Pearson Edexcel
Level 3 GCE

Practice Paper 1

(Time: 1 hour 30 minutes)   Paper Reference **9FM0/3A**

Further Mathematics
Advanced
Paper 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 75. There are 8 questions.
- The marks for each question are shown in brackets
    – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. A tetrahedron has vertices at $A(-1, 3, 2)$, $B(1, -4, 2)$, $C(-1, -5, 6)$ and $D(-7, -2, 2)$.

   Find

   (a) the Cartesian equation of the plane ABC,

   (3)
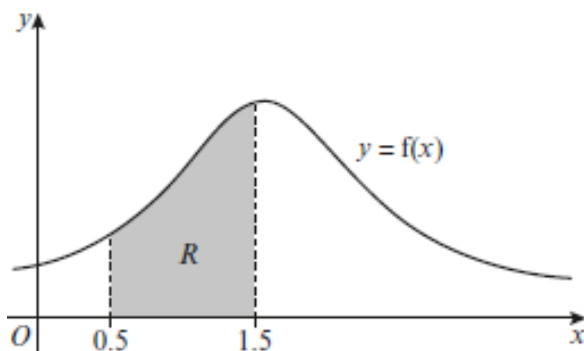
   (b) the volume of tetrahedron $ABCD$.

   (3)

   The normal to the plane ABC through point D intersects the plane at point E.

   (c) Find the angle DCE. Give your answer in radians correct to three significant figures.

   (5)

   (Total for Question 1 is 11 marks)

2.

Figure 1

Figure 1 shows the graph of $y = f(x)$ where $f(x) = \dfrac{1}{4 - 3\sin x}$.

The finite region R is bounded by the curve, the x-axis and the lines $x = 0.5$ and $x = 1.5$.

(a) Use Simpson's rule with 4 intervals to find an approximation for the area of R, giving your answer to 5 decimal places.

(5)

(b) Use the substitution $t = \tan \dfrac{x}{2}$ to find the value of the integral

$$\int_{0.5}^{1.5} f(x) \ dx$$

to 5 decimal places.

(6)

(c) Hence find, correct to 2 decimal places, the percentage error in using the method in part (a), and suggest a way in which the approximation could be improved.

(2)

(Total for Question 2 is 13 marks)

3

3. An extreme sports enthusiast jumps from the top of a cliff attached to a parachute. Her velocity, y m s^{-1}, is related to the distance jumped, x, where x is measured in hundreds of metres, from the top of a cliff. She believes the differential equation used to model the relationship between x and y is

$$xy\frac{dy}{dx} + 3x^2 + y^2 = 0 \qquad [1]$$

(a) Show that the substitution $y = vx$ transforms [1] into the differential equation

$$x\frac{dv}{dx} + \frac{3 + 2v^2}{v} = 0 \qquad [2]$$

(5)

(b) By solving equation [2], find the particular solution to equation [1], given that her velocity is 5 m s^{-1} when she is 100 metres from the top of the cliff.

(8)

(c) Assuming that her velocity reaches zero as she lands, find, according to the model, the height of the cliff.

(2)

(d) By considering your solution to part (b), comment on the suitability of this model for small values of x.

(1)

(Total for Question 3 is 16 marks)

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4. (a) Explain why you cannot use L'Hopital's rule to evaluate $\displaystyle\lim_{x \to 1} \frac{5x^4 - 3x^2 - 1}{11 - 2x - 9x^3}$.

(1)

(b) Use L'Hopital's rule to find $\displaystyle\lim_{x \to 1} \frac{5x^4 - 3x^2 - 2}{11 - 2x - 9x^3}$.

(3)

(Total for Question 4 is 4 marks)

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5. The line L has equation $y = mx + c$, where m and c are constants.

The hyperbola H has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, where a and b are constants.

(a) Given that L is a tangent to H, show that $a^2m^2 = b^2 + c^2$.

(5)

The hyperbola H' has equation $\dfrac{x^2}{26} - \dfrac{y^2}{25} = 1$.

(b) Find the equations of the tangents to H' which pass through the point $(2, 3)$.

(6)

(Total for Question 5 is 11 marks)

6.
$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y = 0.$$

Given that when $x = 0$, $y = \dfrac{dy}{dx} = 1$, find a series solution for y in ascending powers of x, up to and including the term in x^3.

(Total for Question 5 is 9 marks)

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7. Find the set of values of x such that

$$\left|\frac{x}{x+3}\right| < 2 - x$$

expressing your answer in set notation.

(Total for Question 7 is 7 marks)

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8. Given that $y = e^x \sin x$, use Leibnitz's theorem to show that

$$\frac{d^6y}{dx^6} + 8\frac{dy}{dx} = 8y$$

(Total for Question 8 is 4 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 75 MARKS

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