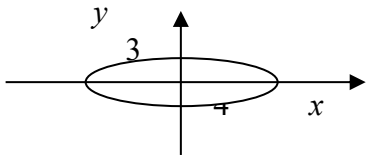
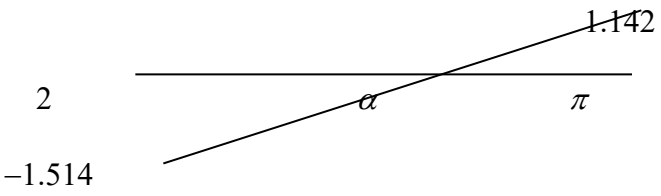
Further Pure Mathematics 1 Practice Paper 3 – mark schemes and answers

Origin of questions:

1. P5 June 2002, Qn 1

Qn	Scheme	Marks
1. (*a*)	 Closed shape 3, 4	B1 **(1)**
(*b*)	$b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2)$	M1
	$e = \frac{\sqrt{7}}{4}$ oe awrt 0.661	A1 **(2)**
(*c*)	Foci are at $(\pm ae, 0)$ use of ae	M1
	$(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ awrt 2.65, 0 is required, ft their e	A1 ft **(2)** **(5 marks)**

2. P4 January 2003, Qn 4

Qn	Scheme	Marks
4. (*a*)	f(2) = –1.514	B1
	f(π) = 1.142 	B1
	$\frac{\pi-\alpha}{\alpha-2}=\frac{1.142}{1.514}$	M1
	$\pi\times 1.514+2\times 1.142=(1.142+1.514)\alpha$	
	$\alpha = 2.65$	A1 **(4)**
(*b*)	f′(x) = 4 cos 2x + 1 k cos 2x + c	M1
	f(2.8) = –0.4625	B1
	f′(2.8) = 4.1023	A1
	$x_2 = 2.8-\frac{(-0.4625)}{4.1023}$	M1
	= 2.91 only	A1 **(9 marks)**

3. FP3 June 2009, Qn 6

Q6 (a)	$\frac{x^2}{a^2}-\frac{(mx+c)^2}{b^2}=1$ and so $b^2x^2-a^2(mx+c)^2=a^2b^2$	M1
	$\therefore\ (b^2-a^2m^2)x^2-2a^2mcx-a^2(c^2+b^2)=0$ Or $(a^2m^2-b^2)x^2+2a^2mcx+a^2(c^2+b^2)=0$ ✳	A1 (2)
(b)	$(2a^2mc)^2=4(a^2m^2-b^2)\times a^2(c^2+b^2)$ $4a^4m^2c^2=-4a^2(b^2c^2+b^4-a^2m^2c^2-a^2m^2b^2)$ $c^2=a^2m^2-b^2$ or $a^2m^2=b^2+c^2$ ✳	M1 A1 (2)
(c)	Substitute (1, 4) into $y = mx+c$ to give $4 = m + c$ and Substitute $a = 5$ and $b = 4$ into $c^2=a^2m^2-b^2$ to give $c^2=25m^2-16$ Solve simultaneous equations to eliminate m or c : $(4-m)^2=25m^2-16$ To obtain $24m^2+8m-32=0$ Solve to obtain $8(3m+4)(m-1)=0$.....m = ..or... $m=1$ or $-\frac{4}{3}$ Substitute to get $c=3$ or $\frac{16}{3}$ Lines are $y=x+3$ and $3y+4x=16$	B1 M1 A1 M1 A1 M1 A1 (7) **[11]**

4. P6 June 2002, Qn 4

Qn	Scheme	Marks
4. (*a*)	$y\frac{d^3y}{dx^3}+\frac{dy}{dx}\frac{d^2y}{dx^2};+2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2};+\frac{dy}{dx}=0$	M1 A1; B1;B1
	marks can be awarded in (b)	
	$\frac{d^3y}{dx^3}=\frac{-3\frac{dy}{dx}\frac{d^2y}{dx^2}-\frac{dy}{dx}}{y}$ or sensible correct alternative	B1 **(5)**
(*b*)	When $x=0$ $\frac{d^2y}{dx^2}=-2$, and $\frac{d^3y}{dx^3}=5$	M1A1, A1 ft
	$\therefore \quad y=1+x-x^2+\frac{5}{6}x^3...$	M1, A1 ft **(5)**
(*c*)	Could use for $x = 0.2$ but not for $x = 50$ as	B1
	approximation is best at values close to $x = 0$	B1 **(2)**
		(12 marks)

(❋) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark; cao = correct answer only; isw = ignore subsequent working

5. FP1 textbook, P191 Qn 7

7 a $v = \frac{ds}{dx} = 2\cos 4x \times 4 + 4\cos 2x \times 2 = 8(\cos 4x + \cos 2x)$

$t = \tan x \Rightarrow \sin 2x = \frac{2t}{1+t^2}, \cos 2x = \frac{1-t^2}{1+t^2}$

$\cos 4x = \cos^2 2x - \sin^2 2x$

$\Rightarrow v = 8\left(\left(\frac{1-t^2}{1+t^2}\right)^2 - \left(\frac{2t}{1+t^2}\right)^2 + \left(\frac{1-t^2}{1+t^2}\right)\right)$

$= \frac{16}{(1+t^2)^2}(1-3t^2)$

b Least value of s occurs at $x = \frac{5\pi}{6}$ and is -4.196 m.

It is a minimum because $\left.\frac{ds}{dx}\right|_{\frac{5\pi}{6}} = 0$ and $\left.\frac{d^2s}{dx^2}\right|_{\frac{5\pi}{6}} > 0$.

6. FP1 textbook, p27 Qn 9

(b) $\frac{\sqrt{2}}{2}$

7. FP1 textbook, P159 Qn 7

$-\frac{1}{2}$

8. P6 June 2002, Qn 7

Qn	Scheme	Marks
7. (*a*)	$\overrightarrow{AB} = 5\mathbf{i} + 3\mathbf{j}$ $\quad \overrightarrow{AC} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ or $\overrightarrow{BC} = -2\mathbf{i} - \mathbf{j} - \mathbf{k}$	
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 3 & 2 & -1 \end{vmatrix} = -3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$	M1, A1
	$\therefore \mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	B1 ft **(3)**
(*b*)	Volume $= \frac{1}{6}\overrightarrow{AD}.(\overrightarrow{AB} \times \overrightarrow{AC})$ $\quad \overrightarrow{AD} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$	B1
	$= {}^1/_6(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}).(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1
	$= {}^{11}/_6$	A1 **(3)**
(*c*)	$\mathbf{r}.(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = (2\mathbf{i} + \mathbf{j}).(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1, A1 ft
	$= -1$	A1 **(3)**
(*d*)	$[\mathbf{i}.(1 - 3\lambda) + \mathbf{j}(2 + 5\lambda) + \mathbf{k}(1 + \lambda)].(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = -1$	M1, A1ft
	$-3 + 9\lambda + 10 + 25\lambda + 3 + \lambda = -1$	
	$35\lambda + 10 = -1 \quad \Rightarrow \lambda = -\frac{11}{35}$	M1
	$\therefore$ E is $\left(\frac{68}{35}, \frac{15}{35}, \frac{94}{35}\right)$	A1 **(4)**
(*e*)	Distance $= -\frac{11}{35}\lvert -3i + 5j + k \rvert = \frac{11\sqrt{35}}{35}$ $\quad$ (✱)	M1 A1 **(2)**
(*f*)	$\lambda = 2 \times \left(-\frac{11}{35}\right) = -\frac{22}{35}$	B1
	$\mathbf{r}_{D'} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + -\frac{22}{35}(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1
	D′ is $\left(\frac{101}{35}, -\frac{40}{35}, \frac{83}{35}\right)$	A1 **(3)**
		(18 marks)

(✱) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark; cao = correct answer only; isw = ignore subsequent working