Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Practice Paper 2

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/4A**

Further Mathematics

Advanced

Paper 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 70. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. (*a*) The point P represents a complex number z in an Argand diagram. Given that

$$|z - 2\mathrm{i}| = 2|z + \mathrm{i}|,$$

(i) find a cartesian equation for the locus of P, simplifying your answer,

(2)

(ii) sketch the locus of P.

(3)

(*b*) A transformation T from the z-plane to the w-plane is a translation $-7 + 11\mathrm{i}$ followed by an enlargement with centre the origin and scale factor 3.

Write down the transformation T in the form

$$w = az + b, \quad a, b \in \mathbb{C}.$$

(2)

(Total for Question 1 is 7 marks)

2.

$$\mathbf{M} = \begin{pmatrix} 4 & 1 & -1 \\ 1 & 0 & 3 \\ 1 & 2 & 1 \end{pmatrix}.$$

(a) Show that -2 is an eigenvalue of $\mathbf{M}$ and find the other two eigenvalues.

(4)

(b) For each of the eigenvalues, find a corresponding eigenvector.

(4)

(c) Find a matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{M}\mathbf{P}$ is a diagonal matrix and write down the diagonal matrix $\mathbf{D}$.

(2)

(Total for Question 2 is 10 marks)

---

3. (a) Explain why the congruence equation $39x \equiv 5 \pmod{600}$ has no solutions.

(4)

(b) Solve the congruence equation $39x = 6 \pmod{600}$.

(3)

(Total for Question 3 is 7 marks)

---

4. The set $G = \{1, 5, 7, 11, 13, 17, 19, 23\}$ forms a group under multiplication modulo 24.

(a) Find the order of each element in this group.

(4)

(b) Explain clearly why this group cannot contain a cyclic subgroup of order 4.

(2)

The elements of H are the complex numbers $\mathrm{e}^{\frac{k\pi \mathrm{i}}{4}}$, where $k = 0, 1, 2, 3, 4, 5, 6, 7, 8$. H forms a group under complex multiplication.

(c) Determine, with reasons, whether $G \cong H$.

(3)

(Total for Question 4 is 9 marks)

---

5. Consider the definite integral $I_n = \displaystyle\int_0^{\ln\sqrt{3}} \tanh^n x \,\mathrm{d}x$, $n \geq 1$.

By rewriting $\tanh^n x$ as $\tanh^{n-2} x \tanh^2 x$ and using the identity $\tanh^2 x = 1 - \mathrm{sech}^2 x$,

(a) show that $I_n = I_{n-2} - \dfrac{1}{(n-1)}\left(\dfrac{1}{2}\right)^{n-1}$, $n \geq 2$.

(6)

(b) Hence show that $\displaystyle\sum_{r=1}^{\infty} \frac{1}{2r}\left(\frac{1}{2}\right)^{2r} = \ln \frac{2}{\sqrt{3}}$.

(You may assume that $\displaystyle\lim_{n\to\infty} \int_0^{\ln\sqrt{3}} \tanh^n x \,\mathrm{d}x = 0$.)

(9)

(Total for Question 5 is 15 marks)

---

6. The arc, in the first quadrant, of the curve with parametric equations $x = \text{sech } t$, $y = \tanh t$, between the points where $t = 0$ and $t = \ln 2$, is rotated completely about the x-axis.

Show that the area of the surface generated is $\dfrac{2\pi}{5}$.

(Total for Question 6 is 6 marks)

---

7. A sequence satisfies the recurrence relation

$$u_n = -2n\,u_{n-1} + 3n(n-1)u_{n-2}\text{, with } u_0 = 1 \text{ and } u_1 = 2.$$

Prove, by induction, that a closed form for this sequence is $u_n = \dfrac{n!}{4}(5 - (-3)^n)$.

(Total for Question 7 is 7 marks)

---

8. The binary operation * is defined on the set $G = \{0, 1, 2, 3\}$ by $a * b = a + 2 + ab \pmod 4$.

(a) Construct a Cayley table for $(G, *)$.

(3)

(b) Determine whether * is associative, justifying your answer.

(3)

(c) Find all solutions to the equation $x * 1 = 2 * x$, for $x \in G$.

(3)

(Total for Question 8 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 70 MARKS