Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 4A Further Pure Mathematics 2

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Question 1

(a) The majority of the candidates knew how to find the characteristic equation; however, a small number did not set = 0 losing a mark.

(b) Many candidates scored the first mark for using the Cayley-Hamilton theorem to write an equation for $\mathbf{A}^3$. A few candidates did not know where to go from there; there were two main options, either replacing $\mathbf{A}^2$ with $7\mathbf{A}+(8+3a)\mathbf{I}$ or substituting in for the matrix **A.**

Equation coefficient of **A** and **I** or using the diagonal matrix to find a value for a.

Candidates who did not use the Cayley-Hamilton theorem and just substituted in for the matrix **A** scored no marks.

Question 2

(a) The easiest approach was to substitute $z = 0$, $z = -4$ and $z = -8\text{i}$ into both sides of the given locus to show it holds. Quite a few candidates made this more complicated by trying to find the Cartesian equation of the locus first and then substitute in.

A few missed showing it holds for the origin and a misread using $z = -4 - 8\text{i}$, scored one mark.

(b) The majority of candidate scored 1 out of 2 marks. They drew a circle that passed through the origin but did not indicate where it crossed the axis. Candidates are reminded to always indicate intercepts with axes to show location.

(c) Candidates appeared to know that $|z| = 4$ represented a circle, centre (0, 0) but errors included radius of 2 and again not indicated where it crossed an axis.

Candidates were still able to score the final mark if they had two overlapping circles and shaded the overlap. Many candidates were able to score this mark.

Question 3

(a) The majority of candidates were able to score 1 out of 2 marks for explaining 2 out of the 3 required terms. Many candidates forgot to explain where $U_1 = 25$ came from or to draw together an overall conclusion.

(b) This was done well by the majority of candidates who demonstrated that they knew how to prove recurrence relations using induction.

The conclusion was done well by most of the candidates remembering to imply if true of $n = k$ then true for $n = k + 1$.

(c) Candidates needed to know how many weeks there were in half a year, and the mark scheme allowed 24, 25, 26 and 27 weeks. Candidates were then able to draw the correct conclusion that it was not a good model.

Question 4

(a) This was done well by virtually all candidates; the only error was not stating the conclusion that $\text{gcd}(168, 66) = 6$.

(b) Again, this was done very well, using their answer to part (a) to find 6 as multiplies of 66 and 168. Many gained full marks.

(c) This part was less well done. Candidates needed to state/show that 10 is not a multiple of 6. A common error was that candidates misread as 6 is not a multiple of 10.

(d) This part was again done well with many candidates starting again and repeating parts (a) and (b) with 28 and 11. Very few realised that they could divide the answer to (b) by 6. They then divided by the multiplicative inverse of 11 and achieved the correct modulus solution.

Another approach was to find 8 as a multiple of 11 and 28 and multiply through by the multiplicative invers of 11.

A few candidates found that $v = 16$ was a solution but did not write as a modulus solution.

Question 5

(i) (a) This part was not done well by many candidates; some common errors were $5^4 + 4\times3\times2$ or $5^4\times4^3$.

(i) (b) Candidates struggled with this question and many left it blank. Candidates need to recognise that there were 5 possible positions for the letters and only 3 selected so $(a)\times{}^5C_3$ and then subtract (a).

(ii) This part was done very well by the candidates. In (a) they were able to select the correct odd digits 1, 3, 5 and 7 using the sum of the digits is a multiple of 9. In (b) again candidates were able to deduce that the correct code was 1357. A few candidates misread mod *ab* as multiplying the values of *a* and *b*.

Question 6

The majority of candidates knew how to find the closed form for a recurrence relation. They were able to form and solve the auxiliary equation. Then wrote down the correct general solution either the Cartesian or polar form. There were some errors with the particular solution, candidates need to use the form $u_n = \alpha n + \beta$, some incorrectly used $u_n = \alpha n$.

Candidates then used $u_0 = 1$ and $u_1 = 4$ to find the values of the constants. There were some numerical slips but the correct method was used.

Question 7

(a) The majority of candidates knew how to find identity element. Some used trial and error and showed that the identity element was – 1 which was fine.

(b) There was less success with this part, setting the operation = –1 and rearranging to find the inverse. A common error was thinking that $xx^{-1} = -1$ when substitutes into the operation, this lost 2 marks.

(c) The least well-done part. Many candidates stated that the inverse is undefined but did not say why. Candidates are reminded to give clear reasoning for their answers. Some candidates showed that $x \bullet -\frac{3}{2} = -\frac{3}{2}$ but concluded therefore it has no inverse instead of all elements are its inverse.

Question 8

(a) The majority of candidates correctly used integration by parts to show the recurrence integration formula. The main error was seen with finding the value of a incorrectly thinking that $-\frac{1}{4}(-2)^{n} = (-2)^{n-2}$ instead of $-(-2)^{n-2}$. Candidates are reminded to look at the form that they are asked to show.

(b) Candidates who were successful with achieving an answer to part (a) were able to use it to find I_2. Due to errors with the value of a many lost the accuracy mark but gained the method marks. Candidates who did not use the answer to part (a) and went straight for integrating did not gain any marks.

Question 9

(a) There was a variety of methods seen to find the value of a, the easiest was that –1 is the middle of 1 and a. Some candidates used distances or form the equation of the circle and eventually found $a = -3$.

(b) This was not very well done, candidates appeared to know that it involves the locus $\arg\left(\frac{z-3-2\mathrm{i}}{z+3-4\mathrm{i}}\right) = \theta$ but struggled to find the angle. Drawing a diagram would have been helpful to candidates. Again, there were many methods to find the angle, using vectors, cross product or trigonometry.

Question 10

(a) Many candidates knew that they needed to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and substituted into the correct surface area formula. Candidates who incorrectly wrote $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(1+\frac{x^2}{9}\right)^{-\frac{1}{2}}$ missing the derivate of $\frac{x^2}{9}$ were unable to score many marks as they would not be able to achieve the required form.

Candidates with the correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ did manage to achieve the correct form but there was an error in taking out 9 which appeared as the numerator instead of the denominator.

The main error was not considering the area of the circles on the ends, some thought that q = 0 and many more just ignored it. Candidates are reminded to check the form of the answer so that they can see if they have missed anything.

(b) This part was less well done with many candidates who did not achieve an answer to (a) not attempting this part, however a few did correctly find $\frac{\mathrm{d}x}{\mathrm{d}u}$ which was good.

A few candidates after finding $\frac{\mathrm{d}x}{\mathrm{d}u}$ when substituting into the formula had it inverted, so they were unable to achieve the correct form of the integral. Candidates who were able to correct achieve $\int \cosh^2 u \, \mathrm{d}u$ used the correct identity to integrate it. The correct limits were then generally used, a few incorrectly used x limits. Not many candidates achieved the correct overall surface area due to earlier errors.

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