Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics

Core Pure Mathematics

Practice Paper 4

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **8** questions in this question paper. The total mark for this paper is **70**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(Total 4 marks)

Mark scheme for Question 1

Examiner Comments

2.

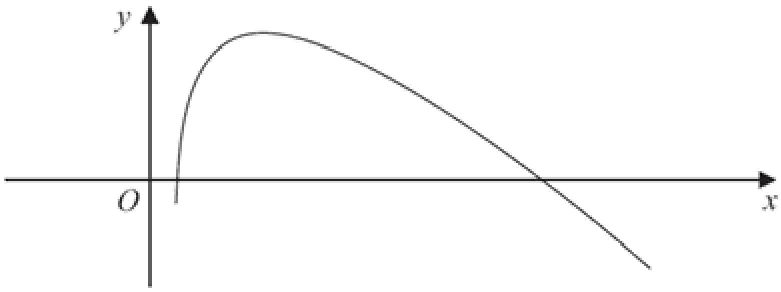

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \text{ arcosh } x - 9x, \qquad x \geq 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r\ln 3 + s\right)$ where p, q, r and s are integers.

(Total 7 marks)

Mark scheme for Question 2

Examiner Comments

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3. The curve C has equation

$$y = \frac{1}{\sqrt{x^2 + 2x - 3}}, \qquad x > 1$$

(a) Find $\int y \, dx$

(3)

The region R is bounded by the curve C, the x-axis and the lines with equations $x = 2$ and $x = 3$. The region R is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated. Give your answer in the form $p\pi \ln q$, where p and q are rational numbers to be found.

(4)

(Total 7 marks)

Mark scheme for Question 3

Examiner Comments

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4. $$f(x) = 5\cosh x - 4\sinh x, \quad x \in \mathbb{R}.$$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$.

(2)

Hence

(b) solve $f(x) = 5$,

(4)

(c) show that $\displaystyle\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x} \, dx = \frac{\pi}{18}$.

(5)

(Total 11 marks)

Mark scheme for Question 4

Examiner Comments

5. (a) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x^2.$$

(8)

(b) Find the particular solution for which, at $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$.

(6)

(Total 14 marks)

[Mark scheme for Question 5](#)

[Examiner Comments](#)

6.

$$y = \ln\left(\frac{1}{1-2x}\right), \qquad |x| < \frac{1}{2}$$

(a) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and $\frac{d^3y}{dx^3}$

(4)

(b) Hence, or otherwise, find the series expansion of $\ln\left(\frac{1}{1-2x}\right)$ about $x = 0$, in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(3)

(c) Use your expansion to find an approximate value for $\ln\left(\frac{3}{2}\right)$, giving your answer to 3 decimal places.

(3)

(Total 10 marks)

[Mark scheme for Question 6](#)

[Examiner Comments](#)

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7. The line l_1 has equation $\frac{x-2}{4} = \frac{y-4}{-2} = \frac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(Total 7 marks)

Mark scheme for Question 7

Examiner Comments

8.

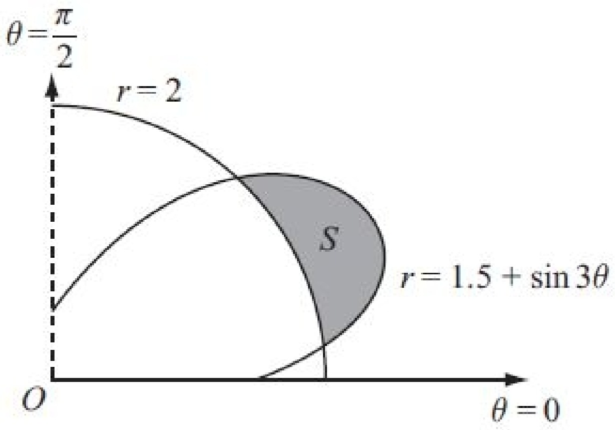


Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

and $$r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

(Total 10 marks)

Mark scheme for Question 8

Examiner Comments

TOTAL FOR PAPER: 70 MARKS

Mark scheme for Question 1

(Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1	$\mathbf{M}=\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$	
	$\det\mathbf{M} = x(4x-11)-(3x-6)(x-2)$	M1
	$x^2+x-12\ (=0)$	A1
	$(x+4)(x-3)\ (=0) \rightarrow x = \ldots$	M1
	$x=-4,\ x=3$	A1
		(4)
		(4 marks)

Mark scheme for Question 2

(Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2	$\frac{dy}{dx}=\frac{40}{\sqrt{(x^2-1)}}-9$	M1A1
	Put $\frac{dy}{dx}=\mathbf{0}$ and obtain $x^2=$**…. (Allow sign errors only)**	dM1
	$x=\frac{41}{9}$	M1A1
	$y = 40\ln\{(\frac{41}{9})+\sqrt{(\frac{41}{9})^2-1}\}-"41"$	M1
	So $y=80\ln 3 - 41$	A1
		(7)
		(7 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	$x^2 + 2x - 3 = (x+1)^2 - 4$	M1
	$\int \frac{1}{\sqrt{(x+1)^2-4}}\,\mathrm{d}x = \operatorname{arcosh}\frac{(x+1)}{2}(+c)$	M1A1
	or $\ln\left((x+1)+\sqrt{(x+1)^2-4}\right)$	
		(3)
(b)	$S = \pi\int y^2\,\mathrm{d}x = \pi\int\left(\frac{1}{\sqrt{x^2+2x-3}}\right)^2\mathrm{d}x$	M1
	$= \int\frac{1}{(x+1)^2-4}\,\mathrm{d}x = \left[\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)\right]$	M1A1
	$= \frac{\pi}{4}\left(\ln\frac{1}{3} - \ln\frac{1}{5}\right) = \frac{\pi}{4}\ln\frac{5}{3}$	A1
		(4)
		(7 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4(a)	$\text{f}(x) = 5\cosh x - 4\sinh x = 5\times\frac{1}{2}(e^x + e^{-x}) - 4\times\frac{1}{2}(e^x - e^{-x})$	M1
	$= \frac{1}{2}(e^x + 9e^{-x})$ *	A1cso
		(2)
(b)	$\frac{1}{2}(e^x + 9e^{-x}) = 5 \Rightarrow e^{2x} - 10e^x + 9 = 0$	M1A1
	So $e^x = 9$ or 1 and $x = \ln 9$ or 0	M1A1
		(4)
(c)	Integral may be written $\int \frac{2e^x}{e^{2x}+9}\,\text{d}x$	B1
	This is $\frac{2}{3}\arctan\left(\frac{e^x}{3}\right)$	M1A1
	Uses limits to give $\left[\frac{2}{3}\arctan 1 - \frac{2}{3}\arctan(\frac{1}{\sqrt{3}})\right] = \left[\frac{2}{3}\times\frac{\pi}{4} - \frac{2}{3}\times\frac{\pi}{6}\right] = \frac{\pi}{18}$ *	DM1 A1cso
		(5)
		(11 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ *or* 1	**M1A1**
	C.F is $Ae^{-\frac{2}{3}x} + Be^x$	**A1ft**
	Let PI $= \lambda x^2 + \mu x + \nu$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into d.e.	**M1**
	Giving $\lambda = -\frac{1}{2}$, $\mu = \frac{1}{2}$ *and* $\nu = -\frac{7}{4}$	**A1A1 A1**
	$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^x$	**A1ft**
		(8)
(b)	Use boundary conditions:	
	$2 = -\frac{7}{4} + A + B$	**M1 A1ft**
	$y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^x$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$	**M1 M1**
	Solve to give A=3/4, $B = 3$ $(\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^x)$	**M1A1**
		(6)
		(14 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	$y=\ln\left(\frac{1}{1-2x}\right)$	
	$y=\ln(1-2x)^{-1}=(\ln 1)-\ln(1-2x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{1-2x}\times -2\left(=\frac{2}{1-2x}\right)$	M1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\times(1-2x)^{-2}\times -2$ $\left(=\frac{4}{(1-2x)^2}\right)$	A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-8\times(1-2x)^{-3}\times -2$ $\left(=\frac{16}{(1-2x)^3}\right)$	A1
		(4)
(b)	$(y_0=0), y'_0=2, y''_0=4, y'''_0=16$	M1
	$(y=)(0)+2x+\frac{4x^2}{2!}+\frac{16x^3}{3!}$	M1
	$y=2x+2x^2+\frac{8}{3}x^3$	A1cao
		(3)
(c)	$\frac{1}{1-2x}=\frac{3}{2}\Rightarrow x=\frac{1}{6}$	B1
	$\ln\left(\frac{3}{2}\right)\approx 2\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)^2+\frac{8}{3}\left(\frac{1}{6}\right)^3$	M1
	$=0.401$	A1cso
		(3)
		(10 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks	AOs
7	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=...$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1),4+6(-2),-6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
			(7 marks)

Notes:

M1: Substitutes the parametric equation of the line into the equation of the plane and solves for λ

A1: Obtains the correct coordinates of the intersection of the line and the plane

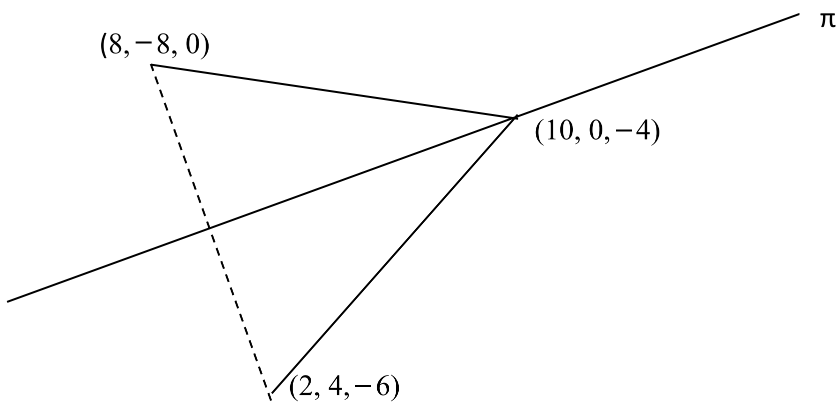
M1: Substitutes the parametric form of the line perpendicular to the plane passing through $(2,\ 4,\ -6)$ into the equation of the plane to find t

M1: Find the reflection of $(2,\ 4,\ -6)$ in the plane

A1: Correct coordinates

M1: Determines the direction of l by subtracting the appropriate vectors

A1: Correct vector equation using the correct notation



Mark scheme for Question 8 (Examiner comment) (Return to Question 8)

Question	Scheme	Marks
8(a)	$1.5 + \sin 3\theta = 2 \rightarrow \sin 3\theta = 0.5 \therefore 3\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6}\right)$	**M1A1**
	and $\therefore \theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$	**A1**
		(3)
(b)	Area $= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5+\sin 3\theta)^2 \, d\theta\right], \; -\frac{1}{9}\pi\times 2^2$	**M1M1**
	$= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25+3\sin 3\theta+\frac{1}{2}(1-\cos 6\theta))d\theta\right]-\frac{1}{9}\pi\times 2^2$	**M1**
	$= \frac{1}{2}\left[(2.25\theta-\cos 3\theta+\frac{1}{2}(\theta-\frac{1}{6}\sin 6\theta))\right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} -\frac{1}{9}\pi\times 2^2$	**M1A1**
	$=\frac{13\sqrt{3}}{24}-\frac{5\pi}{36}$	**M1A1**
		(7)
	(10 marks)	

Further Core Pure Mathematics – Practice Paper 04 – Examiner Report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. The majority of candidates understood what was meant by a singular matrix and gained full marks for this question. The usual sign errors caused some to lose marks and some factorised incorrectly. There were a significant number of candidates who wrote down the solutions to their quadratic with no working shown, possibly using a graphic calculator. Some put the determinant equal to 1 and some went on to find the inverse matrix.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. This was a good source of marks for many candidates with approximately 70% scoring full marks. The differentiation was usually very sound and the loss of marks usually resulted from slips when solving their equation in x and/or slips when substituting into the logarithmic form of arcosh to find the y-coordinate

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. In part (a), the majority of the students began by completing the square correctly and identifying the arcosh form, although a few gave arcosh $\left(\frac{x}{2}\right)$ rather than arcosh$\left(\frac{x+1}{2}\right)$. Some gave the logarithmic equivalent.

   Part (b) was met with less success and it was surprisingly common to see $2\pi\int y^2\mathrm{d}x$ quoted for a volume of revolution and some students attempted to use the formula for surface area. Those who were integrating y^2 could often score the middle two marks by using the correct logarithmic form either from direct integration or by using a substitution or partial fractions.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. This question discriminated well with a good spread of marks. Full marks were gained by 36.5% of the candidates, this was also the modal mark, and only 10% gained 5 or fewer marks. Most candidates were able to complete part (a) correctly. In part (b) most candidates were able to obtain the correct quadratic equation, though not all were able to solve it efficiently or correctly. There were also just a few who did not simplify the answer ln1 to get 0. The integration in part (c) caused a great deal of difficulty, and a wide range of methods were attempted. Few recognised the standard integral. Substituting $u = \mathrm{e}^x$ generally proved successful, with some candidates then substituting tan θ for u, but many clearly invalid attempts were made such as 'splitting' the denominator or using a log, usually a getting some multiple of $\ln(\mathrm{e}^{2x} + 9)$.

   Very badly constructed algebra was seen on occasions, such as candidates trying to get the reciprocal of $\frac{\mathrm{e}^x}{2}+\frac{9\mathrm{e}^{-x}}{2}$, by stating this as being equal to $\frac{2}{\mathrm{e}^x}+\frac{2\mathrm{e}^x}{9}$. Those candidates who integrated correctly nearly always showed sufficient working to justify achieving the given answer.

Examiner comment for Question 5 **(Mark scheme)** **(Return to Question 5)**

5. Candidates found this question more demanding. Mistakes usually centred round the calculation of the Particular Integral and were usually of an algebraic nature. So, for example expanding $-2(px^2 + qx + r)$ as $-2px^2 + qx + r$. This inevitably lost them at least 2 accuracy marks in part (a). The most common mistakes in calculating their Particular Integral were to select a one-term or two-term quadratic. There were very few problems with establishing the Complementary Function part of this question. Most candidates recognised $3m^2 - m - 2 = 0$ as the quadratic equation needed and proceeded to solve it correctly and write the Complimentary Function as $Ae^{(-2/3)x} + Be^x$. Once the solution was found most candidates proceeded to differentiate and substitute correctly. The method marks and follow through accuracy mark in part (b) helped to reward candidates who had gone astray in part (a).

Examiner comment for Question 6 **(Mark scheme)** **(Return to Question 6)**

6. This was a generally well answered, accessible question with the majority of students able to achieve the method marks throughout, and certainly in (b) and (c). The main difficulty students had with this question was with part (a) where many students did not use the chain rule to successfully obtain the first derivative and so lost all accuracy marks in the question. Those who were successful in (a) generally went on to score full marks for the question.

Although there were a number of fully correct solutions to part (a), the initial differentiation of the function caused difficulties for some students. Most did manage to access the method for attempting the chain rule, though sign errors were common. For those who did not achieve the method, many did achieve $\pm\frac{1}{(1-2x)}$ simply forgetting to multiply by $\frac{\mathrm{d}}{\mathrm{d}x}(1-2x)$. Most of the correct attempts arose from first applying the logarithmic index law to achieve the form $y = -\ln(1 - 2x)$ and thus ease the differentiation. Those attempting to apply the chain rule directly were more susceptible to accuracy errors (in sign and/or multiple) creeping in. Those who did the first derivative correctly generally succeeded with the second and third derivatives, albeit sometimes in a very laboured attempt. This generally arose from failing to cancel common factors in the first derivative so proceeding from $y' = \frac{2(1-2x)}{(1-2x)^2}$ and using the quotient rule, obtaining increasingly complicated expressions for the later derivatives.

Other incorrect responses to this part, where no method was gained, often involved long, repeated and complicated attempts which in many cases failed to eliminate log terms from their responses and so included log terms as either multiples or denominators of their expressions.

Only very few student attempt the implicit method, and these were usually well done for the first derivative but were seldom successful in the higher order derivatives if students did not return to an expression in x.

Part (b) was answered well with the majority of students achieving both the method marks. Cases of an incorrect Maclaurin formula were rare. However, working was often not shown, relying on the evaluation of their values for y at $x = 0$ being implied by their expression. Students should be encouraged to show their full working, so that when an incorrect substitution is made the method is still clear; some lost marks here when their values did not imply correct use of $x = 0$. Also, many did not write out the correct formula for the Maclaurin series before substituting, with again the same potential for loss of marks, especially as they were often doing two things at once. Fortunately in this case the values were easy to verify in most cases, and so marks were picked up under implication, but those who showed no working risked losing both marks.

Students who had obtained incorrect derivatives involving log terms in part (a) often found that their coefficient of x^3 was zero. When this occurred, many simply omitted the x^3 term completely, forfeiting the second method mark.

There were a reasonable number of attempts at the alternative method to (b), which were usually successful. This meant that even with a significant loss of marks in part (a) they could achieve full marks in (b), and some of the students who realised they had errors in (a) did opt for this approach.

Some students did not link the series to the function $y = \ldots$ or $\ln(1-2x)^{-1}$ but used an undefined $f(x) = \ldots$ and so lost the final accuracy mark even when all else was correct.

The first two marks of part (c) were gained by most students regardless of what had gone before, with ±0.401 being seen in many attempts. A few who obtained the negative result attempted to apply the modulus to make it positive, rather than checking back through the work to see why they obtain an unexpected negative result in the first place.
Some attempted a rearrangement of the log term first, usually ln(3) − ln(2), which yielded a different, but valid, approximation with values of $x = \frac{1}{3}$ and $x = \frac{1}{4}$ and giving 0.341. Other forms were possible but seldom seen except ln(3) + ln(1/2), but as this required a value $\left(x = -\frac{1}{2}\right)$ outside the domain of the function it was unacceptable. Again in this part it would be advisable for students to show the substitution explicitly so that the method can be secure even if a processing error is made.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. No Examiner's Report available for this question. (Taken from SAMs)

Examiner comment for Question 8 (Mark scheme) (Return to Question 8)

8. The finding of the two values of θ in part (a) was usually correctly done but some candidates then wasted valuable time in taking their two θ values and working in a circle to find the r values - some of which were not equal to 2.

In part (b) the vast majority of the candidates knew how to find the area enclosed between the two radius vectors and the curve simply as one integral. Others chose to split it into one area from 0 to $\frac{\pi}{2}$ minus the area from 0 to $\frac{\pi}{18}$ minus the area from $\frac{5\pi}{18}$ to $\frac{\pi}{2}$ - a long way round.
Most of the candidates decided to use the integration method to find the area of the sector as well. Errors in integration were usually in the use of the double angle formula although most used it correctly. A minority of candidates forgot to square the function before integrating, but where this squaring had been done the subsequent integration of the trigonometric functions was well done. Where candidates fell down was in the careful application of the limits to their functions - writing things more neatly would have helped in a number of cases. Some forgot about the area of the sector of the circle completely. It was, however, pleasing to see many completely correct solutions.