Further Pure Mathematics 1 Practice Paper 2 – mark schemes

Origin of questions:

1. P6 June 2004, Qn 3

(a) $AB = \begin{pmatrix} -4 \\ 3 \\ -4 \end{pmatrix}$ $\quad AC = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ M1

$AB \times AC = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & -4 \\ 2 & -2 & 3 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ A1: One value correct, A1: All correct M1 A1 A1

(4)

(b) $\mathbf{r}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 3 - 4 + 8$ $\quad \mathbf{r}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 7$ M1 A1ft

(2)

(c) $AD\,.\,AB \times AC$ (Attempt suitable triple scalar product) M1

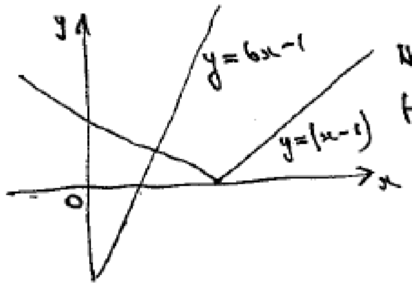
$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$ (if using AD) B1

Volume = $\frac{1}{6}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \frac{1}{6}(2 + 12 - 2) = 2$ M1 A1(cso)

(4)

(10 marks)

2. P4 January 2002, Qn 2

2.	$x \geqslant 1$ and $x - 1 > 6x - 1$ $x < 0$ No values	
	OR  No critical value from $y = x - 1$, $y = 6x - 1$	M1 A1
	$\left.\begin{array}{l} y = 1 - x \\ y = 6x - 1 \end{array}\right\} \rightarrow x = \frac{2}{7}$ as critical value	M1 A1
	Solution set $x < \frac{2}{7}$ [Correct final statement needed for A1 here]	A1 cso (5)

3. FP2 June 2009, Qn 5

Question Number	Scheme		Marks
Q5	$y = \sec^2 x = (\sec x)^2$		
(a)	$\frac{dy}{dx} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$	Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$	B1 aet
	Apply product rule: $\left\{ \begin{matrix} u = 2\sec^2 x & v = \tan x \\ \frac{du}{dx} = 4\sec^2 x \tan x & \frac{dv}{dx} = \sec^2 x \end{matrix} \right\}$		
	$\frac{d^2y}{dx^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$	Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.	M1
		Correct differentiation	A1
	$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$		
	Hence, $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$	Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.	A1 AG
			(4)
(b)	$y_{\frac{\pi}{4}} = \left(\sqrt{2}\right)^2 = \underline{2}$, $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2\left(\sqrt{2}\right)^2(1) = \underline{4}$	Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \underline{4}$	B1
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = 24 - 8 = 16$	Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{d^2y}{dx^2}$.	M1
	$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$	Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct	M1
	$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$		
	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = 24\left(\sqrt{2}\right)^4(1) - 8\left(\sqrt{2}\right)^2(1) = 96 - 16 = 80$	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = \underline{80}$	B1
	$\sec x \approx 2 + 4(x - \frac{\pi}{4}) + \frac{16}{2}(x - \frac{\pi}{4})^2 + \frac{80}{6}(x - \frac{\pi}{4})^3 + \ldots$	Applies a Taylor expansion with at least 3 out of 4 terms ft correctly.	M1
		Correct Taylor series expansion.	A1
			(6)
	$\left\{\sec x \approx 2 + 4(x - \frac{\pi}{4}) + 8(x - \frac{\pi}{4})^2 + \frac{40}{3}(x - \frac{\pi}{4})^3 + \ldots\right\}$		
			[10]

4. P6 June 2003, Qn 8

Question number	Scheme	Marks
8.	(a) $x_0 = 0, y_0 = 1, \left(\frac{dy}{dx}\right)_0 = 0 - 1 = -1$	B1
	$y_1 - y_0 = 0.05(-1) \Rightarrow y_1 = 1 - 0.05 = 0.95$	M1 A1ft
	$x_1 = 0.05, y_1 = 0.95, \left(\frac{dy}{dx}\right)_1 = 0.05^2 - 0.95^2$ $(= -0.9)$	A1 ft
	$y_2 - y_1 = 0.05(-0.9) \Rightarrow y_2 = 0.95 - 0.045 = 0.905$ (A1 is c.s.o.)	M1 A1 (6)
	(b) $\frac{dy}{dx} = x^2 - y^2 \Rightarrow \frac{d^2y}{dx^2} = 2x - 2y\frac{dy}{dx}$	M1 A1
	$\Rightarrow \frac{d^3y}{dx^3} = 2 - 2y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2$ allow at this stage	M1 A1 (4)
	(c) [$y_{x=0} = 1, \left(\frac{dy}{dx}\right)_{x=0} = -1,$] $\left(\frac{d^2y}{dx^2}\right)_{x=0} = 0 - 2(1)(-1) = 2$	B1
	$\left(\frac{d^3y}{dx^3}\right)_{x=0} = 2 - 2(-1)^2 - 2(1)(2) = -4$	B1
	Maclaurin: $y = 1 - x + x^2 - \frac{2}{3}x^3$	M1 A1 (4)
	[Alternative (c) $y = 1 + a_1x + a_2x^2 + a_3x^3$	**[14]**
	$\Rightarrow x^2 - (1 + a_1x + a_2x^2 + a_3x^3)^2 = a_1 + 2a_2x + 3a_3x^2$ B1	
	Compare coeffs $\Rightarrow a_1 = -1;\ a_2 = 1,\ a_3 = -\frac{2}{3}$. B1; M1 A1]	

5. P5 June 2002, Qn 7

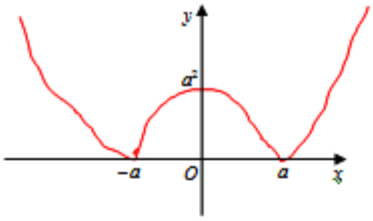
Question Number	Scheme	Marks
7. (*a*)	$\frac{dy}{dx} = -\frac{4}{x^2}$; at $x = 2p$ $\frac{dy}{dx} = -\frac{1}{p^2}$	M1, A1
	Equation of tangent at P, $y - \frac{2}{p} = -\frac{1}{p^2}(x - 2p)$	M1
	$(y = -\frac{1}{p^2}x + \frac{4}{p}$, $p^2 y + x = 4p$ etc)	
	At Q $q^2 y + x + 4q$ Two correct equations in any form	A1
	$(p^2 - q^2)y = 4(p - q)$	M1
	$y = \frac{4}{p+q}$ (✱)	A1
	$x = 4p - \frac{4p^2}{p+q} = \frac{4pq}{p+q}$ (✱)	M1, A1 (8)
(*b*)	$\frac{4pq}{p+q} \times \frac{4}{p+q} = 3$	M1
	$3p^2 - 10pq + 3q^2 = 0$	A1
	$(3p - q)(p - 3q) = 0$	M1
	$q = 3p$, $q = \frac{1}{3}p$	A1, A1 (5)
		(13 marks)

(✱) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

6. FP3 June 2009, Qn 2

Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0 + 5 = 5$	M1 A1 ft (2)
(c)	Area of triangle $OBC = \frac{1}{2}\|5\mathbf{j} + 5\mathbf{k}\| = \frac{5}{2}\sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron $= \frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft (1) **[8]**

7. FP2 June 2009, Qn 8

Q7	$y = \lvert x^2 - a^2 \rvert,\ a > 1$		
(a)		Correct Shape. Ignore cusps.	B1
		Correct coordinates.	B1
			(2)
(b)	$\lvert x^2 - a^2 \rvert = a^2 - x,\ a > 1$		
	$\{\lvert x \rvert > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow x^2 + x - 2a^2 = 0$		
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\lvert x \rvert < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow x = 0, 1$	$x = 0$	B1
		$x = 1$	A1
			(6)
(c)	$\lvert x^2 - a^2 \rvert > a^2 - x,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}$ {or} $x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value	B1 ft
		x is greater than their maximum value	B1 ft
	{or} $0 < x < 1$	For $\{\lvert x \rvert < a\}$, Lowest $< x <$ Highest	M1
		$0 < x < 1$	A1
			(4)
			[12]