Further Pure Mathematics 1 Practice Paper 1 – answers

Exam-style practice: A level

1 **a** $7x + 2y + 4z = 7$ **b** $\frac{104}{3}$ **c** 0.930 radians

2 **a** 0.68778 **b** 0.68795 **c** 0.02% error

3 **a** $xy\frac{dy}{dx} + 3x^2 + y^2 \Rightarrow \frac{dy}{dx} + \frac{3x}{y} + \frac{y}{x}$ (1)

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ (2)

Substituting (2) into (1) gives: $v + x\frac{dv}{dx} + \frac{3}{v} + v = 0$

$\Rightarrow x\frac{dv}{dx} + \frac{3}{v} + 2v = 0 \Rightarrow x\frac{dv}{dx} + \frac{3 + 2v^2}{v} = 0$

b $3x^4 + 2x^2y^2 = 53$

c $x = 2.050 = 205$ metres

d Velocity of jumper tends to infinity as distance from top of the cliff tends to 0. Hence the model is unsuitable for very small values of x.

4 **a** L'Hospital's rule is only applicable for the limits of functions which tend to $\frac{\pm\infty}{\pm\infty}$ or $\frac{0}{0}$

The function given tends to $\frac{1}{0}$, hence L'Hospital's rule is cannot be used.

b $-\frac{14}{29}$

5 **a** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Substitute in $y = mx + c$: $\frac{x^2}{a^2} - \frac{(mx + c)^2}{b^2} = 1$

$\Rightarrow b^2x^2 - a^2(mx+c)^2 = a^2b^2$

$\Rightarrow b^2x^2 - a^2(m^2x^2 + 2mc + c^2) = a^2b^2$

$\Rightarrow (b^2 - a^2m^2)x^2 - 2mca^2x - a^2(c^2 + b^2) = 0$

This is in the form of a quadratic equation.

For $y = mx + c$ to be a tangent, discriminant = 0:

$4m^2c^2a^4 = -4a^2(b^2 - a^2m^2)(c^2 + b^2)$

$\Rightarrow m^2c^2a^2 = -b^4 - b^2c^2 + a^2m^2c^2 + a^2m^2b^2$

$\Rightarrow b^2 + c^2 = a^2m^2$

b $y = x + 1$, $y = -\frac{17}{11}x + \frac{67}{11}$

6 $y = 1 + x - \frac{3x^2}{2} + \frac{2x^2}{3}$

7 $\{x{:}x - \sqrt{6} < x < \sqrt{7} - 1\} \cup \{x{:}x < 1 - \sqrt{7}\}$

8 For $y = e^x \sin x$, let $u = e^x$.

Hence $\frac{d^k u}{dx^k} = e^x$ for all values of k

Let $v = \sin x$, hence $\frac{dv}{dx} = \cos x$, $\frac{d^2v}{dx^2} = -\sin x$,

$\frac{d^3v}{dx^3} = -\cos x$, $\frac{d^4v}{dx^4} = \sin x$, $\frac{d^5v}{dx^5} = \cos x$, $\frac{d^6v}{dx^6} = -\sin x$

Apply Leibnitz's theorem:

$e^x \sin x + 6e^x \cos x - 15e^x \sin x - 20e^x \cos x + 15e^x \sin x$

$+ 6e^x \cos x - e^x \sin x = -8e^x \cos x = \frac{d^6y}{dx^6}$

$8\frac{dy}{dx} = 8e^x(\cos x + \sin x)$

Hence $\frac{d^6y}{dx^6} + 8\frac{dy}{dx} = -8e^x \cos x + 8e^x \cos x + 8e^x \sin x$

$= 8e^x \sin x = 8y$