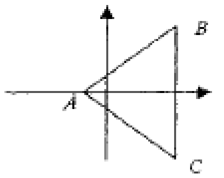
Further Pure Mathematics 2 Practice Paper 3 – mark schemes and answers

Origin of questions:

1. P4 June 2005, Qn 2

2.	(a) $1 - 3\text{i}$ is a root	B1
	$(z-1-3\text{i})(z-1+3\text{i})(z+\alpha) = (z^2-2z+10)(z+\alpha)$	
	$= z^3 + 6z + 20$	
	$10\alpha = 20 \Rightarrow \alpha = 2 \Rightarrow -2$ is a root M1 any complete method	M1 A1 3
	(b)  Position of points only in correct quadrants and negative x-axis	B1 ft 1
	(c) $m_{AB} = \frac{3}{3} = 1, \quad m_{AC} = -1$ Full Method	M1
	$m_{AB} . m_{AC} = -1 \Rightarrow$ triangle is right-angled	A1 2 (6)
	Alternatives	
	$AB^2 + AC^2 = 18 + 18 = 36 = BC^2$	
	Result follows by (converse of) Pythagoras, or any complete method.	M1 A1

2. P6 June 2005, Qn 7

7	(a) Det = $-12 \; -2(2k-8) + 16 = 20 - 4k$ (*) AG	M1A1 (2)
	(b) Cofactors $\begin{pmatrix} -4 & 8-2k & 4 \\ 8-2k & 3k-16 & 2 \\ 4 & 2 & -4 \end{pmatrix}$ [A1 each error]	M1A3
	$\mathbf{A}^{-1} = \dfrac{1}{20-4k}\begin{pmatrix} -4 & 8-2k & 4 \\ 8-2k & 3k-16 & 2 \\ 4 & 2 & -4 \end{pmatrix}$	M1A1√ (6)
	(c) Setting $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$	M1
	$\lambda = -1$	A1 (2)
	(d) Forming equations in x, y and z: $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 8\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1
	$-5x + 2y + 4z = 0,\; 2x + 2z = 8y,\; 4x + 2y - 5z = 0$	A1
	Establishing ratio $x : y : z$: $[x = 2y,\; x = z]$	M1
	Eigenvector $(k)\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$	A1 (4)
		[14]

3. P6 June 2003, Qn 2

2.	$f(1) = 3 \times 7 - 1 = 20$; divisible by 4	B1
	$f(k+1) = (2k+3)7^{k+1} - 1$	B1
	Showing that $f(k+1) = f(k) + 4m$ or equivalent	M1 A1
	e.g. $f(k+1) - f(k) = (2k+3)7^{k+1} - 1 - \{(2k+1)7^{k} - 1\}$	
	$= (12k+20)7^{k} = 4\,(3k+5)7^{k}$	
	If true for $n = k$, then true for $n = k + 1$	M1
	Conclusion , with no wrong working seen.	A1 **[6]**

4. Pearson FP2 textbook, p82, Qn 9

9 a Closure: For every $n, m \in \mathbb{Z}$, $n + m$ is congruent to one of 0, 1, 2, 3, 4, 5 (mod 6), so S is closed.
Identity: $0 + g = g = g + 0$ for all $g \in G$, so 0 is the identity.
Inverse: 0 and 3 are self-inverse; $1^{-1} = 5$, $2^{-1} = 4$
Associativity: $(a + b) + c \equiv_6 a + b + c \equiv_6 a + (b + c)$
So $(S, +_6)$ forms a group.

b All elements can be written in the form 1^k and 5^l for some $k, l \in \mathbb{Z}$, so the group is cyclic with generators 1 and 5.

c $4 \nmid 6$, so by Lagrange's theorem, S cannot contain a subgroup of order 4.

d $\{0, 2, 4\}$

5. P5 June 2002, Qn 4

Question Number	Scheme	Marks
4. (*a*)	$\int x^n \cos x \, dx = x^n \sin x - n\int x^{n-1} \sin x \, dx$	M1, A1
	$= \text{......} \quad -[nx^{n-1}\cos x + \int n(n-1)x^{n-2}\cos x \, dx]$	M1
	Using limits $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$ (✱)	M1, A1 **(5)**
	cso	
(*b*)	$I_0 = \int_0^{\frac{\pi}{2}} \cos x \, dx = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$ at any stage	B1
	$I_6 = \left(\frac{\pi}{2}\right)^6 - 30I_4$	M1
	$= \left(\frac{\pi}{2}\right)^6 - 30\left(\left(\frac{\pi}{2}\right)^6 - 12I_2\right)$	
	$= \left(\frac{\pi}{2}\right)^6 - 30\left(\frac{\pi}{2}\right)^4 + 360\left(\frac{\pi}{2}\right)^2 - 720I_0$	M1
	Hence $I_6 = \left(\frac{\pi}{2}\right)^6 - 30\left(\frac{\pi}{2}\right)^4 + 360\left(\frac{\pi}{2}\right)^2 - 720$	A1 **(4)**
	cao	**(9 marks)**

6. P5 June 2002, Qn 8

8.	(*a*)	$y = 2x^{½}$, $\dfrac{dy}{dx} = x^{-½}$	M1, A1
		$\int_{\dots}^{\dots} 2\pi y\left[1+\left(\dfrac{d}{dx}\right)^2\right]dx = 4\pi\int_{\dots}^{\dots} x^{½}\left[1+\dfrac{1}{x}\right]^{½} dx$	M1
		$= 4\pi\int_0^1 \sqrt{1+x}\, dx$ (✱)	A1 **(4)**
	(*b*)	$S = 4\pi\int_{\dots}^{\dots} \sqrt{1+x}\, dx = \left[4\pi\dfrac{2}{3}(1+x)^{3/2}\right]_{(0)}^{(1)}$	M1, A1
		$= \dfrac{8\pi}{3}\left(2^{3/2}-1\right)$ or any exact equivalent	A1 **(3)**
	(*c*)	$\int\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx = \int\left(1+\dfrac{1}{x}\right)^{\frac{1}{2}} dx$	M1
		$\int\sqrt{\dfrac{x+1}{x}}\, dx$	A1
		Using symmetry, $s = 2\int_0^1 \sqrt{\dfrac{x+1}{x}}\, dx$ (✱)	A1 **(3)**
	(*d*)	$x = \sinh^2\theta$, $\dfrac{dx}{d\theta} = 2\sinh\theta\cosh\theta$ oe	B1
		$I = 2\int\sqrt{\dfrac{1+\sinh^2\theta}{\sinh^2\theta}}.2\sinh\theta\cosh\theta\ d\theta$	M1
		$= 4\int\cosh^2\theta\ d\theta$	A1
		$= 2\int(1+\cosh 2\theta)\ d\theta$	M1
		$= 2\theta + \sinh 2\theta$	A1
		Limits are 0 and arsinh1 $(= \ln(1+\sqrt{2}))$	
		$s = \left[2\theta + 2\sinh\theta\sqrt{1+\sinh^{2\theta}}\right]_0^{\text{arsinh}1}$	
		$= 2\,\text{arsinh}1 + 2\sqrt{1+1^2}$	
		$= 2\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$ (✱)	M1, A1 **(6)**
			(16 marks)

7. **Pearson FP2 textbook, p149, Qn 21**

21 a $S_{n+2} = S_{n+1} + S_n$, $S_1 = 1$, $S_2 = 2$

b $S_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right)$

8. **Pearson FP2 textbook, p186, Qn 19**

19 a $\lambda^3 + 2\lambda^2 - 11\lambda + 6 = 0$

b $\mathbf{A}^3 + 2\mathbf{A}^2 - 11\mathbf{A} + 6\mathbf{I} = \mathbf{0} \Rightarrow \mathbf{A}^3 + 2\mathbf{A}^2 - 11\mathbf{A} = -6\mathbf{I}$

$\Rightarrow \mathbf{A}^2 + 2\mathbf{A} - 11\mathbf{I} = -6\mathbf{A}^{-1}$

c $\mathbf{A}^{-1} = \frac{1}{6}\begin{pmatrix} 6 & -6 & -2 \\ -3 & 6 & 3 \\ 3 & 0 & -1 \end{pmatrix}$