Further Pure Mathematics 1 Practice Paper 4 – mark schemes

Origin of questions:

1. P6 June 2003, Qn 1

$\overrightarrow{AB} \times \overrightarrow{AC} = (\mathbf{b}-\mathbf{a}) \times (\mathbf{c}-\mathbf{a}) = (\mathbf{b} \times \mathbf{c}) - (\mathbf{a} \times \mathbf{c}) - (\mathbf{b} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{a})$	M1 A1
Using $\mathbf{a} \times \mathbf{c} = -\,\mathbf{c} \times \mathbf{a}$ or $\mathbf{b} \times \mathbf{a} = -\,\mathbf{a} \times \mathbf{b}$ or $\mathbf{a} \times \mathbf{a} = \mathbf{0}$	B1
$\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = AB.\,AC \sin\theta = 2 \times$ area of triangle, or equivalent	M1
Final result : $\frac{1}{2} \lvert \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} \rvert$ (*)	A1 cso
	[5 marks]

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2. P6 June 2004, Qn 1

$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = x_0 - \frac{1}{10}{y_0}^2 = 1 - 0.4$ (= 0.6) (Possibly implicit)	B1
$y_1 = 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 + y_0 = (0.1 \times 0.6) + 2 = 2.06$	M1 A1
$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = x_1 - \frac{1}{10}{y_1}^2 = 1.1 - \frac{1}{10}(2.06)^2$ (= 0.67564)	A1ft
$y_2 = 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 + y_1 = 0.067564 + 2.06 = 2.13$ (2 d.p.)	M1 A1
	6 marks

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3. FP1 textbook, p159, Qn 12

12 a $\displaystyle\int \frac{1}{4\cos x - 3\sin x}\,\mathrm{d}x = \int \frac{1}{\frac{4(1-t^2)}{1+t^2} - \frac{6t}{1+t^2}} \times \frac{2}{1+t^2}\,\mathrm{d}t$

$\displaystyle = \int \frac{2}{4 - 4t^2 - 6t}\,\mathrm{d}t = \int \frac{-1}{2t^2 + 3t - 2}\,\mathrm{d}t$

b -0.3429 (4 s.f.)

4. **FP1 textbook, p135, Qns 7 and 9**

7 $\frac{\sqrt{3}}{2} + 1\left(x - \frac{\pi}{6}\right) - \sqrt{3}\left(x - \frac{\pi}{6}\right)^2 - \frac{2}{3}\left(x - \frac{\pi}{6}\right)^3 + \frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^4 + \ldots$

9 $\frac{13}{5} + \frac{12}{5}(x - \ln 5) + \frac{13}{10}(x - \ln 5)^2 + \frac{2}{5}(x - \ln 5)^3 + \frac{13}{120}(x - \ln 5)^4 + \ldots$

5. **P4 January 2003, Qn 5**

5.	(*a*)	$v + x\frac{dv}{dx}$,$= (4 + v)(1 + v)$	M1, M1	
		$x\frac{dv}{dx} = v^2 + 5v + 4 - v$	A1	
		$x\frac{dv}{dx} = (v + 2)^2$ *	A1	**(4)**
	(*b*)	$\int \frac{1}{(v+2)^2}\,dv = \int \frac{1}{x}\,dx$	B1, M1	
		$-\frac{1}{2+v} = \ln x + c$ must have + *c*	M1 A1	
		$2 + v = -\frac{1}{\ln x + c}$	M1	
		$v = -\frac{1}{\ln x + c} - 2$	A1	**(5)**
	(*c*)	$y = -2x - \frac{x}{\ln x + c}$	B1	**(1)**
				(10 marks)

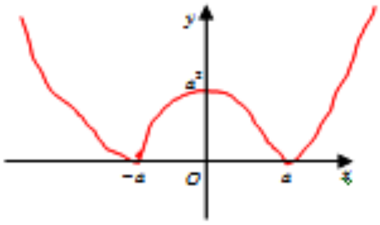
6. FP3 June 2009, Qn 7

Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0$ $(\mu=1$ but this need not be seen$)$.	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix}$ = -6**i**+2**j**-3**k** is perpendicular to both lines and hence to the plane	M1 A1
	The plane has equation **r.n**=**a.n** , which is $-6x + 2y - 3z = -14$,	M1
	i.e. $-6x + 2y - 3z + 14 = 0$.	A1 o.a.e. (4)
OR (b)	**Alternative scheme** Use (1, -1,2) and (α ,-4,0) in equation $ax+by+cz+d=0$ And third point so three equations, and attempt to solve Obtain $6x-2y+3z=$ $(6x-2y+3z)-14=0$	M1 M1 A1 A1 o.a.e. (4)
(c)	$(\mathbf{a}_1 - \mathbf{a}_2) = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	M1
	Use formula $\dfrac{(\mathbf{a}_1 - \mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert} = \dfrac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\sqrt{(36+4+9)}} = \left(\dfrac{-6}{7}\right)$	M1
	Distance is $\dfrac{6}{7}$	A1 (3) **[11]**

7. P5 June 2003, Qn 3

3.	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \frac{1}{2}\int \frac{10}{\sqrt{x^2+\frac{9}{4}}}\,dx$	M1
	$= \frac{10}{2}\,\text{arsinh}\left(\frac{2x}{3}\right) \quad \left(= 5\ln\left[\frac{2x}{3}+\sqrt{\frac{4x^2}{9}+1}\right]\right)$	M1 A1
	$[\ ]_0^5 = 5\ \text{arsinh}\frac{10}{3} \quad \left(= 5\ln\left(\frac{10}{3}+\sqrt{\frac{109}{9}}\right) \approx 9.594\right)$ ft on 5	M1 A1 ft
	Area = 9.594 × 100 = 960 (m^2)	M1 A1
		(7 marks)
	Using a substitution	
	(i) $2x = 3\sinh\theta$; $2\,dx = 3\cosh\theta\,d\theta$	
	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \int \frac{10}{3\cosh\theta}\times\frac{3}{2}\cosh\theta\,d\theta$ complete subs.	M1
	$= 5\int d\theta = 5\ \text{arsinh}\ \frac{2x}{3}$	M1 A1
	then as before,	
	or changing limits to 0 and arsinh $\frac{10}{3}$ (or $\ln\left(\frac{10}{3}+\sqrt{\frac{109}{9}}\right)$) can gain this A1	
	(ii) $2x = 3\tan\theta$; $2\,dx = 3\sec^2\theta\,d\theta$	
	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \int \frac{10}{\sqrt{9\tan^2\theta+9}}\times\frac{3}{2}\sec^2\theta\,d\theta$	M1
	$= 5\int \sec\theta\,d\theta = 5\ln(\sec\theta+\tan\theta)$	M1
	Limits are 0 and arctan $\frac{10}{3}$	A1
	$[\ ]_0^{\arctan\frac{10}{3}} = 5\ln\left(\sqrt{\frac{100}{9}+1}+\frac{10}{3}\right)$ etc	M1 A1ft

8. FP2 June 2009, Qn 7

Q7	$y = \lvert x^2 - a^2 \rvert,\ a > 1$		
(a)		Correct Shape. Ignore cusps. Correct coordinates.	B1 B1
			(2)
(b)	$\lvert x^2 - a^2 \rvert = a^2 - x\ ,\ a > 1$		
	$\{\lvert x \rvert > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow x^2 + x - 2a^2 = 0$		
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\lvert x \rvert < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow x = 0, 1$	$x = 0$ $x = 1$	B1 A1
			(6)
(c)	$\lvert x^2 - a^2 \rvert > a^2 - x\ ,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}$ {or} $x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value x is greater than their maximum value	B1 ft B1 ft
	{or} $0 < x < 1$	For $\{\lvert x \rvert < a\}$, Lowest $< x <$ Highest $0 < x < 1$	M1 A1
			(4)
			[12]