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Edexcel

Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel GCE
In Further Mathematics
Paper 3A Further Pure Mathematics 1

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Question 1

A good question 1, with virtually all the candidate scored full marks on this question, which given all the formulae are in the formula book. The only slips were in part (b) were having extra equations or just writing $= \pm 9$ when the equations of the directrices are $x = \pm 9$.

Question 2

Part (a) was done well by the majority of the candidates. It was good to see that candidates could correctly recall the t formula. They substituted correctly into the left-hand side of the identity and dealt correctly with the fractions. It was pleasing to see that brackets were used especially with $\cos x$. Once the left-hand side was simplified the majority of the candidates worked on the right-hand side to achieve the same simplified expression. Again, it was good to see that the candidates did then draw the correct conclusion to complete the proof.

Part (b) caused more problems for candidates. A few candidates did not recall how to deal with $d\theta$, they replaced $\cos \theta$ with $\frac{1-t^2}{1+t^2}$ but struggled with remembering that $d\theta = \frac{2}{1+t^2} dt$. Once candidates used the correct substitutions, they successfully simplified and recognised that it integrated to the form $\frac{1}{a} \arctan\left(\frac{t}{a}\right)$ which can be found in the formula book. After integrating candidates used the correct limits, most used $t = 1$ and $t = 0$, to achieve the correct exact answer.

Question 3

Candidates needed to consider the two situations when

- $x \geq 0$ solving $2x - 5 > \frac{x}{x-2}$
- $x < 0$ solving $2x - 5 > \frac{x}{-x-2}$

Many candidates multiplying both sides by $(x-2)^2$ considering the first situation, when just multiplying by $(x-2)$ would have sufficed. Candidates found the critical values and then found a required region.

Quite a few candidates did not consider the second situation but managed to score 4 out of the 8 marks.

Candidates who did consider the second situation again multiplying both sides by $(-x-2)^2$ when just multiplying by $(-x-2)$ would have sufficed.

The critical value $\sqrt{5}$ needed to be rejected.

Candidates who found the correct critical values were able to successfully determine the correct regions.

Question 4

Part (a) The majority of candidates were able to identify the normal vector of the plane.

Part (b) The majority of candidates were able to find the cross product

between \mathbf{n} and \mathbf{v}_A and showed that it is 4 times $\begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$.

Part (c) Candidates found this part of the question more difficult. They needed to realise that the landing direction is perpendicular to $\mathbf{n} \times \mathbf{v}_A$ and \mathbf{n} , so needed to find the cross product between their answer to part (b) and

their answer to part (a) so $\begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 25 \end{pmatrix}$. Candidates needed to make sure that

they found the cross product in the correct order to give the correct direction of travel of the plane after landing.

(d) Candidates were able to have a go at writing down a limitation of the model, with the common correct answers were that the plane will not travel in a straight line and the field will not be flat.

Question 5

Part (a) The majority of candidates were able to write down the equations of the asymptotes of the hyperbola, the formula can be found in the formula book.

Part (b) Candidates recognised that they needed to differentiate the parabola and sets equal to the perpendicular gradient of asymptotes in part (a). Most candidates differentiated using implicit differentiation and found the values of y . Once the values of y were found candidates correctly found the corresponding x coordinate and equation of the normal.

The common errors were not reading that the normal to C was required, some candidates set the gradient of the parabola equal to the gradient in part (a) leading to the equation of the tangent to C which are parallel to the asymptotes of H .

Part (c) Candidates who were able to finding equations in part (a) knew that they need to find the points of intersection between their answers to part (b) and the hyperbola. On occasion candidates found the point of intersection between their two answers in part (b) for found the point of intersection with the parabola. Once the coordinates of P and Q were found candidates were able to find the area of the triangle OPQ . The determinate method was often seen.

Question 6

Part (a) The majority of candidates had excellent differentiation skills and were able to find correctly that $\frac{dy}{dx} = \frac{2(1+\ln x)}{x}$ using the product rule and

using either the quotient or product rule to show correctly that $\frac{d^2y}{dx^2} = -\frac{2\ln x}{x^2}$ as required.

Part (b) Again excellent differentiating skills were used to find the third derivative using again the quotient rule or product rule.

Part (c) Candidates knew that they had to evaluate $y(1)$, $y'(1)$, $y''(1)$, and $y'''(1)$ and then applied the Taylors series stated in the question and simplified their answer. A few candidates after finding the series when on and expanded the brackets. The question is asking for candidates to find the Taylor series in ascending powers of $(x-1)$, so they need to leave their answer in this form. This hindered them in answering part (d).

Part (d) Candidates found this part more demanding. Candidates need to use their series found in part (c) and substitute it into the limit in place of $(1+\ln x)^2$. The expression needed to be simplified and candidates needed to leave it as powers of $(x-1)$ so that they could recognising cancelling of

terms leading to a constant + ... $\frac{\frac{1}{3}(x-1)^3 + \dots}{(x-1)^3} = \frac{1}{3}$. The final mark required

candidates to appreciate that limit would be of the form

$\frac{1}{3} + (x-1) + (x-1)^2 + \dots$ so that as x tend to 1 the limit will tend towards $\frac{1}{3}$.

Question 7

Many candidates only managed to score the first mark for find the parametric equation of the straight line. Some candidate recalled that

$\cos \theta = \frac{x}{|a|}$ but used $\cos \theta = \frac{12+9\lambda}{\sqrt{9^2+6^2+2^2}}$ instead of

$\frac{12+9\lambda}{\sqrt{(12+9\lambda)^2 + (16+6\lambda)^2 + (-8+2\lambda)^2}}$. The few candidates who did use the

correct formula solved to find two values of λ leading to two coordinates.

As the direction of cosine was positive the value of $\lambda = -\frac{1}{2}$ was required so

that $12+9\lambda > 0$ leading to coordinates of A are $\left(\frac{15}{2}, 13, -9\right)$ only

Question 8

Many candidates demonstrated very good differentiate equation skills.

Part (a) Done well by the majority of candidates. Recognises that $h = 0.25$ as time was in days, finding the value of the first derivative when $t = 0$ and applying the iterative formula stated in the question. The most common error was in using $h = 6$, thinking that time was in hours, careful reading of the question was required.

Part (b) Candidates were well practiced in transforming differential equations and showed the result successfully. Starting with $\frac{du}{dt} = 3x^2 \times \frac{dx}{dt}$

and using the substitution to replace all the x 's with u 's.

Part (c) Candidates recognised that that needed to find the integrating factor and did so successfully. They multiplied thorough and recognised that they had the derivative of $u \cosh t$ on the left-hand side leading to

$u \cosh t = \int \cosh t + 3 dt$. Candidates correctly integrated and remembered the constant of integration.

Part (d) Candidates who has be successful with part (d) used the initial conditions to find the constant of integration. The substitution was reversed and the particular solution for x was found. Occasionally candidates left their answer as $x^3 = \tanh t + \frac{3t + 27}{\cosh t}$ the question asked for an equation for the concentration of pollutant.

Part (e) candidates who managed to achieve an answer for part (d) found the concentration of pollutant when $t = 0.25$, again a common error was using $t = 6$.

The percentage error was found and it was good to see that the model answer was used as the denominator not the answer to part (a) which was the approximation. Sometimes that final mark was lost due to premature rounding or failing to state that it is an overestimate.

Overall candidates appeared to be well prepared for this exam and were able to have good attempt at the majority of the questions.

