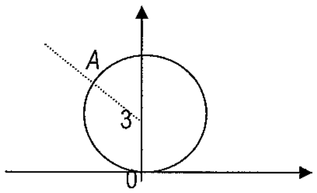
Further Pure Mathematics 2 Practice Paper 4 – mark schemes

Origin of questions:

1. **P6 June 2005, Qn 4**

4	(a)  Circle	M1
	Correct circle.	A1 (2)
	(centre (0, 3), radius 3)	
	(b) Drawing correct **half**-line passing as shown	B1
	Find either *x* or *y* coord of *A*.	M1A1
	$z = -\frac{3\sqrt{2}}{2} + (3 + \frac{3\sqrt{2}}{2})\text{i}$	A1 (4)
	[Algebraic approach, i.e. using $y = 3 - x$ and equation of circle will only gain M1A1, unless the second solution is ruled out, when B1 can be given by implication, and final A1, if correct]	
	(c) $\lvert z - 3\text{i}\rvert = 3 \rightarrow \left\lvert \frac{2\text{i}}{\omega} - 3\text{i} \right\rvert = 3$	M1
	$\Rightarrow \frac{\lvert 2\text{i} - 3\text{i}\omega \rvert}{\lvert\omega\rvert} = 3$	A1
	$\Rightarrow \lvert \omega - {}^2/_3 \rvert = \lvert \omega \rvert$	M1A1
	Line with equation $u = {}^1/_3$ $(x = {}^1/_3)$	A1 (5)

2. P6 June 2002, Qn 5

Question Number	Scheme	Marks
5. (*a*)	$\begin{pmatrix}1&0&4\\0&5&4\\4&4&3\end{pmatrix}\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}6\\-6\\3\end{pmatrix}=3\begin{pmatrix}2\\-2\\1\end{pmatrix}$, $\therefore$ eigenvalue is 3	M1A1, A1 **(3)**
(*b*)	Either $\begin{vmatrix}-8&0&4\\0&-4&4\\4&4&-6\end{vmatrix}=-8(24-16)+4(16)=-64+64=0$	M1 A1 **(2)**
Alt(*b*)	or $\begin{vmatrix}1-\lambda&0&4\\0&5-\lambda&4\\4&4&3-\lambda\end{vmatrix}=0 \Rightarrow$	
	$(1-\lambda)(5-\lambda)(3-\lambda)-16(1-\lambda)-16(5-\lambda)=0$	
	$\Rightarrow (3-\lambda)(\lambda-9)(\lambda+3)=0 \Rightarrow \lambda$ is an eigenvalue	M1 A1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}$ eigenvector $\Rightarrow$ $x+4z=9x$, $5y+4z=9y$, $4x+4y+3z+9z$	M1
	At least two of these equations	
	Attempt to solve $z=2x$, $z=y$, $2x+2y=3z$	A1
	$\therefore \begin{pmatrix}1\\2\\2\end{pmatrix}$	A1 **(3)**
(*c*)	Make e.vectors unit to obtain $\mathbf{P}=\begin{pmatrix}\frac{2}{3}&\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{2}{3}&-\frac{2}{3}\end{pmatrix}$ columns in	M1, A1ft
	any order	
	$\mathbf{D}=\begin{pmatrix}3&0&0\\0&9&0\\0&0&\lambda_3\end{pmatrix}$, where $\lambda_3=-3$, **P** and **D** consistent	M1, A1, B1 **(5)**
		(13 marks)

3. Further Maths Syllabus B, June 1981, Paper 2 Qn 8

Proof

4. Pearson FP2 textbook, p62, Qn 14

14 **a** $3 \times (9 \times 11) = 3 \times 3 = 9$; $(3 \times 9) \times 11 = 11 \times 11 = 9$

b Cayley table is

$\times$	1	3	9	11
1	1	3	9	11
3	3	9	11	1
9	9	11	1	3
11	11	1	3	9

<u>Closure:</u> All entries in the table are in M.
<u>Identity:</u> The row and column corresponding to 1 are the same as the column and row headings, so 1 is the identity.
<u>Inverses:</u> $1^{-1} = 1$, $3^{-1} = 11$, $9^{-1} = 9$
Associativity: By part **a**
So $(M, \times)$ is a group.

c $3^1 = 3$, $3^2 = 9$, $3^3 = 11$, $3^4 = 1$, so M is a cyclic group with generator 3. 11 is also a generator of $(M, \times)$.

5. P5 June 2003, Qn 7

Question Number	Scheme	Marks
7. (*a*)	$I_n = \left[x^n e^x\right]_0^1 - n\int_0^1 x^{n-1} e^x \, dx = e - nI_{n-1}$ (*)	M1 A1 (2)
	cso	
(*b*)	$J_n = \left[-x^n e^{-x}\right]_0^1 + n\int_0^1 x^{n-1} e^{-x} \, dx$	M1 A1
	$= -e^{-1} + nJ_{n-1}$	A1 (3)
(*c*)	$J_2 = -e^{-1} + 2J_1$	
	$J_1 = -e^{-1} + J_0$	M1
	J_2 <u>and</u> J_1	
	$= -e^{-1} + \int_0^1 e^{-x} \, dx$	
	$= -e^{-1} + (1 - e^{-1})$ $(= 1 - 2e^{-1})$	A1
	$J_2 = -e^{-1} + 2(1 - 2e^{-1}) = 2 - \frac{5}{e}$ (*)	A1 (3)
(*d*)	$\int_0^1 x^n \cosh x \, dx = \int_0^1 x^n \left(\frac{e^x + e^{-x}}{2}\right) dx = \frac{1}{2}(I_n + J_n)$ (*)	B1 (1)
(*e*)	$I_2 = e - 2I_1 = e - 2(e - I_0) = 2I_0 - e$	
	$= 2\int_0^1 e^x \, dx - e = 2[e - 1] - e$ $(= e - 2)$	M1 A1
	$\frac{1}{2}(I_2 + J_2) = \frac{1}{2}(e - 2 + 2 - \frac{5}{e}) = \frac{1}{2}(e - \frac{5}{e})$	M1 A1 (4)
		(13 marks)

6. P5 June 2004, Qn 4

4.	$\frac{dx}{dt} = \sinh t - 1 \qquad \frac{dy}{dt} = \sinh t + 1$	M1 A1
	$\frac{d^2x}{dt^2} = \cosh t \qquad \frac{d^2y}{dt^2} = \cosh t$	M1 A1
	Use $\rho = \left\| \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}} \right\|, = \frac{[(\sinh t - 1)^2 + (\sinh t + 1)^2]^{\frac{3}{2}}}{\sinh t \cosh t - \cosh t - \sinh t \cosh t - \cosh t}$ $(=(-)\sqrt{2}\cosh^2 t)$	M1, A1
	When t = ln3, cosh t = 5/3 (or sinh t = 4/3)	M1A1
	$\therefore \rho = (-)\sqrt{2} \times \frac{25}{9}$	A1 **(9)**

7. Pearson FP2 textbook, p145, Qn 15

15 a $u_1 = 7, u_2 = 29, u_3 = 133, u_4 = 641$

b Basis: $u_1 = 5 + 2 = 7$; Assumption: $u_k = 5k + 2k$

Induction: $u_{k+1} = 5(5^k + 2^k) - 3(2^k) = 5^{k+1} + 2k^{k+1}$

So if the closed form is valid for $n = k$, it is valid for $n = k + 1$.

Conclusion: $u_n = 5^n + 2^n$ for all $n \in \mathbb{Z}^+$.

8. Pearson FP2 textbook, p40, Qn 29

29 a $\binom{n}{3} = \frac{n!}{(n-3)!3!}$ **b** 2^n

c $\sum_{r=0}^{n} \binom{n}{r} = \sum_{r=0}^{n}$(Number of subsets of S with r elements)

= total number of subsets of S

$= 2^n$