Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Practice Paper 4

(Time: 1 hour 30 minutes)

Paper Reference **9FM0/4A**

Further Mathematics
Advanced
Paper 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 77. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. A complex number z is represented by the point P in the Argand diagram. Given that

$$|z - 3\mathrm{i}| = 3,$$

(a) sketch the locus of P.

(2)

(b) Find the complex number z which satisfies both $|z - 3\mathrm{i}| = 3$ and $\arg(z - 3\mathrm{i}) = \frac{3}{4}\pi$.

(4)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{2\mathrm{i}}{w}.$$

(c) Show that T maps $|z - 3\mathrm{i}| = 3$ to a line in the w-plane, and give the cartesian equation of this line.

(5)

(Total for Question 1 is 11 marks)

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2. $$\mathbf{A} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 3 \end{pmatrix}.$$

(a) Verify that $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and find the corresponding eigenvalue.

(3)

(b) Show that 9 is another eigenvalue of $\mathbf{A}$ and find the corresponding eigenvector.

(5)

(c) Given that the third eigenvector of $\mathbf{A}$ is $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{D}.$$

(5)

(Total for Question 2 is 13 marks)

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3. Prove, by induction, that $3^{2n} + 7$ is divisible by 8 for all positive integers n.

(Total for Question 3 is 8 marks)

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4. Consider the set $M = \{1, 3, 9, 11\}$ under multiplication modulo 16. For the purposes of this question, denote this multiplication by $\times$.

(a) Show that $3 \times (9 \times 11) = (3 \times 9) \times 11$.

(2)

(b) Show that $(M, \times)$ is a group.

(5)

(c) Show that this group is cyclic and write down all possible generators of this group.

(3)

(Total for Question 4 is 10 marks)

5.

$$I_n = \int_0^1 x^n e^x \, dx \quad \text{and} \quad J_n = \int_0^1 x^n e^{-x} \, dx, \quad n \geq 0.$$

(*a*) Show that, for $n \geq 1$,

$$I_n = e - nI_{n-1}.$$

(2)

(*b*) Find a similar reduction formula for J_n .

(3)

(*c*) Show that $J_2 = 2 - \dfrac{5}{e}$.

(3)

(*d*) Show that $\displaystyle\int_0^1 x^n \cosh x \, dx = \tfrac{1}{2}(I_n + J_n)$.

(1)

(*e*) Hence, or otherwise, evaluate $\displaystyle\int_0^1 x^2 \cosh x \, dx$, giving your answer in terms of e.

(4)

(Total for Question 5 is 13 marks)

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6. The curve C has parametric equations

$$x = \cosh t - t, \qquad y = \cosh t + t.$$

Find the exact value of the radius of curvature of C at the point where $t = \ln 3$.

(Total for Question 6 is 9 marks)

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7. A sequence $u_1, u_1, u_1, u_1, \ldots$, is defined by $u_{n+1} = 5u_n - 3(2^n)$, with $u_1 = 7$.

(a) Find the first four terms of the sequence.

(1)

(b) Prove, by induction for $n \in \mathbb{Z}^+$, that $u_n = 5^n + 2^n$.

(7)

(Total for Question 7 is 8 marks)

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8. A set S contains n distinct elements.

(a) Write an expression for the number of different subsets of S containing three elements.

(1)

(b) Write an expression for the total number of different subsets of S.

(1)

(c) By considering subsets, or otherwise, show that $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.

(3)

(Total for Question 8 is 5 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 77 MARKS

Guide:

1. P6 June 2005, Qn 4

2. P6 June 2002, Qn 5

3. Further Maths Syllabus B, June 1981

4. Pearson FP2 textbook, p62, Qn 14

5. P5 June 2003, Qn 7

6. P5 June 2004, Qn 4

7. Pearson FP2 textbook, p145, Qn 15

8. Pearson FP2 textbook, p40, Qn 29