Specimen Paper 9FM0/01: Core Pure Mathematics 1 Mark scheme

Question	Scheme	Marks	AOs
1	$6(1+2\sinh^2 x)+4\sinh x=7$ and rearranges to quadratic form OR substitutes correct exponential identifies and rearranges to quartic in e^x, $\cosh 2x=\frac{e^{2x}+e^{-2x}}{2}$ and $\sinh x=\frac{e^x-e^{-x}}{2}$ used.	M1	3.1a
	$12\sinh^2 x+4\sinh x-1=0$ OR $3e^{4x}+2e^{3x}-7e^{2x}-2e^x+3=0$	A1	1.1b
	$(6\sinh x-1)(2\sinh x+1)=0\Rightarrow \sinh x=\ldots$ OR $(e^{2x}+e^x-1)(3e^{2x}-e^x-3)=0\Rightarrow e^x=\ldots.$	M1	1.1b
	$\sinh x=\frac{1}{6}$ or $\sinh x=-\frac{1}{2}$ OR $e^x=\frac{-1\pm\sqrt{5}}{2}$ or $e^x=\frac{1\pm\sqrt{37}}{6}$	A1	1.1b
	$x=\ln\left(a+\sqrt{1+a^2}\right)$ where a is one of their $\sinh x$ values OR undoes exponentials using ln	M1	1.2
	$x=\ln\left(\frac{1+\sqrt{37}}{6}\right)$, $x=\ln\left(\frac{-1+\sqrt{5}}{2}\right)$	A1	1.1b
		(6)	
(6 marks)			

Notes:

M1: Identifies a correct approach to solving the problem, either through use of identity or definition of hyperbolics

A1: Reaches a correct quadratic in $\sinh x$ or a correct quartic in e^x.

M1: Solves their quadratic/quartic, may just see answers from calculator.

A1: Correct values for $\sinh x$ or e^x found.

M1: Correct process of reaching x from their solutions in $\sinh x$ or e^x.

A1: Correct answers as exact simplified logarithms, and no others (in the alternative the negative exponential cases must have been rejected).

Question	Scheme	Marks	AOs
2	Profit in 2017 is $0.99\times£39.15\text{m} = £38.7585\text{m}$	B1	2.2a
	Let x = number of visitors to park A in 2016, y = number of visitors to park B in 2016 and z = number of visitors to park C in 2016. So $0.5x + 1.25y + 1.15z = 1.35\times10^6$ $30x + 26y + 33z = 39.15\times10^6$ $15x + 32.5y + 37.95z = 38.7585\times10^6$	M1 A1	3.1b 1.1b
	Hence $\begin{pmatrix} 0.5 & 1.25 & 1.15 \\ 30 & 26 & 33 \\ 15 & 32.5 & 37.95 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.35\times10^6 \\ 39.15\times10^6 \\ 38.7585\times10^6 \end{pmatrix}$	M1 A1	3.1a 1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.5 & 1.25 & 1.15 \\ 30 & 26 & 33 \\ 15 & 32.5 & 37.95 \end{pmatrix}^{-1}\begin{pmatrix} 1.35\times10^6 \\ 39.15\times10^6 \\ 38.7585\times10^6 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 400331.75... \\ 593257.41... \\ 355010.74... \end{pmatrix}$	A1	1.1b
	So in 2016 park A had 400 000 visitors, park B had 590 000 visitors and park C had 360 000 visitors to 2 s.f.	A1ft	3.2a
		(8)	

(8 marks)

Notes:

B1: Deduces correct total profit for 2017.
M1: Attempts to set up equations in the three unknowns using the information given.
A1: At least two equations correct, with appropriate variables defined.
M1: Sets up a matrix equation from their three equations.
A1: Correct matrix equation (or equivalent).
M1: Solves the equation using the inverse of their matrix (found from calculator or otherwise) to obtain at least one value of x, y or z.
A1: Correct answer as a vector.
A1ft: Interprets their answer in the context of the question with figures quoted to 2 s.f. Withhold this mark if answers not given to 2 s.f.

Note Accept equivalents throughout with the rows in a different order.

Note If the $\times10^6$ is missing throughout, allow the Ms but not the As as they have not interpreted the context (but if units are later sorted out and correct answer reached, full marks can be awarded).

Note $\begin{pmatrix} 0.5 & 1.25 & 1.15 \\ 30 & 26 & 33 \\ 15 & 32.5 & 37.95 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4916... & 0.0576... & -0.0650... \\ 3.6871... & -0.0098... & -0.1031... \\ -3.3519... & -0.0143... & 0.1403... \end{pmatrix}$

Question	Scheme	Marks	AOs
3(a)	Correct overall strategy employed, eg. $A=2\times\left(\frac{1}{2}\int_{2\pi}^{3\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta-\frac{1}{2}\int_{4\pi}^{5\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta+\frac{1}{2}\int_{0}^{\pi}\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta\right)$	M1	3.1a
	Evidence of use of $\frac{1}{2}\int\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta$	B1	1.1a
	$\int\sin^2\left(\frac{\theta}{6}\right)\mathrm{d}\theta=\int\frac{1}{2}\left(1-\cos\left(\frac{\theta}{3}\right)\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\left(\theta-3\sin\left(\frac{\theta}{3}\right)\right)$	A1	1.1b
	$A=\left(2\times\frac{1}{2}\right)\times\frac{1}{2}\left[\left((3\pi-0)-\left(2\pi-\frac{3\sqrt{3}}{2}\right)\right)-\left(\left(5\pi+\frac{3\sqrt{3}}{2}\right)-\left(4\pi+\frac{3\sqrt{3}}{2}\right)\right)+\left(\left(\pi-\frac{3\sqrt{3}}{2}\right)-(0)\right)\right]$	M1	2.1
	$=\frac{\pi}{2}$	A1	1.1b
		(6)	
(b)	Area of painting on wall = (area curve)×(12/(width of curve))^2 with their area and width.	M1	3.1a
	Width of curve $\left(=\sin\left(\frac{3\pi}{6}\right)+\sin\left(\frac{2\pi}{6}\right)\right)=1+\frac{\sqrt{3}}{2}=1.866...$	B1	1.1b
	So as two coats needed, total area of paint required $=2\times\frac{\pi}{2}\times41.354$ =129.92… m^2	M1	2.2a
	So 5 tins of paint will be needed.	A1	3.2a
		(4)	
			(10 marks)

Notes:

(a)

M1: A correct attempt to find the correct area, splitting into suitable required sections and attempting the integration, combining correctly. There are many variations that could be used so check carefully the strategy is correct.

B1: Applies the area formula to the curve with any limits. The ½ may be implied if a correct overall formula is used.

M1: Applies the double angle formula to set up the integral

A1: Correct integration of $\sin^2(\theta/6)$

M1: Applies limits correctly to all of their integrals and combines and simplifies.

A1: Correct area from a fully correct method.

(b)

M1: Full attempt to find the appropriate scaling factor (must be squaring) to scale the area found in (a) to the area required for the wall painting.

B1: Correct width of logo in the curve found.

M1: Area of paint required for two coats found.

A1: Correct number of tins identified. Must be an integer answer.

Question	Scheme	Marks	AOs
4	$\int_0^3 \frac{kx}{x^2+6}\,dx + \int_3^\infty \frac{k}{x^2-4}\,dx = \frac{1}{4} \Rightarrow k\left([\ldots]_0^3 + [\ldots]_3^\infty\right) = \frac{1}{4} \Rightarrow k = \ldots$	M1	3.1a
	$(k)\int_0^3 \frac{x}{x^2+6}\,dx = (k)\left[\frac{1}{2}\ln\left(x^2+6\right)\right]_0^3 = (k)\left(\frac{1}{2}\ln(15) - \frac{1}{2}\ln(6)\right)$	M1 A1	1.1b 1.1b
	$(k)\int \frac{1}{x^2-4}\,dx = (k)\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)$	M1	1.1b
	$\int_3^\infty \frac{1}{x^2-4}\,dx = \lim_{t\to\infty}\left[\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)\right]_3^t$ $= \frac{1}{4}\left(\lim_{t\to\infty}\ln\left(\frac{t-2}{t+2}\right) - \ln\left(\frac{1}{5}\right)\right) = \frac{1}{4}\ln 5$	M1 A1	3.1a 1.1b
	So $\frac{k}{2}\ln\left(\frac{15}{6}\right) + \frac{k}{4}\ln 5 = \frac{1}{4} \Rightarrow k\ln\left(\frac{5}{2}\right)^2 + k\ln(5) = 1$ so $k = \frac{1}{\ln\left(\frac{125}{4}\right)}$	M1 A1	2.1 1.1b
		(8)	
			(8 marks)

Notes:

M1: For a complete overall method. Correct expression for the total area formed, a sum of two areas with an attempt made for both areas in terms of *k* and with area set equal to $\frac{1}{4}$.

M1: Correct form for integral of left hand part (with or without *k*) and attempts to apply the limits. Accept $a\ln\left(x^2+6\right)$.

A1: Integral correct, with limits applied correctly. No need to combine lns at this stage.

M1: Correct form integral for the right hand part (with or without *k*), quoted directly or uses partial fractions. Accept $A\ln\left(\frac{x-2}{x+2}\right)$ or $A\ln(x-2) - B\ln(x+2)$ from an attempt at partial fractions.

M1: Applies the limits on the improper integral, 3 as lower limit and *t* as upper with $t \to \infty$ to obtain a value for the integral. If partial fractions used, the log terms will need combining first.

A1: Fully correct integral for right hand section, as in scheme or equivalent, with or without *k*.

M1: Adds the results of both the integrals and equates to $\frac{1}{4}$, including *k*, and uses correct log laws to combine their log terms to find *k*.

A1: Correct answer. Accept any equivalents in the correct form, e.g. $\frac{1}{\ln(31.25)}$ or statement that $a = \frac{125}{4}$ etc.

Question	Scheme	Marks	AOs
5(a)	$\overrightarrow{OC}=0.8\mathbf{k}$, $\overrightarrow{OB}=3\mathbf{i}+0.8\mathbf{k}$ and $\overrightarrow{OD}=1.2\mathbf{j}+1.5\mathbf{k}$, or $\overrightarrow{CB}=3\mathbf{i}$, and $\overrightarrow{CD}=1.2\mathbf{j}+0.7\mathbf{k}$	B1	3.3
	So plane has equation $\mathbf{r}$ = their $\overrightarrow{OC}$ + their $\lambda\overrightarrow{CB}$ + their $\mu\overrightarrow{CD}$ (oe) OR $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}).(3\mathbf{i})=0$ and $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}).(1.2\mathbf{j}+0.7\mathbf{k})=0$ leading to a =..., b =... and c =... (may use vector product)	M1	1.1b
	Equation is $\mathbf{r}=0.8\mathbf{k}+\lambda(3\mathbf{i})+\mu(1.2\mathbf{j}+0.7\mathbf{k})$ OR normal is $\mathbf{n}=p(7\mathbf{j}-12\mathbf{k})$	A1	1.1b
	$x=3\lambda$, $y=1.2\mu$ and $z=0.8+0.7\mu \Rightarrow 70y-120z=-96$ OR $(0.8\mathbf{k}).(7\mathbf{j}-12\mathbf{k})=-9.6 \Rightarrow d=-9.6$	M1	1.1b
	Equation is $\mathbf{r}.(7\mathbf{j}-12\mathbf{k})=-9.6$ (or a multiple e.g. $\mathbf{r}.(70\mathbf{j}-120\mathbf{k})=-96$)	A1	2.5
		(5)	
(b)	Full attempt to find the minimum distance from the centre of the base rectangle to the plane – e.g. using the distance formula for closest point, or first finding the intersection point then finding the distance. Must have correct starting point (1.5,0.6,0).	M1	3.1b
	E.g. Minimum distance $=\dfrac{\lvert 0\times1.5+7\times0.6+(-12)\times0+9.6\rvert}{\sqrt{0^2+7^2+(-12)^2}}=\ldots$	M1	3.4
	= 0.993 m or 99.3 cm or 993 mm (to 3 s.f.) Accept awrt.	A1	1.1b
		(3)	
(c)	E.g. the boards will not have negligible thickness, which should be taken into account in the model, or wooden boards will bow and so not form planes.	B1	3.5b
		(1)	
			(9 marks)

Notes:

(a) Accept use of column vectors throughout.

B1: Identifies three points on or two vectors in the plane that can be used to set up the model.

M1: Attempts a plane equation with their vectors OR attempts to find a normal vector using scalar (or cross) product.

A1: Correct plane equation OR correct normal vector (any multiple).

M1: Solves $x=3\lambda$, $y=1.2\mu$ and $z=0.8+0.7\mu$ to find equation x, y and z. OR Applies $\mathbf{r}.\mathbf{n}=d$ with a point on the line and their $\mathbf{n}$ to find d.

A1: Correct equation of plane in the correct form $\mathbf{r}.\mathbf{n}=d$, as shown or a multiple thereof.

(b)

M1: See scheme. Alternative methods can be used (e.g find p required for $\mathbf{r}=1.5\mathbf{i}+0.6\mathbf{j}+p(7\mathbf{j}-12\mathbf{k})$ to intersect the plane).

M1: Uses the model to attempt the minimum distance from any point to the plane, or an attempt to find the value of p for the point of intersection for the minimum distance.

A1: Correct answer awrt 993 mm or equivalent in m or cm.

(c) B1: Any reasonable limitation about the boards - e.g. those in the scheme.

5(a) Alt	Sets up equation of plane as $ax+by+c=d$	B1	3.3
	Identifies at least three points on the plane and substitutes in to the equation to form simultaneous equations. E.g. (3,0,0.8), (0,0,0.8), (0,1.2,1.5) and (3,1.2,1.5) give $3a+0.8c=d$ $0.8c=d$ $1.2b+1.5c=d$ $3a+1.2b+1.5c=d$ Note may use d = 1 with only 3 equations.	M1	1.1b
	Solves to find correct corresponding values. E.g. With d = 1, c = 1.25, a = 0 and b = $-\frac{35}{48}$ (so accept any appropriate multiples)	A1	1.1b
	Forms plane equation in correct form with their values. E.g. $-\frac{35}{48}y+\frac{5}{4}z=1(\Rightarrow 35y-60z=-48)\Rightarrow \mathbf{r}\cdot\mathbf{n}=d$	M1	1.1b
	Equation is $\mathbf{r}\cdot(35\mathbf{i}-60\mathbf{j})=-48$ (or any multiple)	A1	2.5
		(5)	

(a) Alt

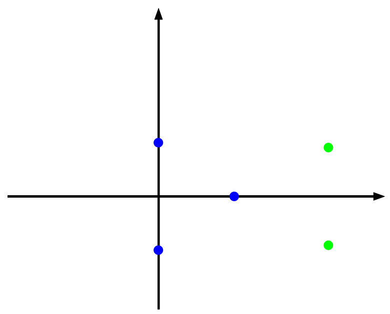
B1: Sets up appropriate Cartesian plane equation for the model.

M1: Identifies at least three points on the plane and forms simultaneous equations using them in the general equation.

A1: Solves the equations to find correct values for the coefficients (may be a common multiple of the ones shown).

M1: Uses their coefficients in their Cartesian equation to form an equation for the plane in the correct form.

A1: Correct equation of plane in the correct form **r**.**n** = d, as shown or a multiple thereof.

Question	Scheme	Marks	AOs
6(a)	Attempts sum of roots of f (= −3/k) and product of roots of g (= 9/m) and uses them to form a relationship between k and m.	M1	3.1a
	So $-3/k = 9/m$	A1	1.1b
	Sum of roots of g is $2/m \Rightarrow 2/m$ is a root of g as the other roots have no real part. OR root on imaginary axis has form $\alpha\mathrm{i}$, and substituting in g and equating real and imaginary terms gives $2\alpha^2 - 9 = 0$ & $3\alpha - m\alpha^3 = 0$	B1	3.1a
	$\mathrm{g}(2/m) = 0 \Rightarrow m(2/m)^3 - 2(2/m)^2 + 3(2/m) - 9 = 0 \Rightarrow m = \ldots \quad (m = 2/3)$ OR $\alpha^2 = \frac{9}{2} \neq 0 \Rightarrow m = \frac{3}{\alpha^2} = \ldots \left(= \frac{2}{3}\right)$	M1	1.1a
	So $\mathrm{g}(x) = 0 \Rightarrow \left(\frac{2}{3}(x-3)\left(x^2 + \frac{9}{2}\right) = 0 \Rightarrow\right) \; x = 3, \pm\frac{3\sqrt{2}}{2}\mathrm{i}$	M1	1.1b
	$k = -2/9$, $\mathrm{f}(x) = 0 \Rightarrow x = \dfrac{-3 \pm \sqrt{3^2 - 4(-2/9)(-11)}}{2(-2/9)} = \ldots$	M1	2.2a
	$x = 3, \pm\frac{3\sqrt{2}}{2}\mathrm{i}, \frac{27}{4} \pm \frac{3\sqrt{7}}{4}\mathrm{i}$	A1	1.1b
		(7)	
(b)	 Correct roots for f plotted (shown green). Correct roots for g plotted (shown blue).	B1ft B1ft	1.1b 1.1b
		(2)	
			(9 marks)

Notes:

(a)

M1: Identifies sum of roots of f or product of roots of g correctly

A1: Correct equation between k and m

B1: Realises that g having roots on the imaginary axis means the sum of roots is equal to the only real root of the equation or forms correct simultaneous equations after substituting $\alpha\mathrm{i}$ into g

M1: Uses factor theorem with their real root to find m or solves their equations to find m

M1: Uses their m to solve g(x). May just see answers from calculator, or can factorise or complete the square.

M1: Deduces the correct value for k and solves f(x) using it. May just see answers from calculator, or can factorise or complete the square.

A1: All five roots correct – may not all be listed in one line, as long as the roots of g and f are clear. Accept exact equivalents.
(b) **B1ft:** Correct roots for f plotted, follow through as long as they are complex. If answers to (b) are correct these should be further from the imaginary axis than the real root of g. **B1ft:** Correct roots for g plotted, follow through their roots as long as two are on the imaginary axis.

Question	Scheme	Marks	AOs
7(i)	When $n = 1$, LHS $= \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$, RHS $= \begin{pmatrix} 3\times1+1 & -1 \\ 9\times1 & 1-3\times1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$. So the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$, so $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^k = \begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$	M1	2.4
	Then $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^{k+1} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$ or $\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 4(3k+1)-9k & -4k-(1-3k) \\ 9(3k+1)-18k & -9k-2(1-3k) \end{pmatrix}$ or $\begin{pmatrix} 4(3k+1)-9k & -(3k+1)+2k \\ 36k+9(1-3k) & -9k-2(1-3k) \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3(k+1)+1 & -(k+1) \\ 9(k+1) & 1-3(k+1) \end{pmatrix}$	A1	1.1b
	Hence the result is true for $n = k+1$. Since it is <u>true for $n = 1$</u>, and <u>if true for $k = n$ then true for $n = k+1$</u>, thus by mathematical induction the <u>result holds for all n</u> ∈ •	A1 cso	2.4
		(6)	
(ii)	(a) $2^2 = 4 \nless 4 = 2^2$ OR $3^2 = 9 \nless 8 = 2^3$ OR $4^2 = 16 \nless 16 = 2^4$	B1	1.1b
	(b) The statement $2k+1 < k^2$ is not true for all positive integers.	B1	1.1b
	(c) The statement in line 4 is true for positive integers $k > 2$ so the induction hypothesis is true for $n > 2$. So the induction holds from any base case greater than 2.	M1	2.3
	Since the result is true for $n = 5$ as $5^2 = 25 < 32 = 2^5$ and $2k+1 < k^2$ also true for $k > 5$ so the induction holds with base case $n = 5$.	A1	2.4
	But not true for $n = 2$, 3 or 4 as $2^2 = 4 \nless 4 = 2^2$ and $3^2 = 9 \nless 8 = 2^3$ and $4^2 = 16 \nless 16 = 2^4$. Hence true for $n = 1$ and for n Ö 5	A1	2.1
		(5)	
		(11 marks)	

Notes:

(a)

B1: Shows the general form holds for $n = 1$.

M1: Makes the inductive assumption, assume true for $n = k$.

M1: Attempts the multiplication either way.

A1: Correct matrix in terms of k.

A1: Rearranged into correct form to show true for $k + 1$.

A1: Completes the inductive argument conveying **all** three underlined points or equivalent at some point in their argument.

(b)(i)

B1: Provides a suitable counter example using $n = 2$, 3 or 4. Accept = in place of $\nless$ as long as there is a suitable conclusion with it.

(b)(ii)

B1: Identifies the error as in the scheme or equivalent (e.g. $k^2 + 2k + 1 < 2k^2$ is not always true).

(b)(iii)

M1: Identifies that the induction is valid as long as $2k+1 < k^2$ is true which happens for kÖ3 (accept any value greater than 3 for this mark).

A1: Correct base case of 5 and explains the proof given holds for integers greater than or equal to 5.

A1: Complete argument correct. All positive integers satisfying the inequality identified, with demonstration that 2, 3 and 4 do not.

Question	Scheme	Marks	AOs
8(a)(i)	Container contains 3+0.25t −0.125t = 3 + 0.125t litres after t minutes	B1	3.3
(ii)	Rate of contaminant out $=0.125\times\dfrac{m}{3+0.125t}$ mg per minute	M1	3.3
	Rate of contaminant in = 0.25 × $(5-e^{-0.1t})$ mg per minute	B1	2.2a
	$\dfrac{dm}{dt}=\dfrac{5-e^{-0.1t}}{4}-\dfrac{m}{24+t}$ *	A1*	1.1b
		(4)	
(b)	Rearranges to form $\dfrac{dm}{dt}+\dfrac{m}{24+t}=\dfrac{5-e^{-0.1t}}{4}$ and attempts integrating factor (may be by recognition).	M1	3.1a
	I.F. = $\left(e^{\int\frac{1}{24+t}\,dt}=e^{\ln(24+t)}\right)=24+t$	A1	1.1b
	$(24+t)m=\dfrac{1}{4}\int(24+t)(5-e^{-0.1t})\,dt=\dfrac{1}{4}\int 120+5t-24e^{-0.1t}-te^{-0.1t}\,dt=..$	M1	3.1a
	$=\dfrac{1}{4}\left(120t+\dfrac{5t^2}{2}-\dfrac{24e^{-0.1t}}{-0.1}+\dots\right)$	A1	1.1b
	$\int te^{-0.1t}\,dt=t\dfrac{e^{-0.1t}}{-0.1}-\int 1\times\dfrac{e^{-0.1t}}{-0.1}\,dt=t\dfrac{e^{-0.1t}}{-0.1}-\dfrac{e^{-0.1t}}{(-0.1)^2}$	M1 A1	1.1b 1.1b
	So $(24+t)m=\dfrac{5}{8}t^2+30t+85e^{-0.1t}+\dfrac{5}{2}te^{-0.1t}+c$		
	When t = 0, m = 0 as initially no contaminant in the container, so 0 = 0 + 0 + 85 + 0 + c ⇒ c = −85	M1	3.4
	$m=\dfrac{1}{24+t}\left(\dfrac{5}{8}t^2+30t+85e^{-0.1t}+\dfrac{5}{2}te^{-0.1t}-85\right)$	A1	2.2b
		(8)	
(c)	When t = 30 m = 25.65677... and V = 6.75, hence the concentration is 3.80 mg per litre.	M1	3.4
	This resembles the measured value very closely and could easily be explained by minor inaccuracies in measurements, so the model seems to be suitable over this timeframe.	A1	3.5a
		(2)	
			(14 marks)

Notes:

(a)(i)

B1: A correct expression for the volume, may be unsimplified.

(ii)

M1: Expresses the amount of contaminant out in terms of m and t.
B1: Correct interpretation for amount of contaminant entering the container.
A1*: Puts all the components together to form the correct differential equation.

(b)

M1: Identifies the problem as a first order linear problem requiring integrating factor (by finding it or by recognition.

A1: Correct integrating factor

M1: Multiplies through by the IF, expands brackets on RHS and attempts the integration.

A1: Correct integration for first three terms.

M1: Integration by parts used on the $te^{-0.1t}$ term.

A1: Correct integration by parts.

M1: Uses the initial conditions to find the constant of integration – must have a constant of integration for this mark to be awarded.

A1: Correct expression for m, need not be simplified.

(c)

M1: Calculates the concentration from the model at $t = 30$

A1: Correct concentration found and uses it to make a comment on the validity of the model.