Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Further Mathematics
Core Pure Mathematics

Practice Paper 2

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **8** questions in this question paper. The total mark for this paper is **70**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Solve the equation

$$2\cosh^2 x - 3\sinh x = 1$$

giving your answers in terms of natural logarithms.

(Total 6 marks)

Mark scheme for Question 1

Examiner comment

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2. (i) The complex number w is given by

$$w = \frac{p - 4\text{i}}{2 - 3\text{i}}$$

where p is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are real constants.
Give your answer in its simplest form in terms of p.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of p.

(2)

(ii) The complex number z is given by

$$z = (1 - \lambda\text{i})(4 + 3\text{i})$$

where λ is a real constant.
Given that

$$|z| = 45$$

find the possible values of λ
Give your answers as exact values in their simplest form.

(3)

(Total 8 marks)

Mark scheme for Question 2

Examiner comment

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3. Given that $y = \operatorname{artanh}\frac{x}{\sqrt{(1+x^2)}}$

show that $\frac{dy}{dx} = \frac{1}{\sqrt{(1+x^2)}}$

(Total 4 marks)

[Mark scheme for Question 3]

[Examiner comment]

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4. Find the general solution of the differential equation

$$\sin x\frac{dy}{dx} - y\cos x = \sin 2x\sin x,$$

giving your answer in the form $y = f(x)$.

(Total 8 marks)

[Mark scheme for Question 4]

[Examiner comment]

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5. The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(4)

(Total 11 marks)

[Mark scheme for Question 5]

[Examiner comment]

6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 27e^{-x}$$

(6)

(b) Find the particular solution that satisfies $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

(6)

(Total 12 marks)

[Mark scheme for Question 6]

[Examiner comment]

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7. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda\,(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2

(2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree.

(3)

(Total 8 marks)

[Mark scheme for Question 7]

[Examiner comment]

8.

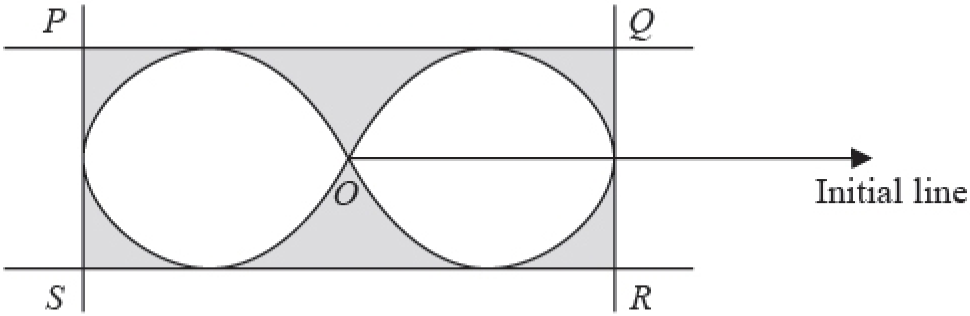

Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \qquad \text{where } -\frac{\pi}{4} < \theta \leqslant \frac{\pi}{4}, \frac{3\pi}{4} < \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, SR, PS and QR are tangents to C, where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

(a) Find the total area enclosed by the curve C, shown unshaded inside the rectangle in Figure 1.

(4)

(b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1.

(9)

(Total 13 marks)

[Mark scheme for Question 8]

[Examiner comment]

TOTAL FOR PAPER: 70 MARKS

Further Core Pure Mathematics – Practice Paper 02 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1	$2\left(1+\sinh^2 x\right)-3\sinh x=1$	**M1**
	$2\sinh^2 x-3\sinh x+1=0$	**A1**
	$(2\sinh x-1)(\sinh x-1)=0$	**M1**
	$\sinh x$ or $\frac{e^x-e^{-x}}{2}=\frac{1}{2}$ or 1	**A1**
	$x=\ln\frac{1}{2}\left(1+\sqrt{5}\right),\ \ln\left(1+\sqrt{2}\right)$	**A1A1**
	Allow equivalent answers e.g. $\ln\left(\frac{1}{2}+\sqrt{\frac{5}{4}}\right), \ln\left(\frac{1}{2}+\sqrt{1+\frac{1}{4}}\right)$ and allow awrt 3SF accuracy e.g. $\ln 1.62,\ \ln 2.41$	
		(6)
		(6 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(i)(a)	$w = \dfrac{p-4\text{i}}{2-3\text{i}} \qquad \arg w = \dfrac{\pi}{4}$	
	$w = \dfrac{(p-4\text{i})}{(2-3\text{i})} \times \dfrac{(2+3\text{i})}{(2+3\text{i})}$	**M1**
	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$	**A1A1**
		(3)
(i)(b)	$\left\{\arg w = \dfrac{\pi}{4} \Rightarrow\right\}$ $2p + 12 = 3p - 8$ o.e seen anywhere.	**M1**
	$\Rightarrow p = 20$	**A1**
		(2)
(ii)	$Z = (1 - \lambda\text{i})(4 + 3\text{i})$ and $\lvert z \rvert = 45$	
	$\sqrt{1+\lambda^2}\ \sqrt{4^2+3^2}$	**M1**
	$\sqrt{1+\lambda^2}\ \sqrt{4^2+3^2} = 45$	**A1**
	$\{\lambda^2 = 9^2 - 1 \Rightarrow\}\ \lambda = \pm 4\sqrt{5}$	**A1**
		(3)
		(8 marks)

Mark scheme for Question 3 **(Examiner comment)** **(Return to Question 3)**

Question	Scheme	Marks
3	$\frac{\mathrm{d}y}{\mathrm{d}x}=\underline{\left(\frac{1}{1-\frac{x^2}{1+x^2}}\right)}\cdot\underline{\underline{\left(\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}\right)}}$ NB $\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}=\frac{1}{(1+x^2)^{\frac{3}{2}}}$	**<u>M1M</u>** **A1**
	$=\frac{1}{\sqrt{1+x^2}}$ **ag**	**A1**
		(4)
		(4 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4	$\sin x\dfrac{dy}{dx} - y\cos x = \sin 2x\sin x$	
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x\sin x}{\sin x}$	M1
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \sin 2x$	
	Integrating factor $= e^{\int -\frac{\cos x}{\sin x}dx} = e^{-\ln\sin x}$	dM1 A1
	$= \dfrac{1}{\sin x}$	A1
	$\left(\dfrac{1}{\sin x}\right)\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$	
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	M1
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\, dx$	
	$\dfrac{y}{\sin x} = 2\sin x + K$	ddd M1
	$y = 2\sin^2 x + K\sin x$	A1cao
		(8)
		(8 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$	
	$\cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$	
	$= 2\cos n\theta$	M1A1
		(2)
(b)	$(z + z^{-1})^5 = 32\cos^5\theta$	B1
	$(z + z^{-1})^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1A1
	$32\cos^5\theta = (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$	A1
		(5)
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta(8\cos^4\theta + 1) = 0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1
		(4)
		(11 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=27e^{-x}$	
	$m^2+2m+10\ (=0) \Rightarrow m=$	**M1**
	$m=-1\pm 3i$	**A1**
	$(y=)e^{-x}(A\cos 3x+B\sin 3x)$ or $(y=)Ae^{(-1+3i)x}+Be^{(-1-3i)x}$	**A1**
	$y=ke^{-x}, y'=-ke^{-x}, y''=ke^{-x}$	**M1**
	$e^{-x}(k-2k+10k)=27e^{-x} \Rightarrow k=3$	**A1**
	$y=e^{-x}(A\cos 3x+B\sin 3x+3)$ or $y=Ae^{(-1+3i)x}+Be^{(-1-3i)x}+3e^{-x}$	**B1ft**
		(6)
(b)	$x=0, y=0 \Rightarrow A=(-3)$	**M1**
	$y'=-e^{-x}(A\cos 3x+B\sin 3x+3)+$ $e^{-x}(3B\cos 3x-3A\sin 3x)$	**M1A1**
	$x=0, y'=0 \Rightarrow B=0$	**M1A1**
	$y=e^{-x}(3-3\cos 3x)$ oe	**A1**
		(6)
		(12 marks)

Mark scheme for Question 7 [(Examiner comment)] [(Return to Question 7)]

Question	Scheme	Marks	AOs
7(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\bullet\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	**M1**	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	**M1**	1.1b
	$=\sqrt{29}$	**A1**	1.1b
		(3)	
(b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=\ldots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\ldots$	**M1**	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	**A1**	2.2a
		(2)	
(c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	**M1**	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	**M1**	2.1
	So angle between planes $\theta=52^\circ$ *	**A1***	2.4
		(3)	
		(8 marks)	

Notes:

(a)

M1: Realises the need to and so attempts the scalar product between the normal and the position vector

M1: Correct method for the perpendicular distance

A1: Correct distance

Notes for Question 5 *continued*
(b) **M1:** Recognises the need to calculate the scalar product between the given vector and both direction vectors **A1:** Obtains zero both times and makes a conclusion
(c) **M1:** Calculates the scalar product between the two normal vectors **M1:** Applies the scalar product formula with their 11 to find a value for $\cos\theta$ **A1*:** Identifies the correct angle by linking the angle between the normal and the angle between the planes

Mark scheme for Question 8 (Examiner comment) (Return to Question 8)

Question	Scheme	Marks
8(a)	$A = (4\times)\ \int_0^{\frac{\pi}{4}} \frac{9}{2}\cos 2\theta\,\mathrm{d}\theta$	M1A1
	$= 18\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$	M1
	$9\left[\sin\frac{\pi}{2} - 0\right] = 9$	A1
		(4)
(b)	$r = 3(\cos 2\theta)^{½}$	
	$r\sin\theta = 3(\cos 2\theta)^{½}\sin\theta$	M1
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$	M1 dA1
	At max/min: $\frac{-3\sin 2\theta\sin\theta}{(\cos 2\theta)^{\frac{1}{2}}} + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta = 0$	M1
	$\sin 2\theta\sin\theta = \cos 2\theta\cos\theta$	
	$2\sin^2\theta\cos\theta = (1 - 2\sin^2\theta)\cos\theta$	
	$\cos\theta\,(1 - 4\sin^2\theta) = 0$	
	$(\cos\theta = 0)\quad \sin^2\theta = \frac{1}{4}$	
	$\sin\theta = \pm\frac{1}{2}\qquad \theta = \pm\frac{\pi}{6}$	M1A1
	$r\sin\frac{\pi}{6} = 3\left(\cos\frac{\pi}{3}\right)^{\frac{1}{2}}\times\frac{1}{2} = \frac{3\sqrt{2}}{4}$	B1
	$\therefore$ length $PS = \frac{3\sqrt{2}}{2}$, (length $PQ = 6$)	
	Shaded area $= 6\times\frac{3\sqrt{2}}{2} - 9,\ = 9\sqrt{2} - 9$ oe	M1A1
		(9)
		(13 marks)

Further Core Pure Mathematics – Practice Paper 02 – Examiner Report -

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. The vast majority of students correctly used the identity $\cosh^2 x = 1 + \sinh^2 x$ to obtain a quadratic in sin hx. Most then used the logarithmic form of arsinh to obtain the final answers. Some students wrote sin hx in terms of exponentials and proceeded to solve the resulting quadratics in e^x and sometimes ended up with extra solutions that were not rejected. A significant number of students attempted to solve the given equation by expressing it in terms of exponentials. Such solutions usually stopped once a quartic in e^x was reached. Quite often, students who adopted this approach, realised that any progress would be difficult and so resorted to using the identity $\cosh^2 x = 1 + \sinh^2 x$.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. Almost all candidates knew to multiply top and bottom by the complex conjugate of the denominator, and most did this correctly, though there were occasional errors in both numerator and denominator. The final mark in part (i)(a) was sometimes lost due to failure to separate the real and imaginary parts, as the question required. In part (b), relatively few candidates were able to immediately equate the numerators of the real and imaginary parts. Most used $\tan \pi/4 = 1$ involving them in extra work. In part (ii) it was very rare to see use of the modulus of the product as the product of the moduli. Instead candidates worked out the product of the two complex numbers, sometimes incorrectly, and then attempted to obtain the modulus of that product, usually successfully. Most were then able to progress to a correct conclusion, though a worrying minority thought that $\sqrt{(25 + \lambda^2)} = (5 + \lambda)$.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. The vast majority of the students identified the given function as being composite and applied the chain rule in determining $\frac{dy}{dx}$. Most of the students used the product rule to differentiate $\frac{x}{\sqrt{1+x^2}}$ and the algebraic processing of the terms involved within both this and the quotient rule, when used, was generally of a good standard. Nearly all of the students correctly differentiated artanh x but the final stages of the solution caused some problems and a number of errors were seen in processing the terms down to the printed result. There were a number of students who initially rearranged the given function to make tanh y the subject and then used implicit differentiation and achieved equal success with those students who used the chain rule.

Examiner comment for Question 4 **(Mark scheme)** **(Return to Question 4)**

4. This question was well answered by candidates and statistics showed that at least 50% of the candidature scored 7 or more marks out the 8 available for this question. Some of these candidates usually lost the final accuracy mark as they either missed out the constant of integration or usually incorrectly manipulated $\frac{y}{\sin x} = 2\sin x + K$ to give $y = 2\sin^2 x + K$

Most candidates were able to divide all terms in the differential equation by $\sin x$ to achieve an equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$ and most attempted to find the integrating factor. Some candidates wrote down $\mathrm{e}^{\int \frac{\cos x}{\sin x} dx}$ as their integrating factor instead of $\mathrm{e}^{\int -\frac{\cos x}{\sin x} dx}$. A few candidates struggled to integrate $-\frac{\cos x}{\sin x}$ correctly, but a significant number simplified $\mathrm{e}^{-\ln \sin x}$ incorrectly to $\sin x$. At this stage, some candidates did not use their integrating factor correctly to achieve an equation of the form $\frac{\mathrm{d}}{\mathrm{d}x}(y \times \text{their I.F.}) = \sin 2x \times$ their I.F. and so lost the remaining four marks. There were also a significant number of candidates who struggled to simplify $\sin 2x \operatorname{cosec} x$ to $2\cos x$.

Examiner comment for Question 5 **(Mark scheme)** **(Return to Question 5)**

5. Another well answered question with most candidates knowing how to approach both proofs. Some candidates opted to find an expression for $\cos 5\theta$ then $\cos 3\theta$ by expanding $(\cos\theta + \mathrm{i}\sin\theta)^n$ and equating real parts, then substituted these into the right hand side of the proof. This was more complicated than expanding $\left(z + \frac{1}{z}\right)^n$ and many candidates simply found an expression for $\cos 5\theta$ and then were unable to complete the proof. In part (c) many candidates did not recognise the significance of $\cos^4\theta = -\frac{1}{8}$ and either failed to mention it at all or, in some cases, ignored the minus sign and solved $\cos^4\theta = \frac{1}{8}$ to obtain extra (incorrect) solutions.

Examiner comment for Question 6 **(Mark scheme)** **(Return to Question 6)**

6. This was a classic textbook example of a second order differential equation. A large number of students tackled this question successfully. Some preferred to give the solution in terms of $A\mathrm{e}^{(-1+3\mathrm{i})x} + B\mathrm{e}^{(-1-3\mathrm{i})x}$ rather than $\mathrm{e}^{-x}(A\cos 3x + B\sin 3x)$. Some mistakes occurred in finding the Particular Integral by not using $y = k\mathrm{e}^{-x}$ and finding k. A small number of students lost the final accuracy mark in both part (a) and part (b) by not writing their solutions with "$y =$".

Examiner comment for Question 7 **(Mark scheme)** **(Return to Question 7)**

7. No Examiner's Report available for this question. (Taken from SAMs)

Examiner comment for Question 8 (Mark scheme) (Return to Question 8)

8. It was surprising how few candidates realised that integrating from 0 to $\frac{\pi}{4}$ and multiplying by 4 was the easiest way to deal with part (a), but almost all candidates did manage to obtain the correct answer. Part (b) could be approached in a number of ways and most candidates knew to differentiate $r \sin\theta$. It was surprising that fewer students did not realise that by differentiating $r^2 \sin^2\theta$ they could have simplified the algebra. The product and chain rules were generally used correctly but many candidates did not seem sufficiently confident with manipulating trigonometric expressions to be able to see their derivative through to a solution. Slips in accuracy led to equations becoming overcomplicated and the candidates were unable to recover. Poor handwriting and poor presentation did not help when trying to work out what some candidates were doing. Many candidates overcomplicated the final stage of this question by not realising that they could substitute $\theta = 0$ into r to find the width of the rectangle. Instead, time was wasted solving $\frac{\mathrm{d}}{\mathrm{d}\theta}(r \cos \theta) = 0$. It is worth reminding candidates that communicating their method is very important. Many candidates wrote minimal working and produced an answer for which, if incorrect, their working made it difficult to ascertain how the answer had been derived.