b)	$z^3 = 16^3(\cos(\frac{2\pi}{3}) + \mathrm{i}\sin(\frac{2\pi}{3}))^3 = 16^3(\cos 2\pi + \mathrm{i}\sin 2\pi)$ =4096 or 16^3	M1A1
		(2)
6(c)	$w = 16^{\frac{1}{4}}(\cos(\frac{2\pi}{3}) + \mathrm{i}\sin(\frac{2\pi}{3}))^{\frac{1}{4}} = 2(\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right))$ $\left(= \sqrt{3} + \mathrm{i}\right)$	M1 A1ft
	OR $-1+\sqrt{3}\mathrm{i}$ OR $-\sqrt{3}-\mathrm{i}$ OR $1-\sqrt{3}\mathrm{i}$	M1A2 (1,0)
		(5)
		(10 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(i)	$15+2x-x^2=16-(x-1)^2$	**B1**
	$\int \frac{1}{\sqrt{16-(x-1)^2}}\,dx=\arcsin\left(\frac{x-1}{4}\right)$	**M1A1**
	$\left[\arcsin\left(\frac{x-1}{4}\right)\right]_3^5=\arcsin 1-\arcsin\frac{1}{2}$	**dM1**
	$=\frac{\pi}{3}$	**A1**
		(5)
(ii)(a)	$5\cosh x-4\sinh x=5\left(\frac{e^x+e^{-x}}{2}\right)-4\left(\frac{e^x-e^{-x}}{2}\right)$	**B1**
	$=\frac{e^x+9e^{-x}}{2}$ or $\frac{e^x}{2}+\frac{9e^{-x}}{2}$	**M1**
	$=\frac{e^{2x}+9}{2e^x}$ *	**A1***
		(3)
(ii)(b)	$u=e^x \Rightarrow \frac{du}{dx}=e^x$	**B1**
	$\int \frac{2e^x}{e^{2x}+9}\,dx=\int \frac{2u}{u^2+9}\cdot\frac{du}{u}$	**M1**
	$=\frac{2}{3}\arctan\left(\frac{u}{3}\right)(+c)$	**dM1**
	$=\frac{2}{3}\arctan\left(\frac{e^x}{3}\right)(+c)$	**A1**
		(4)
		12 marks)

Mark scheme for Question 8 (Examiner comment) (Return to Question 8)

Question	Scheme	Marks
8(a)	$m^2 + 6m + 9 = 0 \qquad m = -3$	M1
	C.F. $x = (A + Bt)e^{-3t}$	A1
	P.I. $x = P\cos 3t + Q\sin 3t$	B1
	$\dot{x} = -3P\sin 3t + 3Q\cos 3t$ $\ddot{x} = -9P\cos 3t - 9Q\sin 3t$	M1
	$(-9P\cos 3t - 9Q\sin 3t) + 6(-3P\sin 3t + 3Q\cos 3t) + 9(P\cos 3t + Q\sin 3t) = \cos 3t$	M1
	$-9P + 18Q + 9P = 1$ and $-9Q - 18P + 9Q = 0$	M1
	$P = 0$ and $Q = \dfrac{1}{18}$	A1
	$x = (A + Bt)e^{-3t} + \dfrac{1}{18}\sin 3t$	A1ft
		(8)
(b)	$t = 0$: $x = A = \dfrac{1}{2}$	B1
	$\dot{x} = -3(A + Bt)\mathrm{e}^{-3t} + B\mathrm{e}^{-3t} + \dfrac{3}{18}\cos 3t$	M1
	$t = 0$: $\dot{x} = -3A + B + \dfrac{1}{6} = 0 \qquad B = \dfrac{4}{3}$	M1A1
	$x = \left(\dfrac{1}{2} + \dfrac{4t}{3}\right)\mathrm{e}^{-3t} + \dfrac{1}{18}\sin 3t$	A1
		(5)
(c)	$t \approx \dfrac{59\pi}{6}$ (≈ 30.9)	B1
	$x \approx -\dfrac{1}{18}$	B1ft
		(2)
		15 marks)

Further Core Pure Mathematics – Practice Paper 05 – Examiner report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. Almost all students knew they were required to multiply through by the conjugate of the denominator in question (a), but some lost out accuracy to sign errors or not collecting terms correctly. There were a number of students who did not present the solution in the required form. In question (b) there were some confused attempts with the argument of a complex number, but most students realised that the real and imaginary parts had to be equal and made some progress. If accuracy was lost in question (a) then this impacted on the accuracy marks awarded here too.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. This question was well done by many candidates although there were two particular places in the question where marks were lost. In part (a) virtually all successfully substituted $x = 4$ into $f(x)$ but many failed show sufficient working to justify that $f(4) = 0$. Many incorrectly assumed that $3(4)^3 - 6(4)^2 - 7(4) - 4 = 0$ was enough.

In part (b) many candidates successfully established the quadratic factor using either long division, comparing coefficients or inspection and went on to solve the resulting quadratic by using the formula or completing the square. A small number made an attempt at factorising.

There were a surprising number of cases where the final mark was lost when candidates failed to give the real root as well as the complex ones or confused solving with factorising. It was quite common to see $(x-4)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$ as a conclusion.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. Almost all candidates gained the first two marks for this question with a small minority, however, using $r\cos\theta$ instead of $r\sin\theta$. Differentiation using the chain or product rule was usually successful and a correct quadratic in $\cos\theta$ was often obtained. The majority made a valid attempt at solving their quadratic and gave a correct value for $\cos\theta$. Unfortunately, some candidates thought that their answer was a value for θ and then could not gain the method mark for finding a value for r. Again, some did not realise that their value for r was, in fact, the length OP. Of these, many found x and y and went on to use Pythagoras to obtain the same answer they had started with. However, a number of candidates resorted to decimals and lost the last accuracy mark.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. Most candidates were well prepared and calculated the correct integrating factor and used it effectively. A few candidates omitted to divide through by x and others could not deal with $e^{5\ln x}$. Integration by parts was almost always recognised and usually done well. A few candidates forgot to include a constant of integration and lost marks for their final answer.

Examiner comment for Question 5 **(Mark scheme)** **(Return to Question 5)**

5. In part (a), the vast majority of the students correctly identified the direction vector of the line and thus confidently formed the vector equation of the line, usually in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.

In part (b), the substitution of the line, once expressed in parametric form, into the given equation of the plane and a solution determined for λ, giving the required point, was well known by many of the students and most scored full marks in this part.

However, part (c), a significant minority did not use this point to determine the perpendicular distance of P from the plane as requested, with many resorting to using the result given in the formula book, which scored no marks.

Examiner comment for Question 6 **(Mark scheme)** **(Return to Question 6)**

6. In part (a) long division and comparing coefficients were each used to good effect and errors were rare. Part (b) resulted in many good answers. Some however felt that 3, and even $x + 3$, was a root and others omitted the real root completely. Some confused roots with factors. It was disappointing at this level to see many candidates failing to solve a quadratic correctly. Candidates should be advised to quote the quadratic formula before using it to ensure that they earn the method mark. Completion of the square was often more successful in this question than use of the formula. In part (c) some included an x in their answer and others found a product instead of a sum. The vast majority earned this follow through mark however.

Examiner comment for Question 7 **(Mark scheme)** **(Return to Question 7)**

7. A well–answered question by the majority with full marks commonly awarded.

Part (a) was the most likely to cause problems and a small number of candidates failed to realise the need to complete the square and sometimes tried to factorise the quadratic expression. The negative coefficient of x^2 was handled wrongly by some, with $(x - 1)^2 - 16$ instead of $16 - (x - 1)^2$ seen, leading to an arcosh expression after integration. A small number had the alternative $16 - (1 - x)^2$ but tended to make a sign error when integrating. Those who obtained the correct integrand invariably proceeded correctly. Although it was a standard integral that most dealt with directly, substitutions of $u = x + 1$ and $4 \sin \theta = x - 1$ were quite common and were usually used successfully.

Part (ii) (a) was well answered by almost all candidates. Only a very small number had any errors in their proof, usually from a sign error when combining a subtraction of fractions. The given answer was occasionally miscopied.

Good scoring was also seen in part (ii) (b), with only a few making errors with the substitution (usually producing a numerator of $2u^2$ in the integrand). An arctan term was produced by most although logarithmic expressions or attempts with incorrect partial fractions were occasionally seen. Some candidates lost the “2” during integration. The most common error was to fail to replace u with e^x in the final line of the answer.

Examiner comment for Question 8 **(Mark scheme)** **(Return to Question 8)**

8. Stronger candidates found part (a) routine and often made good progress. However, some got their variables mixed up in the complementary function and others got the wrong sign on the power of e. Most stated the correct standard form of the particular integral and completed the necessary calculations successfully to find their coefficients. A common error was then to ascribe them to the wrong trigonometric term. Part (c) was a challenge to the majority of candidates and was rarely completed successfully even by otherwise capable candidates.