Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Practice Paper 4

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

Further Mathematics

Advanced

Paper 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- The total mark for this part of the examination is 75. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. Referred to a fixed origin O, the position vectors of three non-collinear points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. By considering $\overrightarrow{AB} \times \overrightarrow{AC}$, prove that the area of ΔABC can be expressed in the form $\frac{1}{2}\left|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}\right|$.

(Total for Question 1 is 5 marks)

2.

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{10}y^2 = x, \qquad y = 2 \text{ at } x = 1.$$

Use the approximation $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$, with a step length of 0.1 to estimate the values of y at $x = 1.1$ and $x = 1.2$, giving your answers to 2 decimal places.

(Total for Question 2 is 6 marks)

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3. Using the substitution $t = \tan \frac{x}{2}$,

(a) show that the integral $\displaystyle\int \frac{1}{4\cos x - 3\sin x}\,\mathrm{d}x$ can be written as $\displaystyle\int \frac{-1}{2t^2 + 3t - 2}\,\mathrm{d}t$.

(3)

(b) Hence evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{4\cos x - 3\sin x}\,\mathrm{d}x$.

(5)

(Total for Question 3 is 8 marks)

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4. Find the Taylor series expansion of

(a) $\sin 2x$ about $\frac{\pi}{6}$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^4$,

(4)

(b) $\cosh x$ about $x = \ln 5$ in ascending powers of $(x - \ln 5)$ up to and including the term in $(x - \ln 5)^4$.

(5)

(Total for Question 4 is 9 marks)

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5. (*a*) Use the substitution $y = vx$ to transform the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x + y)(x + y)}{x^2}, \; x > 0 \qquad \text{(I)}$$

into the equation

$$x\frac{\mathrm{d}v}{\mathrm{d}x} = (2 + v)^2. \qquad \text{(II)}$$

(4)

(*b*) Solve the differential equation II to find v as a function of x.

(5)

(*c*) Hence show that

$$y = -2x - \frac{x}{\ln x + c}, \text{ where } c \text{ is an arbitrary constant,}$$

is a general solution of the differential equation I.

(1)

(Total for Question 5 is 10 marks)

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6. The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(*a*) the value of α,

(4)

(*b*) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants.

(4)

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(*c*) find the shortest distance between the lines l_1 and l_2 .

(3)

(Total for Question 6 is 11 marks)

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7. The hyperbola C has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

(*a*) Show that an equation of the normal to C at the point P $(a \sec t, b \tan t)$ is

$$ax \sin t + by = (a^2 + b^2) \tan t.$$

(6)

The normal to C at P cuts the x-axis at the point A and S is a focus of C. Given that the eccentricity of C is $\frac{3}{2}$, and that $OA = 3OS$, where O is the origin,

(*b*) determine the possible values of t, for $0 \leq t < 2\pi$.

(8)

(Total for Question 7 is 14 marks)

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8. (*a*) Sketch the graph of $y = |\, x^2 - a^2 \,|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes.

(2)

(*b*) Solve $|\, x^2 - a^2 \,| = a^2 - x$, $a > 1$.

(6)

(*c*) Find the set of values of x for which $|\, x^2 - a^2 \,| > a^2 - x$, $a > 1$.

(4)

(Total for Question 8 is 12 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 75 MARKS

Origin of questions:

1. **P6 June 2003, Qn 1**
2. **P6 June 2004, Qn 1**
3. **FP1 textbook, p159, Qn 12**
4. **FP1 textbook, p135, Qns 7 and 9**
5. **P4 January 2003, Qn 5**
6. **FP3 June 2009, Qn 7**
7. **P5 June 2003, Qn 3**
8. **FP2 June 2009, Qn 7**