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Surname

Other names

Pearson Edexcel
Level 3 GCE

Centre Number

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Candidate Number

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Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference

9FM0/02

You must have:

Mathematical Formulae and Statistical Tables, calculator

Total Marks

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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. The roots of the equation

$$x^3 - 8x^2 + 28x - 32 = 0$$

are α , β and γ

Without solving the equation, find the value of

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(ii) $(\alpha + 2)(\beta + 2)(\gamma + 2)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(8)

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Question 1 continued

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(Total for Question 1 is 8 marks)

2. The plane Π_1 has vector equation

$$\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1 (3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2 (2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree. (3)

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Question 2 continued

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(Total for Question 2 is 8 marks)

3. (i)

$$\mathbf{M} = \begin{pmatrix} 2 & a & 4 \\ 1 & -1 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

where a is a constant.

(a) For which values of a does the matrix \mathbf{M} have an inverse?

(2)

Given that \mathbf{M} is non-singular,

(b) find \mathbf{M}^{-1} in terms of a

(4)

(ii) Prove by induction that for all positive integers n ,

$$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$$

(6)

Question 3 continued

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(Total for Question 3 is 12 marks)

4. A complex number z has modulus 1 and argument θ .

(a) Show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta, \quad n \in \mathbb{Z}^+$$

(2)

(b) Hence, show that

$$\cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$$

(5)

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Question 4 continued

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(Total for Question 4 is 7 marks)

5.

$$y = \sin x \sinh x$$

- (a) Show that $\frac{d^4 y}{dx^4} = -4y$ (4)
- (b) Hence find the first three non-zero terms of the Maclaurin series for y , giving each coefficient in its simplest form. (4)
- (c) Find an expression for the n th non-zero term of the Maclaurin series for y . (2)

Question 5 continued

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(Total for Question 5 is 10 marks)

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6. (a) (i) Show on an Argand diagram the locus of points given by the values of z satisfying

$$|z - 4 - 3\mathbf{i}| = 5$$

Taking the initial line as the positive real axis with the pole at the origin and given that

$$\theta \in [\alpha, \alpha + \pi], \text{ where } \alpha = -\arctan\left(\frac{4}{3}\right),$$

- (ii) show that this locus of points can be represented by the polar curve with equation

$$r = 8 \cos \theta + 6 \sin \theta \tag{6}$$

The set of points A is defined by

$$A = \left\{ z : 0 \leq \arg z \leq \frac{\pi}{3} \right\} \cap \left\{ z : |z - 4 - 3\mathbf{i}| \leq 5 \right\}$$

- (b) (i) Show, by shading on your Argand diagram, the set of points A .

- (ii) Find the **exact** area of the region defined by A , giving your answer in simplest form.

(7)

7. At the start of the year 2000, a survey began of the number of foxes and rabbits on an island.

At time t years after the survey began, the number of foxes, f , and the number of rabbits, r , on the island are modelled by the differential equations

$$\frac{df}{dt} = 0.2f + 0.1r$$

$$\frac{dr}{dt} = -0.2f + 0.4r$$

- (a) Show that $\frac{d^2f}{dt^2} - 0.6\frac{df}{dt} + 0.1f = 0$ (3)
- (b) Find a general solution for the number of foxes on the island at time t years. (4)
- (c) Hence find a general solution for the number of rabbits on the island at time t years. (3)

At the start of the year 2000 there were 6 foxes and 20 rabbits on the island.

- (d) (i) According to this model, in which year are the rabbits predicted to die out?
- (ii) According to this model, how many foxes will be on the island when the rabbits die out?
- (iii) Use your answers to parts (i) and (ii) to comment on the model. (7)

Question 7 continued

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Question 7 continued

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(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS