Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

A level Further Mathematics
Core Pure Mathematics

Practice Paper 1

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **68**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that 4 and 2i − 3 are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

(Total 6 marks)

[Mark scheme for Question 1]

[Examiner comment]

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2. (a) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(5)

(b) Hence solve for $0 \leq \theta \leq \frac{\pi}{2}$

$$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$$

giving your answers as exact multiples of π.

(5)

(Total 10 marks)

[Mark scheme for Question 2]

[Examiner comment]

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3. (a) A sequence of numbers is defined by

$$u_1 = 8$$

$$u_{n+1} = 4u_n - 9n, \; n \geq 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

(Total 10 marks)

Mark scheme for Question 3

Examiner comment

4. A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line. The vertical displacement, x metres, of the top of the capsule below its initial position at time t seconds is modelled by the differential equation,

$$m\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x = 200\cos t, \quad t \geqslant 0$$

where m is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30 000N.

Taking the value of g to be 10 ms^{-2} and assuming the capsule is at its maximum permissible weight,

(a) (i) explain why the value of m is 3

(ii) show that a particular solution to the differential equation is

$$x = 40\sin t - 20\cos t$$

(iii) hence find the general solution of the differential equation.

(8)

(b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

(4)

(Total 12 marks)

[Mark scheme for Question 4]

[Examiner comment]

5.

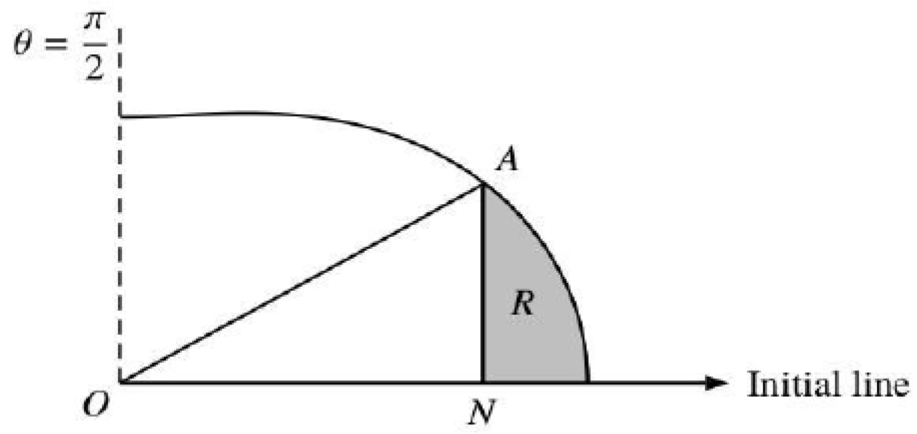

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(Total 9 marks)

Mark scheme for Question 5

Examiner comment

6. (i)

$$p\frac{dx}{dt}+qx=r \qquad \text{where } p,\ q \text{ and } r \text{ are constants}$$

Given that $x = 0$ when $t = 0$

(a) find x in terms of t

(4)

(b) find the limiting value of x as $t \to \infty$

(1)

(ii)

$$\frac{dy}{d\theta}+2y=\sin\theta$$

Given that $y = 0$ when $\theta = 0$, find y in terms of θ

(7)

(Total 12 marks)

Mark scheme for Question 6

Examiner comment

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7. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2-10x+34}\,dx = k\pi,$

giving the value of the fraction k,

(5)

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2-10x+34)}}\,dx = \ln(A+\sqrt{n}),$

giving the values of the integers A and n.

(4)

(Total 9 marks)

Mark scheme for Question 7

Examiner comment

TOTAL FOR PAPER: 68 MARKS

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$x^3 + ax^2 + bx - 52 = 0$, $a, b \in \mathbb{R}$	
	$-2i - 3$	**B1**
		(1)
(b)	$(x - (2i - 3))(x - \text{"}(-2i - 3)\text{"}); = x^2 + 6x + 13$ or	**M1**
	$x = -3 \pm 2i \Rightarrow (x + 3)^2 = -4; = x^2 + 6x + 13\ (= 0)$	**A1**
	$(x - 4)(x - (2i - 3)); = x^2 - (1+2i)x + 4(2i-3)$	
	$(x - 4)(x - \text{"}(-2i - 3)\text{"}); = x^2 - (1-2i)x + 4(-2i-3)$	
	$(x - 4)(x^2 + 6x + 13)\ \{= x^3 + ax^2 + bx - 52\}$	**M1**
	$a = 2,\ b = -11$ or $x^3 + 2x^2 - 11x - 52$	**A1A1**
		(5)
		(6 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	$\cos 6\theta = \text{Re}[(\cos\theta + i\sin\theta)^6]$	
	$(\cos\theta + i\sin\theta)^6 =$ $= c^6 + 6c^5is + 15c^4i^2s^2 + 20c^3i^3s^3 + 15c^2i^4s^4 + 6ci^5s^5 + i^6s^6$	**M1**
	$\cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 + s^6$	**M1A1**
	$= c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$	**M1**
	$\cos 6\theta = c^6 - 15c^4 + 15c^6 + 15c^2(1 - 2c^2 + c^4) - (1 - 3c^2 + 3c^4 - c^6)$	
	$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$*	**A1cso**
		(5)
(b)	$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$ $\Rightarrow 2\cos 6\theta - 1 = 0 \therefore \cos 6\theta = \frac{1}{2}$ or 0.5	**M1A1**
	$\cos 6\theta = \frac{1}{2} \Rightarrow (6\theta =)\ \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}$	**M1A1** **A1**
		(5)
		(10 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	$u_1 = 8$ given $n = 1 \Rightarrow u_1 = 4^1 + 3(1) + 1 = 8 \quad (\therefore \text{ true for } n = 1)$	B1
	Assume true for $n = k$ so that $u_k = 4^k + 3k + 1$	
	$u_{k+1} = 4\left(4^k + 3k + 1\right) - 9k$	M1
	$= 4^{k+1} + 12k + 4 - 9k = 4^{k+1} + 3k + 4$	A1
	$= 4^{k+1} + 3\left(k+1\right) + 1$	A1
	If true for $n = k$ then true for $n = k + 1$ and as true for $n = 1$ true for all n	A1cso
		(5)
(b)	**Condone use of *n* here.**	
	$lhs = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ $rhs = \begin{pmatrix} 2(1)+1 & -4(1) \\ 1 & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	B1
	Assume $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$	
	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 6k+3-4k & -8k-4+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix}$	A1
	$= \left(\begin{pmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{pmatrix}\right)$	
	$= \begin{pmatrix} 2\left(k+1\right)+1 & -4\left(k+1\right) \\ k+1 & 1-2\left(k+1\right) \end{pmatrix}$	A1
	If true for $m = k$ then true for $m = k + 1$ and as true for $m = 1$ true for all m	A1cso
		(5)
		(10 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks	AOs
4(a)(i)	Weight = mass × g $\Rightarrow m = \frac{30\,000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	**M1**	3.3
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 40\cos t + 20\sin t,\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -40\sin t + 20\cos t$	**M1**	1.1b
	$3(-40\sin t + 20\cos t) + 4(40\cos t + 20\sin t)$ $+40\sin t - 20\cos t = \ldots$	**M1**	1.1b
	$= 200 \cos t$ so PI is $x = 40 \sin t - 20 \cos$	**A1***	2.1
	or		
	Let $x = a\cos t + b\sin t$ $\frac{\mathrm{d}x}{\mathrm{d}t} = -a\sin t + b\cos t,\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -a\cos t - b\sin t$	**M1**	1.1b
	$4b - 2a = 200,\ -2b - 4a = 0 \Rightarrow a = \ldots, b = \ldots$	**M1**	2.1
	$x = 40 \sin t - 20 \cos t$	**A1***	1.1b
(iii)	$3\lambda^2 + 4\lambda + 1 = 0 \Rightarrow \lambda = -1, -\frac{1}{3}$	**M1**	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t}$	**A1**	1.1b
	$x = PI + CF$	**M1**	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	**A1**	1.1b
		(8)	
(b)	$t = 0, x = 0 \Rightarrow A + B = 20$	**M1**	3.4
	$x = 0, \frac{\mathrm{d}x}{\mathrm{d}t} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40\cos t + 20\sin t = 0$ $\Rightarrow A + \frac{1}{3}B = 40$	**M1**	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	**A1**	1.1b
	$t = 9 \Rightarrow x = 33\text{m}$	**A1**	3.4
		(4)	
			(12 marks)

Question 4 notes:	
(a)(i)	
M1:	Correct explanation that in the model, $m = 3$
(ii)	
M1:	Differentiates the given PI twice
M1:	Substitutes into the given differential equation
A1*:	Reaches $200\cos t$ and makes a conclusion
or	
M1:	Uses the correct form for the PI and differentiates twice
M1:	Substitutes into the given differential equation and attempts to solve
A1*:	Correct PI
(iii)	
M1:	Uses the model to form and solve the auxiliary equation
A1:	Correct complementary function
M1:	Uses the correct notation for the general solution by combining PI and CF
A1:	Correct General Solution for the model
(b)	
M1:	Uses the initial conditions of the model, $t = 0$ at $x = 0$, to form an equation in A and B
M1:	Uses $\frac{dx}{dt} = 0$ at $x = 0$ in the model to form an equation in A and B
A1:	Correct PS
A1:	Obtains 33m using the assumptions made in the model

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks	AOs
5	$4+\cos 2\theta=\frac{9}{2}\Rightarrow \theta=\ldots$	M1	3.1a
	$\theta=\frac{\pi}{6}$	A1	1.1b
	$\frac{1}{2}\int(4+\cos 2\theta)^2 \mathrm{d}\theta=\frac{1}{2}\int\left(16+8\cos 2\theta+\cos^2 2\theta\right)\mathrm{d}\theta$	M1	3.1a
	$\cos^2 2\theta=\frac{1}{2}+\frac{1}{2}\cos 4\theta\Rightarrow A=\frac{1}{2}\int\left(16+8\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\left[16\theta+4\sin 2\theta+\frac{\sin 4\theta}{8}+\frac{\theta}{2}\right]$	A1	1.1b
	Using limits 0 and their $\frac{\pi}{6}$: $\frac{1}{2}\left[\frac{33\pi}{12}+2\sqrt{3}+\frac{\sqrt{3}}{16}-(0)\right]$	M1	1.1b
	Area of triangle $=\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{81}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}$	M1	3.1a
	Area of $R=\frac{33\pi}{24}+\frac{33\sqrt{3}}{32}-\frac{81\sqrt{3}}{32}$	M1	1.1b
	$=\frac{11}{8}\pi-\frac{3\sqrt{3}}{2}\left(p=\frac{11}{8},\ q=-\frac{3}{2}\right)$	A1	1.1b
		(9 marks)	

Notes:

M1: Realises the angle for A is required and attempts to find it
A1: Correct angle
M1: Uses a correct area formula and squares r to achieve a 3TQ integrand in $\cos 2\theta$
M1: Use of the correct double angle identity on the integrand to achieve a suitable form for integration
A1: Correct integration
M1: Correct use of limits
M1: Identifies the need to subtract the area of a triangle and so finds the area of the triangle
M1: Complete method for the area of R
A1: Correct final answer

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(i)(a)	$xe^{\frac{qt}{p}} = \int \frac{r}{p} e^{\frac{qt}{p}} dt$	M1
	$xe^{\frac{qt}{p}} = \frac{r}{q} e^{\frac{qt}{p}} (+c)$	dM1
	$t = 0,\ x = 0,\ c = -\frac{r}{q}$	ddM1
	$xe^{\frac{qt}{p}} = \frac{r}{q} e^{\frac{qt}{p}} - \frac{r}{q}$ $x = \frac{r}{q} - \frac{r}{q} e^{-\frac{q}{p}t}$	A1
		(4)
(i)(b)	$t \to \infty,\ e^{-\frac{q}{p}t} \to 0, \qquad (x \to) \frac{r}{q}$	B1
		(1)
(ii)	$ye^{2\theta} = \int e^{2\theta} \sin\theta \, d\theta$	M1A1
	$ye^{2\theta} = \left[-e^{2\theta}\cos\theta\right] + 2\int e^{2\theta}\cos\theta \, d\theta$ Or $\left[\frac{1}{2}e^{2\theta}\sin\theta\right] - \frac{1}{2}\int e^{2\theta}\cos\theta \, d\theta$	M1
	$\left[-e^{2\theta}\cos\theta\right] + 2\left\{\left[e^{2\theta}\sin\theta\right] - 2\int e^{2\theta}\sin\theta \, d\theta\right\}$ Or $\frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{2}\left[\frac{1}{2}e^{2\theta}\cos\theta + \frac{1}{2}\int e^{2\theta}\sin\theta d\theta\right]$	M1
	$\left(ye^{2\theta} =\right) - e^{2\theta}\cos\theta + 2e^{2\theta}\sin\theta - 4\int e^{2\theta}\sin\theta \, d\theta$ Or $\frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{4}e^{2\theta}\cos\theta - \frac{1}{4}\int e^{2\theta}\sin\theta \, d\theta$	A1
	$ye^{2\theta} = -e^{2\theta}\cos\theta + 2e^{2\theta}\sin\theta - 4ye^{2\theta} + c$ or $ye^{2\theta} = \frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{4}e^{2\theta}\cos\theta - \frac{1}{4}ye^{2\theta} + c$	dM1

Question	Scheme	Marks
6(ii) *continued*	$y\mathrm{e}^{2\theta} = \int \mathrm{e}^{2\theta}\sin\theta\ \mathrm{d}\theta = \frac{1}{5}\mathrm{e}^{2\theta}(2\sin\theta - \cos\theta)(+c)$	
	$\theta = 0, y = 0 \Rightarrow C = \frac{1}{5}$	
	$y = \frac{1}{5}(2\sin\theta - \cos\theta) + \frac{1}{5}\mathrm{e}^{-2\theta}$	**A1cso**
		(7)
		(12 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme		Marks
7(a)	$x^2 - 10x + 34 = (x-5)^2 + 9$ so $\frac{1}{x^2-10x+34} = \frac{1}{(x-5)^2+9} = \frac{1}{u^2+9}$		**B1**
	(mark can be earned in either part (a) or (b))		
	$I = \int \frac{1}{u^2+9}du = \left[\frac{1}{3}\arctan\left(\frac{u}{3}\right)\right]$	$I = \int \frac{1}{(x-5)^2+9}du$ $= \left[\frac{1}{3}\arctan\left(\frac{x-5}{3}\right)\right]$	**M1A1**
	Uses limits 3 and 0 to give $\frac{\pi}{12}$	Uses limits 8 and 5 to give $\frac{\pi}{12}$	**dM1** **A1**
			(5)
(b)	$I = \ln\left(\left(\frac{x-5}{3}\right) + \sqrt{\left(\frac{x-5}{3}\right)^2 + 1}\right)$ or $I = \ln\left(\frac{x-5+\sqrt{(x-5)^2+9}}{3}\right)$ or $I = \ln\left((x-5) + \sqrt{(x-5)^2+9}\right)$		**M1A1**
	Uses limits 5 and 8 to give $\ln(1+\sqrt{2})$		**dM1** **A1**
			(4)
			(9 marks)

Further Core Pure Mathematics - Practice Paper 01 - Examiner Report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. Many candidates were confused by the root 2i − 3, written with the imaginary part first, and therefore wrote down 2i + 3 as their third root. They could then score, at best, only 2 marks out of 5. Of those with the correct third root many then scored full marks. Most firstly multiplied out the two brackets containing the complex roots, followed by $(x - 4)$. Others found the correct quadratic factor by using − (sum) and product of roots and just a few used $(x + 3)^2 = -4$. The latter two approaches were less error prone than multiplying out the brackets. It was not uncommon to see pairs of brackets with the wrong signs, due to using $(x + \text{root})$. Other approaches were rarely seen.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. Being a 'show that' question, detailed working was required to gain full marks and this was seen from most students.

Most students answered the first part of this question successfully navigating their way correctly through real and imaginary parts and the expansion of brackets involving $(1 - \cos^2\theta)$. Some students used $(\cos^2\theta - 1)$ by mistake compromising the last two marks of question (a). It was surprising to see that having expanded $(c + \text{i}s)^6$ is using the Binomial Expansion, most students then expanded $(1 - \cos^2\theta)^3$ 'long hand'.

Some students began by expanding $\left(z+\frac{1}{z}\right)^n$ without giving any indication of what z might be representing. Most of those who did replace $z^n+\frac{1}{z^n}$ with $2\cos n\theta$ were successful in obtaining the desired result after some further work.

Most students reached the correct equation $\cos 6\theta = \frac{1}{2}$ and solved this correctly. Those who ended up with the wrong equation (e.g. $\cos 6\theta = 1$ or $\cos 6\theta = 0$) could still gain 2 out of the available 5 marks. Students were confident proceeding to obtain three correct solutions in the required range although a number left their final answer with only one or two angles.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. In general, the methods required for mathematical induction were well understood, but the specific requirements of this question were missed by many candidates. Statements were often ones that had been learned, rather than being used in the appropriate context. The conclusions were often ill-conceived, particularly when defining the values for which the proof was valid.
In part (a) some candidates validated the result for $n = 2$ rather than $n = 1$. Some candidates used $u_{k+1} = 4u_k - 9(k + 1)$ and a few wrote that $4(4^k) = 16^k$, but the most common error here was not taking the expression $4^{k+1} + 3k + 4$ any further and not formally proving that it is true for $n = k + 1$.

Part (b) was more successful than part (a), although a few candidates did not show sufficient working when multiplying out their matrices to justify being awarded full marks for their solution.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. No Examiner's Report available for this question. (Taken from SAMs)

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. No Examiner's Report available for this question. (Taken from SAMs)

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. This was one of the more challenging questions on the paper, along with this question; particularly part (ii) which tested students' integration techniques. The use of unknown constants in part (i) was also a difficulty for some students. However, nearly all were confident enough to have an attempt at this question, even if incomplete. The question as a whole seemed to work as a good discriminator between students' different levels of ability, with only a small minority achieving full marks.

In part (i) it is unfortunate that there was an omission of constraints given on the variables p, q and r, but this did not seem to cause problems for students, with very few (if any) cases of students not assuming they are all positive (as was the intention of the question).

Part (a) was generally well done, although there were many students who did not use the initial conditions to find the constant of integration. This was perhaps due to the fact that there were already "unknown constants" in the equation, and so the presence of a constant of integration was just one more. The subtle difference of the role of the constants in the question and the role of the constant of integration (to be found in terms of the constants of in the question given the initial condition) was lost on these students. Many of the students who stopped at the general solution part (i) but who had a good attempt at part (ii) did attempt the constant in the second part.

The majority of students used an integrating factor method to answer this part of the question. A few did identify that the variables could be separated, and usually went on to successfully complete the integration, although for some the separation attempt was poor, yielding equations with a single term denominator. A very small number of students applied an auxiliary equation method (with successful outcome). For those using the integrating factor approach, most were successful in completing the first stage, although there were still a number of students who did not multiply the right-hand side by the integrating factor, or missed the variable t from the index of their term. There were also occasional errors in algebraic manipulation or mixing up the constants, with miscopying of the index between lines being a not infrequent error; students need to mind their p's and q's.

In part (b) the idea of a limiting value seemed to be well understood by the majority of students, along with the fact that expressions of the form e^{-kx} approach 0 as $x \to \infty$. Almost all students with a correct answer to part (i)(a) went on to gain this mark, even those who had not evaluated the constant of integration.

Part (ii) was a discriminator for the paper.

The majority of the students used the intended integrating factor method, and almost all of these reached $ye^{2\theta} = \int e^{2\theta} \sin\theta d\theta$. After this point the levels of progress made varied considerably, as this type of integral seemed to be beyond the scope of many students, although applications of integration by parts twice are within the C4 specification.

Of those who reached this stage, only a minority stopped and made no more progress, not even attempting the integral, while a similar number made incorrect approaches (eg substitutions which lead nowhere), or attempted a reduced simplicity integral (eg just integrating the sine term to yield $-e^{2\theta}\cos\theta$. The majority of students did, however, attempt integration by parts (not necessarily always correctly, with sign errors, or errors with the constant multiples occurring frequently) at least once.

For most, though, they stopped after the first application, not recognising where to go next, or they then regressed to an over simplification (similar to above). Of the minority who realised a second application of parts was necessary, a small number reversed the roles of the parts and so ended up back at the beginning, resulting in giving up or trying a different method, but most did apply the correct way a second time. For those doing so most did then recognise the original integral within their expression and proceed to replace by $ye^{2\theta}$, rearrange and hence find the expression. Of the students successfully completing the method, a few had made sign errors and ended up dividing through, for example, by 3 instead of 5, but most did achieve the correct expression and usually included a constant of integration at this stage. However, some divided through by the exponential before including the constant, and thus ended up with the incorrect answer. The inclusion, and attempt to find a constant of integration, was much more in evidence on this part than in part (i), even if incorrectly carried out.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. This was probably the most accessible question on the paper with over 75% of the candidates gaining full marks and only just under 15% failing to get at least 7 marks. It was done consistently well and it was clear from the scripts that candidates were usually comfortable with these integrations and the work flowed well with little crossing out.

Almost all candidates were able to write the quadratic in the form $(x - 5)^2 + 9$ and went on to quote the correct arctan or arcsinh functions often in terms of x; however, many substituted $(x - 5) = u$ first. Errors, when they did occur, usually resulted from not applying the change of limits correctly, or having the wrong value of k in $k\arctan\left(\frac{x-5}{3}\right)$ but errors from weaker candidates included writing $\frac{1}{(x-5)^2+9}$ as $\frac{1}{(x-5)^2}+\frac{1}{9}$ or writing the given integrals as $\int(x^2-10x+34)^{-1}dx$ and $\int(x^2-10x+34)^{-\frac{1}{2}}dx$ respectively and trying to find a purely algebraic result. In part (b) the quickest way was to use arcsinh u between limits 0 and 1 leading to arcsinh $1 = \ln(1 + \sqrt{2})$. There were sometimes errors in selecting the correct ln form for $\text{arsinh}\left(\frac{x-5}{3}\right)$ or $\text{arsinh}\left(\frac{u}{3}\right)$, and those who used the ln form with limits 3 and 8 to give $\ln(3 + \sqrt{18}) - \ln 3$ sometimes made errors in simplifying, but generally it was successfully managed.