Pearson Edexcel

Examiners' Report

Principal Examiner Feedback

Summer 2023

Pearson Edexcel GCE

A Level Further Mathematics (9FM0)

Paper 3A Further Pure Mathematics 1

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Overview

The paper proved to be highly accessible, with the questions being well structured to give numerous access points or checking points for students, with many excellent responses seen. The final question, on vectors, was the only one that caused significant problems. The evidence suggests this was more due to students simply opting not to answer the question than issues of timing, as the majority of responses did offer attempts for the question.

There was a lot of good algebraic work on display by students, who seemed very comfortable with the majority of the topics on the specification.

Question 1

This proved to be an accessible start to the paper for the majority of students with many fully correct solutions seen. Most achieved either full marks or lost just one mark in total, either the level of accuracy in part (a) or the explanation of part (b) being the cause of the loss of a mark.

In part (a) it was rare to see incorrect applications of Simpson's Rule, most seeming well drilled in the process. Occasional incorrect, or omission of the h, did occur, as well as a few cases of the 2 and 4 times multiples being mixed up, or a repeated value or wrong number of ordinate, but none of these were common. An error seen more often was to have their calculator in degree mode when attempting to find the required five ordinates, thus producing an answer far away from the given exact answer. Another common error was to lose the accuracy mark because the answer was not given to the required three significant figures.

In order to access part (b) a correct attempt resulting in a value rounding to 3.87 needed to have been made in part (a). Those who had wildly incorrect answers from (a) should have realised such answers would not befit the method and check their work, so were not given credit.

In commenting about the accuracy of the estimate in part (b), a percentage error was the most popular item of evidence to use. A few did use one of the other viable comments allowed, such as reference to "correct to two significant figures", "correct to one decimal place" or "accurate as only out by 0.015". However, there were a few cases where just "accurate" or similar was stated, and these were not deemed sufficient for credit. Some tried to explain how it would be more accurate with shorter steps, without making a specific comment about their result, not appreciating the demand of the question.

Question 2

This question, though lengthy, was well structured to give plenty of entry points for students, with part (a) proving the most challenging part, but many able to pick up from part (b) onwards and achieve high scores.

Part (a) was a test of students' skills in applying the chain rule and product rule to form linking equations between derivatives and proved challenging for many in being able to correctly apply the chain rule when taking the second derivative in part (ii). Very often the only lost marks in the question came in part (a), most commonly the two marks of (ii).

There were some very convoluted answers to this part and it was difficult at times to distinguish t from h and occasionally x, as handwriting was not always clear. This was presumably due to the students themselves being confused as to which variables to be using in which places to make the proof work. However, most students were able apply a correct chain rule for the first derivative equation in part (i). A variety of approaches were seen, some starting with one of the forms of the chain rule before substituting for $\frac{\mathrm{d}t}{\mathrm{d}h}$ and rearranging, others opting to differentiate the given transformation with respect to h and use the chain rule. Mostly this was clear, though some who basically started with the given result with $t = \mathrm{e}^x$ substituted did not provide sufficient evidence for the proof.

In part (a)(ii) students who differentiated the $t\frac{\mathrm{d}h}{\mathrm{d}t}$ with respect to t were generally the most successful, provided they applied the chain rule to the other side, as the result came out directly. But only relatively few took this approach with attempts to differentiate with respect to x more prevalent, and often failing to use the chain rule and either grinding to a halt or forcing the proof. Students were not always clear what they were differentiating with respect to, often differentiating one side with respect to t and the other side with respect to x but realising they were doing so. There were also numerous attempts to differentiate with respect to h seen, from a rearranged, and false reciprocal laws on $\frac{\mathrm{d}^2t}{\mathrm{d}h^2}$ attempted. It was clear that this part was the least comfortable for students in this question.

However, as both results were given in part (a), part (b) was a very routine operation of substitution into the equation (I). This was correctly done by nearly all students, with only occasional slips noted, and very few non-attempts.

Part (c), solving of a second order differential equation, is a very expected topic, and so is well rehearsed and largely carried out correctly. Forming and solving the auxiliary equation was usually done well, although some students neglected to show full working, with some going directly to the complementary function - deducible from the given answer, and so lost marks. Others showed the auxiliary equation first before stating the complementary function, and gained credit, while some, after solving the auxiliary equation, went directly to a complementary function in terms of t (either $At + Bt^2$ or sometimes $A\mathrm{e}^t + B\mathrm{e}^{2t}$ was seen), losing accuracy marks. Aside from these, errors of an incorrect form of the complementary function were very rare.

Likewise, where students correctly worked in terms of x to start with, errors in the form of the particular integral were uncommon, this particular case being a very standard one. Where errors did occur, the most likely incorrect forms were either $\mathrm{e}^{\lambda t}$ or just e^{3t}, the latter unable to access further marks. Most were able to carry out the procedure of differentiating twice and substituting to find any constants in their particular integral and achieve the method, and

there were very many fully correct attempts. Those who worked in x throughout would then successfully convert into an equation in t, though there was often some confusion in the naming of the constants if their A and B were the wrong way round in the original complementary function. Some explained well that they could simply be renamed as they are arbitrary constants, but it was more common to see "correction" backtracked. Those who simply left them the other way around were awarded full credit.
There were numerous attempts to work in t throughout, and form a particular integral λt^2, clearly with the aim of the given answer in sight. These often went wrong as students would generally substitute into the transformed equation, for which the particular integral was incorrect, and difficulties would arise when trying to equate to e^{3x}, leading to forcing of the result. Also, this belied a lack of realisation a full quadratic should really be used for this type of particular integral, as terms may need to cancel. This was the case even among those who went back to equation (I) to substitute into. Such attempts at working directly with t did not satisfy the demand of the question, and were often poorly done. Students should heed words such as "hence", which not only specify particular methods but are also there as guidance to the correct route through a question.

Part (d) was the most successfully answered part, often being correct even when earlier work had gone wrong. The process of finding a particular solution was well demonstrated, especially since the general solution had been given, so students could simply pick up from part (d). The most common error in the process was to apply the second boundary condition at $t = 1$ instead of $t = 2$, losing the last three marks, but this was not very common. There was some poor arithmetic in evidence with otherwise excellent solutions for the rest of the question marred by an inability to substitute values in formulas accurately, or to solve incorrectly for the constants, while some who had their A and B the other way to that given in the question sometimes mixed up the order when substituting back in to the equation.

There were mistakes seen in evaluating the solution at $t = 5$ and the final mark was lost by those who omitted to give the necessary units of the answer, but again this was uncommon. It would be advisable for students to show their substitutions into equations to gain method marks, as there were cases where final values did not follow the students' equation and could not be awarded the method mark for substitution into the equation as it was not seen or implied.

Question 3

Another very well answered question with many scoring full marks. A score of 4/7 also proved a common scoring profile, which could arise in a couple of ways - either all four critical values correct, but incorrect form of solution set, or just two of the critical values from one equation, with −6, as another and a correct form for the solution set using this.

Most students recognized the need to multiply by $(x + 6)$, or its square, and solve the resulting equation, with the majority successfully finding the two values for x from this branch. Those who multiplied by $(x + 6)^2$ were sometimes then confused by the solution $x = -6$, not recognizing that that was the equation of the asymptote, but treating it as one of the critical values, and some would go on to use this as part of the solution set, while neglecting the other branch (see paragraph 1). Often an inequality would be given immediately following the solution, though usually it was also reiterated in a final solution.

Most students then identified the need to multiply by $-(x + 6)$ and those that used the quadratic formula successfully gained the correct values of x, though some only gave the decimal values. Attempts at multiplying by $(x - 6)$ and even $(6 - x)$, and sometimes all four cases, were also seen among some students, who tried desperately to come out with a solution, and often result in multiple answers making it difficult to decide what the students final answer was. Students should bring the working together at the end of such piecemeal attempts to make their final answer clear.

A few students attempted to square both sides and were less successful. Those that did manage to gain the correct quartic were usually unable to progress to the exact values required to gain full credit, though 5 marks were available for decimal solutions for the exact values.

Students found the creation of the solution set more problematic and were not always able to relate the four values of x to the diagram to identify form of the solution set and the value to reject. Those that did refer to the diagram or had drawn a number line to aid them were more successful in gaining full marks. Where all four critical values and −6 were all found, and the correct form of the solution set deduced, there was still some difficulty in selecting the correct values to use. Though the −6 was often rejected, so was the $-\frac{27}{5}$, with a common incorrect answer being $x < \frac{-7-\sqrt{57}}{2}, -6 < x < 2$.

Question 4

Once again an accessible question at all levels, and one which proved a good discriminator across grades.

Part (a) required students to find the eccentricity of the given ellipse. The formula required was well known, and in the formula booklet, and most used it successfully to give the correct answer. A common error seen here was to give $\pm\sqrt{7}/4$ as an answer rather than the required positive value, while a few also failed to simplify $\sqrt{16}$ to 4, and likewise only a small number used a and b in place of a^2 and b^2 in the formula.

Part (b) was also well done with most students able to find the gradient correctly, usually either by parametric or by implicit differentiation, and then find the normal gradient to substitute in to get the correct equation. Occasional slips did occur, but in many cases students were able to identify the error and backtrack to correct when their answer did not match the given one - a useful checking point for students.

It was in part (c) that difficulties began to arise, though most students found the equation for OQ correctly. Many made this more complicated than necessary, by adopting the form $y+3\sin\theta=-\dfrac{3\sin\theta}{4\cos\theta}\left(x-4\cos\theta\right)$ rather than appreciating the line went through the origin, so has intercept zero. Unfortunately, a number of such candidates made errors in simplifying, or when substituting into the equation of the normal, such as losing one of the 4's, to end up with incorrect coefficients in the coordinates, but of the correct form. These would lose both accuracy marks in (c) and the accuracy mark in (d), which required correct coordinates to have been found. It was rare, but not unknown, for only one coordinate to be correct.

Another common error was to lose the 'x' when substituting for the line OQ into the given equation of the normal equation to find the intersection points, resulting in incorrect coordinates. These students did not appear to realise that the values they were getting were too complicated to be correct as they would not give the ellipse required for part (d). Such students could not gain the method mark in part (d) from forms of the coordinates which were not for an ellipse.

Success in part (d) was dependent on whether they had successfully found intersection points in part (c) correctly, or at least of the correct form. As such many did not make much progress in this part, though those with correct form for the coordinates usually scored the method mark. Many students did not appreciate the necessity to give evidence that their point from part (c) did in fact lie on an ellipse rather than just finding the eccentricity, though many did first attempt to find the equation in Cartesian form using trigonometric identities, and so tacitly evidenced it was an ellipse, and could score the A. Familiarity with the standard parametric form from the formula booklet was not in evidence, and some thought they needed to show a is bigger than b before they could proceed. Finding the eccentricity from coordinates of the correct form was usually completed successfully, and in fact often did give the same value for e from incorrect coordinates due to "double errors" cancelling, meaning students were unaware of their error. However, the final mark was only permitted for a fully correct solution with at least some evidence of having an ellipse for the new locus, and so was not a difficult and discriminating, mark to achieve.

Question 5

Another accessible question, which was liked more than the preceding question, with full or high marks being common (neglecting to return to x at the end of (b) being the main cause of the latter score profile).

In part (a) the majority of students were able to replace successfully sin x and cos x with the t equivalents, with only very few cases of incorrect formulae used. Sometimes this was the only mark scored, but the majority were able to progress further by identifying a correct derivative statement and the dx to gain a complete integral in terms of t. They then were generally successful in rearranging, to gain the printed answer, and full marks in part (a) was very common. Occasionally slips in the derivative statement, or a rearrangement of it, did occur, while some did not identify a linking equation for dx and dt at all and were unable to progress. The style of proof, however, was a lot less satisfactory, with many making all the substitutions in one go, not having made their dx equation clear, and having to rely on implication to score the marks. A good, logical layout should be encouraged, and the best solutions were those that identified the relevant formulae first, before substituting. The algebra to simplify to the required form was good, with few forced proofs. Having the answer to aim for aided the students, who could correct when they spotted an error as they did not achieve the result.

Part (b) was less accessible with many either not attempting or not being aware of the correct method to proceed and considering other (incorrect) methods, integration for functions in the form of 1/g(t), with logarithm expressions being frequently seen. Attempts at partial fractions were also noted. This type of integral is standard for the specification and it was surprising how many did not seem to be expecting one.

Those that did complete the square were usually successful but there were some errors in coefficients when taking out the factor of $\frac{1}{3}$, but usually a correct form had been seen. The completing of the square was sometimes seen to have been done separately to the integral initially, and some students then went on to solve their equation rather than relate it to the integral, and so lost all the marks.

Those students who did complete the square and apply it to the integral were usually successful in recognising the arctan form but a large number lost the final two marks for failing to replace t with tan(x/2) in their answer. Others had fractions inverted, giving an answer with arctan (ax) rather than arctan(x/a), and some forgot to square root so had arctan(x/a^2).

Question 6

There were some discriminating aspects in this question, though also many accessible marks across grades, and there were many very good attempts. The last two marks were the most challenging, in understanding the requirement for the limit to exist, but many were able to progress through to the latter parts.

Part (a) was completed well with most students successfully differentiating using the chain and product rules. Most attempted the main approach achieving a correct expression and cancelling exponential terms to simplify to the correct solution, and the method was apparent. For the minority who applied the log summation law first, the differentiation became easier but they did not always show all the steps leading to the given answer, and forfeited marks if a correct differentiation step was not shown. However, most were diligent in showing the fraction before cancelling.

In part (b) most students were successful in differentiating for the second derivative and gaining the B mark, but errors in coefficients and powers for sec and tan for the third and fourth derivatives caused many students to lose either one or two marks depending on the error. Some simply made errors that resulted in the wrong coefficients, but gained derivatives of the correct form and losing just the accuracy, but many made errors in applying the chain rule and product rule, losing one of the terms, and therefore lost the method mark here. Often the 3 was missing from sec $3x$ or tan $3x$ or they had forgotten to apply the product rule correctly and a tan $(3x)$ was missing from their final derivative. Another, less common error seen was to have functions of x rather than $3x$.

Many students used up time and effort applying trigonometric identities, which did not generally make the expressions easier to differentiate and sometimes introduced unnecessary errors. The majority of those who found $\frac{d^4y}{dx^4}$ correctly did so without much rearranging.

In part (c) students generally understood what was required for the Maclaurin expansion formula, errors in part (b) meant that many could not access full marks in this section. Most did evaluate all of their derivatives at zero, though some stopped at the third derivative, particular where errors meant they had three non-zero terms by that stage. Substitution into a correct Maclaurin formula usually followed, though a few, as is common, made errors with the factorial divisors. Some students substituted the wrong derivative values into the Maclaurin's series with the fact that the third derivative was zero, appearing to cause some confusion. Many incorrect derivative functions from (b) led to the correct series, due to some trigonometric terms evaluating to zero or 1, but these were not permitted the accuracy to match the score of those with incorrect derivatives which did not give the correct coefficients.

The majority of students successfully completed part (d) giving the expansion in full. A few students lost the mark by losing the power of k in some of their terms, having incorrect signs or having factorials in the numerators.

Most students attempted part (e) but some failed to use the subtraction law of logarithms and were unable to progress. Those that did correctly substitute in their expansions usually went on to consider a value for k, though many instead attempted to find the limit, not the condition for its existence. Of those attempting to find k many used the x^2 coefficient of the expansions and failed to find the correct value of k, usually $k = \pm 3$. Where the correct answer was attained, there was often no good explanation given for it, but just a statement $k = 2$, with no clear indication where it arose from, but these were permitted the mark. Clear explanations were rare to see.

Question 7

Easily the most challenging question of the paper, with many students unable to make progress, especially in the latter parts. However, there were still many who were able to successfully complete the whole question, so there was little evidence time was an issue, but mainly the low scores were due to very poor attempts with students unable to determine what was required. Vectors is, and will continue to be, one of the most demanding subjects for students. However, the majority were able to make some progress in places, though only the higher-grade students could access part (d).

The techniques involved throughout seemed to be well known to many, with formulae quoted for various parts. However, in many cases these were not always employed to good effect, but sometimes just stated without the students working out the relevant vectors needed to use in them. It would help the understanding of a solution if students clearly stated which vectors they were using in their attempted solution.

The method of evaluating cross products was well demonstrated by the majority, but there were a lot of arithmetic errors, sign errors and poor notation during the workings and simplifications, even for students who knew the required steps. As the question progressed, workings became scrappier as students struggled to find their way through, and possibly hurried to finish the paper, with many attempting dot and cross products of any vector they could find just in case it helped.

Part (a) was the best attempted overall, with many students able to obtain an expression for the vector linking the given point A to a general point on the line and proceed to set the distance equal to 15 to find the two values of the parameter. However, some attempted this with just a general point on the line. Others attempted to work out the cross product in the given line equation, using a general point (x, y, z), and attempted to use these in a distance formula to form an equation in x, y or z. These seldom proceeded far enough to attain marks in the alternative method.

Most students who started with the appropriate main scheme method achieved at least three of the four marks, but errors in expanding brackets and arithmetic errors cause many to lose the A mark, while others substituted the parameter back into expression for the A to line vector, and mixed up B and C with AB and AC.

Part (b) was done reasonably well with many fully correct solutions seen, and many scoring both methods where errors had led to incorrect coordinates for B and C. Common errors were to use vectors OA and OB rather than AB and AC, while some gave the answer in a vector form rather than in the required Cartesian form. There were also a few students who gave the plane in the alternative vector form shown in the alternative method, but did not attempt to convert into Cartesian form, so scored no marks.

Part (c) saw a mixed set of responses. Though many did score full marks, or at least the method marks following earlier error, there were numerous errors made. Some did not attempt DA, but attempted the triple product with OD and their normal vector of (b), while others were able to get the first couple of marks but forgot to use ± 147, so didn't achieve two answers, or omitted the $\frac{1}{6}$ entirely from the formula. Of those who successfully carried out the method, many realised the normal vector from part (b) could be used to speed up work, but there was also a considerable number who either reworked this anyway, or used a different vector combination (e.g. used BD as in the scheme) and required another cross product to be used. Of the students who neglected to find the Cartesian equation in part (b), many did proceed to find the relevant triple product in this part and obtain the marks.

By part (d), many students had given up and offered no solution at all. For those who did persevere, the most common approach was for students to quote and attempt to apply the formula for the shortest distance. Clearly many had memorised this in anticipation. However, working out which vectors to use did prove a challenge for many, and this part did discriminate very well at the top-grade range. Where incorrect answers to part (c) had been found, the first three marks were still attainable in this part and were scored by many students.

Identifying a vector connecting A and the line was often, but not always, done correctly with the first shown in the mark scheme the prevalent one. Likewise, good progress with the required cross product was often made, and once achieved the completion of the method usually followed. But correct answers were relatively rare, as slips in earlier working, an incorrect α being very common, prevented the correct answer being found.

Other approaches were seen to this part, by those who either had not learned or remembered the shortest distance formula, and some students were able to work their way to a solution through the alternatives shown in the scheme. These tended to be longer winded and prone to errors in calculation, or required dependence on calculators solving equations, but showed good ability to think through a situation to come up with an approach to the answer.

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