Specimen Paper 9FM0/3A: Further Pure Mathematics 1 Mark scheme

Question	Scheme		Marks	AOs
1	$\frac{dx}{d\theta} = a\sec\theta\tan\theta,\ \frac{dy}{d\theta} = b\sec^2\theta$	$k\frac{x}{a^2} - n\frac{y}{b^2}\frac{dy}{dx} = 0$ where $k > 0$ and $n > 0$	M1	2.1
	$\frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}\left(= \frac{b\sec\theta}{a\tan\theta} = \frac{b}{a\sin\theta}\right)$	$\frac{dy}{dx} = \frac{b^2x}{a^2y} = \frac{b\sec\theta}{a\tan\theta}$	A1	1.1b
	$y - b\tan\theta = \left(\frac{b\sec\theta}{a\tan\theta}\right)(x - a\sec\theta)$		M1	1.1b
	$ya\tan\theta = xb\sec\theta - ab$*		A1*	2.1
			(4)	
			(4 marks)	

Notes:

M1: Differentiates in an attempt to find $\frac{dy}{dx}$

either differentiates x and y and divides $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$

or achieves $k\frac{x}{a^2} - n\frac{y}{b^2}\frac{dy}{dx} = 0$ where $k > 0$ and $n > 0$

A1: Correct expression for $\frac{dy}{dx}$

M1: Uses $y - b\tan\theta =$ 'their $\frac{dy}{dx}$' $(x - a\sec\theta)$

A1*: Uses correct algebra and trig identities to achieve the correct equation of the tangent.

Question	Scheme	Marks	AOs
2(a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$ or $\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}$	M1	1.1b
	area of triangle $OBC = \frac{1}{2}\lvert 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k} \rvert = \frac{1}{2}\sqrt{50} = \frac{5}{2}\sqrt{2}$ o.e.	A1	2.2a
		(2)	
(b)	$\mathbf{a} \,.\, (\mathbf{b} \times \mathbf{c}) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} = 0 + 5 + 0 = 5$	M1	1.1b
	volume of tetrahedron $OABC = \frac{1}{6} \times \lvert 5 \rvert = \frac{5}{6}$	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)
M1: Attempts the vector product $\mathbf{b} \times \mathbf{c}$, with at least two correct terms.
A1: Deduces area of triangle $OBC = \frac{1}{2}\lvert \mathbf{b} \times \mathbf{c} \rvert$

(b)
M1: Attempt at the triple scaler product $\mathbf{a} \,.\, (\mathbf{b} \times \mathbf{c})$
A1: Deduces volume of tetrahedron $OABC = \frac{1}{6}\lvert \mathbf{a} \,.\, (\mathbf{b} \times \mathbf{c}) \rvert$

Question	Scheme	Marks	AOs
3(a)	$4\tan x + 3\cot\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$ $= 4\tan x + \frac{3}{\tan\left(\frac{x}{2}\right)}\left(1 + \tan^2\left(\frac{x}{2}\right)\right)$ $= 4\left(\frac{2t}{1-t^2}\right) + \frac{3}{t}(1 + t^2)$	M1	2.1
	$\left(\frac{8t}{1-t^2}\right) + \frac{3(1+t^2)}{t} = 0$ $8t^2 + 3(1+t^2)(1-t^2) = 0$ or $\frac{8t^2+3(1+t^2)(1-t^2)}{t(1-t^2)} = 0$	M1	1.1b
	$3t^4 - 8t^2 - 3 = 0$ *	A1*	1.1b
		(3)	
(b)	Solves quadratic for t^2 by factorising, quadratic formula, calculator $(3t^2+1)(t^2-3) = 0$, $t^2 = \frac{8 \pm \sqrt{(-8)^2-4(3)(-3)}}{2(3)}$ $t^2 = 3,\ t^2 = -\frac{1}{3}$ leading to value for *t* = ….	M1	3.1a
	$t = \pm\sqrt{3}$	A1	1.1b
	Finds two correct values for $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$	M1	1.1b
	All correct values for $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a)

M1: Expresses $3\cot\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$ in terms of $\tan\left(\frac{x}{2}\right)$ and uses *t* substitutions to obtain an expression in terms of *t* only

M1: Multiplies through by *t* and $(1 - t^2)$ or forms a common denominator

A1*: $3t^4 - 8t^2 - 3 = 0$ cso

(c)

M1: Solve the quadratic in t^2 leading to a value for *t*

A1: Correct values for $t = \pm\sqrt{3}$

M1: Finds two correct values for $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

A1: All correct values for $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

Question	Scheme	Marks	AOs
4	Solves $x^2 - 2x - 2 = 0$ or $x^2 + 2x - 2 = 0$	M1	1.1b
	Solves both $x^2 - 2x - 2 = 0$ and $x^2 + 2x - 2 = 0$	M1	3.1a
	$x = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$ and $x = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$	A1	1.1b
	Deduces the required roots are $x = 1 + \sqrt{3}$ and $x = -1 + \sqrt{3}$	M1	2.2a
	e.g. $\{x \in \mathbb{R}: x < -1 + \sqrt{3}\} \cup \{x \in \mathbb{R}: x > 1 + \sqrt{3}\}$	A1	2.5
		(5)	
			(5 marks)

Notes:

M1: Solves either $x^2 - 2x - 2 = 0$ or $x^2 + 2x - 2 = 0$
M1: Complete strategy to identify and solve all relevant equations and gets two critical values
A1: Correct exact values for x, may be unsimplified
M1: Deduces that the larger roots are required in each case
A1: Correct set of values given and correct set notation form

Question	Scheme	Marks	AOs
5(i)	$\mathbf{b} \times \mathbf{a}$ is perpendicular to $\mathbf{a}$ (and/or $\mathbf{b}$)	M1	2.4
	Therefore $\mathbf{a}.(\mathbf{b} \times \mathbf{a}) = 0$	A1	1.1b
		(2)	
(ii)	$\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \Rightarrow \mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$	M1	3.1a
	As $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$ then $\mathbf{a}$ is parallel to $(\mathbf{b} - \mathbf{c})$ therefore $\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$	A1	2.4
		(2)	
			(4 marks)

Notes:

(i)
M1: Reasoning that $\mathbf{b} \times \mathbf{a}$ is perpendicular to $\mathbf{a}$
A1: $\mathbf{a}.(\mathbf{b} \times \mathbf{a}) = 0$

(ii)
M1: Collecting on to one side and factorising
A1: Reasoning: as $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$ then $\mathbf{a}$ is parallel to $(\mathbf{b} - \mathbf{c})$ therefore $\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$

Question	Scheme	Marks	AOs
6(a)	$x_0 = 0, \quad y_0 = 1, \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 0 - 1 = -1$	B1	1.1b
	$y_1 \approx y_0 + h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 1 + 0.05(-1) = \ldots$	M1	1.1b
	= 0.95	A1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 0.05^2 - 0.95^2 = -0.9$ $y_2 \approx y_1 + h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 0.95 + 0.05(-0.9) = \ldots$	M1	2.1
	= 0.905	A1	1.1b
		(5)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y^2 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x - 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	2.1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - \lambda y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \pm \mu\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ or substituting in $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y^2$ so that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x - 2yx^2 + 2y^3$ $\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 \pm \alpha xy \pm \beta x^2\frac{\mathrm{d}y}{\mathrm{d}x} + \delta y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	1.1b
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ or $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - 4xy - 2x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 6y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	1.1b
		(3)	
(c)	$x = 0, y = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -1$	M1	2.2a
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2(0) - 2(1)(-1) = 2$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - 2(1)(2) - 2(-1)^2 = -4$	M1 A1	1.1b 1.1b
	$y = y(0) + y'(0)x + \frac{y''(0)x^2}{2} + \frac{y'''(0)x^3}{6} + \ldots$	M1	2.5
	Series solution $y = 1 - x + x^2 - \frac{2}{3}x^3$	A1	1.1b
		(5)	
			(13 marks)

Notes:

(a)

B1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = -1$

M1: Applies the approximation formula with y_0 and their value for $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0$

A1: $y_1 = 0.95$

M1: Finds $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1$ and applies the approximation formula with their values for y_1 and $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1$

A1: $y_2 = 0.905$

(b)

B1: Differentiates to $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x - 2y\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Differentiates to the form $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - \lambda y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \pm \mu\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ where $\lambda > 0, \mu \neq 0$

or substituting in $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y^2$ so that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x - 2yx^2 + 2y^3$

$\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 \pm \alpha x\frac{\mathrm{d}y}{\mathrm{d}x} \pm \beta x^2\frac{\mathrm{d}y}{\mathrm{d}x} + \delta y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ where $\delta > 0, \alpha, \beta \neq 0$

A1: $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ or $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - 4xy - 2x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 6y^2\frac{\mathrm{d}y}{\mathrm{d}x}$

(c)

B1: Deduces the value for $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$

M1: Finds the values of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

A1: Correct values for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

M1: Substitutes into the correct formula and mathematical language, allow factorial notation

A1: Correct series, must start with $y = \ldots$

Question	Scheme	Marks	AOs
7(a)	$A = 1000\left(1+\frac{5}{1200}\right)^{12} = 1051.16*$	B1*	1.1b
		(1)	
(b)	Let $y = \left(1+\frac{r}{100n}\right)^n$ so $\ln y = \ln\left(1+\frac{r}{100n}\right)^n = n\ln\left(1+\frac{r}{100n}\right)$	M1	3.1a
	$\lim_{n\to\infty} \ln y = \lim_{n\to\infty} n\ln\left(1+\frac{r}{100n}\right) = \lim_{n\to\infty} \frac{\ln\left(1+\frac{r}{100n}\right)}{{}^{1}/_{n}}$	M1	2.1
	$= \lim_{n\to\infty}\left[\frac{{}^{-r}/_{100n^2}}{\left(1+\frac{r}{100n}\right)} \div \frac{-1}{n^2}\right]$	dM1 A1	1.1b 1.1b
	$= \lim_{n\to\infty} \frac{r/100}{1+\frac{r}{100n}} = \frac{r}{100}$	A1	1.1b
	$\lim_{n\to\infty}\left(1+\frac{r}{100n}\right)^n = \lim_{n\to\infty} y = \lim_{n\to\infty} \mathrm{e}^{\ln y} = \mathrm{e}^{\lim_{n\to\infty} \ln y} = \mathrm{e}^{\frac{r}{100}}*$	A1*	2.1
		(6)	
(c)	$\lim_{n\to\infty}\left(1+\frac{5}{100n}\right)^n = \mathrm{e}^{0.05}$	B1	3.4
	Therefore $\lim_{n\to\infty} 1000\left(1+\frac{5}{100n}\right)^n = 1000\mathrm{e}^{0.05}$	M1	2.2a
	Student has £1051.27 in their saving account after one year	A1	3.2a
		(3)	
			(10 marks)

Notes:

(a)

B1*: Using $P = 1000$, $r = 5$ and $n = 12$ to show $A = 1051.16$

(b)

M1: Taking ln's to express $\ln\left(1+\frac{r}{100n}\right)^n = n\ln\left(1+\frac{r}{100n}\right)$

M1: Expressing the limit as a quotient $\lim_{n\to\infty} \ln y = \lim_{n\to\infty} \frac{\ln\left(1+\frac{r}{100n}\right)}{{}^{1}/_{n}}$

dM1: Applies L'Hospital's rule and attempts to differentiate both the numerator and denominator. Depends on previous method mark

A1: Correct differentiation $\lim_{n\to\infty}\left[\frac{{}^{-r}/_{100n^2}}{\left(1+\frac{r}{100n}\right)} \div \frac{-1}{n^2}\right]$ simplified or un-simplified

A1: Correct answer for the limit

A1*: Fully correct proof with all mathematical notation cso

(c)

B1: Uses model and the result from part (b) $\lim_{n\to\infty}\left(1+\frac{0.05}{n}\right)^n = \mathrm{e}^{0.05}$

M1: Deduces that the amount will be $\lim_{n\to\infty} 1000\left(1+\frac{0.05}{n}\right)^n = 1000\mathrm{e}^{0.05}$

A1: Give answer in pounds to 2 decimal places £1051.27

Question	Scheme	Marks	AOs
8(a)	$x = wt \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}w}{\mathrm{d}t}t + w$	B1	1.1b
	$t\left(\frac{\mathrm{d}w}{\mathrm{d}t}t + w\right) + 2t^2(wt) = wt(2t+1)$	M1	2.1
	$t^2\frac{\mathrm{d}w}{\mathrm{d}t} + tw + 2t^3w = 2wt^2 + wt$ $t^2\frac{\mathrm{d}w}{\mathrm{d}t} + 2t^3w = 2wt^2$ Leading to $\frac{\mathrm{d}w}{\mathrm{d}t} + 2tw - 2w = 0$ *	A1*	1.1b
		(3)	
(b)	$\int \frac{1}{w}\,dw = 2\int (1-t)\,dt$	M1	3.1a
	$\ln w = 2t - t^2\ (+c)$	M1	1.1b
	Uses correct exponential and ln work to reach $w = \mathrm{e}^{2t-t^2+c} = \mathrm{e}^{2t-t^2}\mathrm{e}^{c} = A\mathrm{e}^{2t-t^2}$	A1	2.1
	Displacement from O is given by $x = wt = \ldots$	M1	3.4
	$x = At\mathrm{e}^{2t-t^2}$*	A1*	2.2a
		(5)	
(c)	Uses $x = 10$ when $t = 2$ to find the value of A, $10 = 2A$ and achieves $x = 5t\mathrm{e}^{2t-t^2}$	B1	3.4
		(1)	
(d)	Sets $\frac{\mathrm{d}x}{\mathrm{d}t} = 0 \Rightarrow 2t^2x = x(2t+1) \Rightarrow 2t^2 = 2t+1$ or differentiates $x \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 5\mathrm{e}^{2t-t^2} + 5t(2-2t)\mathrm{e}^{2t-t^2} = 0$ to form and solve a quadratic equation.	M1	3.1a
	$t = \frac{1 \pm \sqrt{3}}{2}$	A1	1.1b
	$x = 5\left(\frac{1+\sqrt{3}}{2}\right)\mathrm{e}^{2\left(\frac{1+\sqrt{3}}{2}\right)-\left(\frac{1+\sqrt{3}}{2}\right)^2}$	dM1	1.1b
	Maximum displacement = awrt 16.2 m or 162 cm	A1	3.2a
		(4)	
(e)	$x = 5t\mathrm{e}^{2t-t^2} = 5t\mathrm{e}^{2t}\mathrm{e}^{-t^2}$ or $\frac{5t\mathrm{e}^{2t}}{\mathrm{e}^{t^2}}$	M1	3.4
	As $t \longrightarrow \infty$, $\mathrm{e}^{-t^2} \to 0$ or $\mathrm{e}^{2t-t^2} \to 0$ $\therefore$ displacement from O tends to 0 or $\mathrm{e}^{t^2} \longrightarrow \infty$ $\therefore$ displacement from O tends to 0	A1	2.4
		(2)	
			(15 marks)

Notes:
(a) **B1:** Correct derivative $\frac{dx}{dt}$ **M1:** Substitutes in their $\frac{dx}{dt}$ **A1*:** Completely correct proof
(b) **M1:** Separates the variables correctly, with d*w* and d*t* the correct positions. **M1:** Integrates both sides to the form $\ln w = f(t)$ with or without +c **A1:** Uses correct exponential and ln work to reach $w = e^{2t - t^2 + c} = e^{2t - t^2}e^c = Ae^{2t - t^2}$ Must have + c and a correct intermediate stage. **M1:** Links their equation $w = f(t)$ to the solution of the model equation correctly. For *x* = *t* 'their *w*' **A1*:** Deduces the correct general equation for the distance
(c) **B1:** Uses *x* = 10 when *t* = 2 to find the correct value of *A*
(d) **M1:** Sets $\frac{dx}{dt} = 0$ into the differential equation, or uses the product rule to differentiate *x* and sets $\frac{dx}{dt} = 0$, to form and solve a quadratic equation. **A1:** Correct value(s) for *t* **dM1:** Dependent of previous method mark, substitutes their value of *t* to find a value for x. **A1:** Maximum displacement = awrt 16.2 m or 162 cm
(e) **M1:** Using the model, separates the exponential terms **A1:** Reason ∴ displacement from *O* tends to 0

Question	Scheme	Marks	AOs
9(a)	$\frac{a}{e} = \frac{8}{3}\sqrt{3}$	B1	3.4
	Uses $a = 2b$ and $b^2 = a^2(1 - e^2)$ Either $b^2 = 4b^2(1 - e^2)$ or $\frac{a^2}{4} = a^2(1 - e^2)$	M1	3.1a
	Either $\frac{1}{4} = 1 - e^2$ then $e = \frac{\sqrt{3}}{2}$ so $a = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{8}{3}\sqrt{3}\right)$ or $e = \frac{3}{8\sqrt{3}} = \frac{a\sqrt{3}}{8}$ then $\frac{1}{4} = \left(1 - \frac{3a^2}{64}\right)$ leading to $a = \ldots$	M1	2.1
	$a = 4$	A1	1.1b
	$b = \frac{a}{2} = 2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	M1	1.1b
	$\frac{x^2}{16} + \frac{y^2}{4} = 1$	A1	1.1b
		(6)	
(b)	Foci $(\pm ae, 0)$, $x = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$	M1	3.1b
	Water features at $\left(2\sqrt{3}\,, 0\right)$ and $\left(-2\sqrt{3}\,, 0\right)$	A1	3.4
		(2)	
(c)	Uses $PS = ePN$, leading to $2 = \frac{\sqrt{3}}{2}\,PN$	M1	3.4
	$PN = \frac{4}{\sqrt{3}} = \frac{4}{3}\sqrt{3}$	A1	1.1b
	$x = \frac{8}{3}\sqrt{3} - \frac{4}{3}\sqrt{3} = \frac{4}{3}\sqrt{3}$	M1	1.1b
	$\frac{\left(\frac{4}{3}\sqrt{3}\right)^2}{16} + \frac{y^2}{4} = 1$ leading to a value for y	M1	1.1b
	Uses symmetry to find all 4 points $\left(\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right)$, $\left(\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right)$, $\left(-\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right)$ and $\left(-\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right)$	A1	3.4
		(5)	

	Alternative 9(c) Solves $\left(x - '2\sqrt{3}'\right)^2 + y^2 = 4$ or $\left(x + '2\sqrt{3}'\right)^2 + y^2 = 4$ with the equation of their ellipse and follow through on their foci to find a value of *x* or *y*	M1	3.4
	Any correct value of *x* or *y*	A1	1.1b
	Uses symmetry to find another value of *x* or *y* or Solves $\left(x - '2\sqrt{3}'\right)^2 + y^2 = 4$ and $\left(x + '2\sqrt{3}'\right)^2 + y^2 = 4$ with the equation of their ellipse and follow through on their foci to find a value of *x* or *y*	M1	1.1b
	Finds a complete point	M1	1.1b
	Finds all 4 points $\left(\frac{4}{3}\sqrt{3},\ \frac{2}{3}\sqrt{6}\right), \left(\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right), \left(-\frac{4}{3}\sqrt{3},\ \frac{2}{3}\sqrt{6}\right)$ and $\left(-\frac{4}{3}\sqrt{3},\ -\frac{2}{3}\sqrt{6}\right)$	A1	3.4
		(5)	
			(13 marks)

Notes:

(a)
B1: Using half the length equals the *x* coordinate of the directrix
M1: Uses $a = 2b$ and $b^2 = a^2(1 - e^2)$
M1: For a complete method to find a value for *a*
A1: For $a = 4$
M1: Finding the value for *b* and substituting the values of *a* and *b* into the equation of an ellipse
A1: Correct equation of the ellipse, must square out *a* and *b*.

(b)
M1: For realising that the foci for the ellipse are required and finds *x*-coordinate of focus $x = ae$
A1: Finds both coordinates for the water features

(c)
M1: Uses focus directrix property with *PS* = 2 and their value for e
A1: Correct distance for *PN*
M1: Using *x* = directrix – *PN*
M1: Substitutes value for *x* into their equation of the ellipse to find a value for *y*
A1: All 4 correct points

(c) Alternative
M1: Solves simultaneously their equation of the ellipse and a circle with centre (‘their foci’, 0) and radius 2. Finds at least one value for *x* or *y*
A1: A correct value of $x = \pm\frac{4}{3}\sqrt{3}$ or $y = \pm\frac{2}{3}\sqrt{6}$
M1: Or uses symmetry to find another values of *x* or *y*. Or solves simultaneously their equation of the ellipse and both circles with centre (‘their foci’, 0) and radius 2. Finds at least one value of *x* or *y*.
M1: Finds a complete coordinate
A1: All 4 correct points