Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Practice Paper 3

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/4A**

Further Mathematics
Advanced
Paper 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 75. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. Given that $z = z_1 + z_2$, where $z_1 = 4 + 3i$ and $|z_2| = 3$,

(a) sketch, in an Argand diagram, the locus of z as z_2 varies.

(3)

(b) Find the maximum value of arg z, giving your answer to the nearest degree.

(4)

(Total for Question 1 is 7 marks)

2.

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & k \end{pmatrix}.$$

(*a*) Show that det $\mathbf{A} = 20 - 4k$.

(2)

(*b*) Find $\mathbf{A}^{-1}$.

(6)

Given that $k = 3$ and that $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$,

(*c*) find the corresponding eigenvalue.

(2)

Given that the only other distinct eigenvalue of $\mathbf{A}$ is 8,

(*d*) find a corresponding eigenvector.

(4)

(Total for Question 2 is 14 marks)

3.

$$f(n) = (2n + 1)7^n - 1.$$

Prove by induction that, for all positive integers n, f(n) is divisible by 4.

(Total for Question 3 is 6 marks)

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4. (a) Show that the set $S = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6 is a group.

(5)

(b) Show that the group is cyclic and write down its generators.

(3)

(c) Explain why $(S, +_6)$ cannot contains subgroup of order 4.

(3)

(d) Find the subgroup of $(S, +_6)$ that contains exactly three elements.

(1)

(Total for Question 4 is 12 marks)

5.

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \ \mathrm{d}x, \quad n \geq 0.$$

(*a*) Prove that $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}, \ n \geq 2.$

(5)

(*b*) Find an exact expression for I_6.

(4)

(Total for Question 5 is 9 marks)

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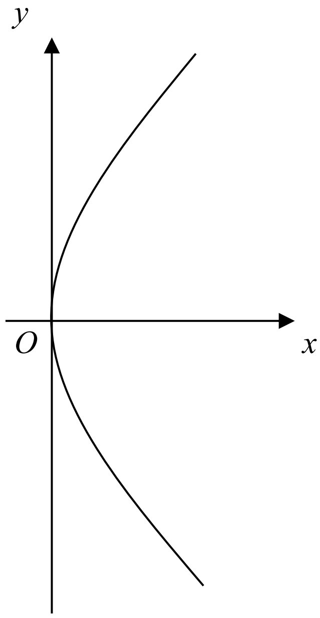
6. **Figure 1**



The curve C shown in Fig. 1 has equation $y^2 = 4x$, $0 \leq x \leq 1$.

The part of the curve in the first quadrant is rotated through 2π radians about the x-axis.

(*a*) Show that the surface area of the solid generated is given by

$$4\pi \int_0^1 \sqrt{(1+x)}\ \mathrm{d}x.$$

(4)

(*b*) Find the exact value of this surface area.

(3)

(*c*) Show also that the length of the curve C, between the points $(1, -2)$ and $(1, 2)$, is given by

$$2\int_0^1 \sqrt{\left(\frac{x+1}{x}\right)}\ \mathrm{d}x.$$

(3)

(*d*) Use the substitution $x = \sinh^2 \theta$ to show that the exact value of this length is

$$2[\sqrt{2} + \ln(1 + \sqrt{2})].$$

(6)

(Total for Question 6 is 16 marks)

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7. Messages are transmitted over a network using two types of signal packet. Type A signal packets require 1 microsecond to transmit and type B signal packets require 2 microseconds to transmit. The packets are transmitted consecutively with no gaps between them.

The number of different messages consisting of sequences of these two types of signal packet that can be sent in microseconds is denoted with S_n.

(a) Write a recurrence relation for S_{n+2} in terms of S_{n+1} and S_n. State the initial conditions for your recurrence relation.

(3)

(b) Solve your recurrence relation to find an expression for Sn in terms of n.

(5)

(Total for Question 7 is 7 marks)

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8. Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 3 & -3 \end{pmatrix},$$

(a) find the characteristic equation of $\mathbf{A}$.

(3)

(b) Show that $\mathbf{A}^2 + 2\mathbf{A} - 11\mathbf{I} = -6\mathbf{A}^{-1}$.

(3)

(c) Hence find $\mathbf{A}^{-1}$.

(3)

(Total for Question 8 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 80 MARKS

Guide:

1. P4 June 2005, Qn 2
2. P6 June 2005, Qn 7
3. P6 June 2003, Qn 2
4. Pearson FP2 textbook, p82, Qn 9
5. P5 June 2002, Qn 4
6. P5 June 2002, Qn 8
7. Pearson FP2 textbook, p149, Qn 21
8. Pearson FP2 textbook, p186, Qn 19