Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 02 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	Area $=\frac{1}{2}\int_0^{\pi} r^2\ \mathrm{d}\theta=\frac{1}{2}\int_0^{\pi} 4\left(\sinh\theta+\cosh\theta\right)\ \mathrm{d}\theta$	**B1**	1.1b
	$=2\left[\cosh\theta+\sinh\theta\right]_0^{\pi}$	**M1**	1.1b
	$=2\left(\cosh\pi+\sinh\pi-\cosh 0-\sinh 0\right)$ $=2\left(\frac{\mathrm{e}^{\pi}+\mathrm{e}^{-\pi}}{2}+\frac{\mathrm{e}^{\pi}-\mathrm{e}^{-\pi}}{2}-1-0\right)$	**M1**	3.1a
	$=2\mathrm{e}^{\pi}-2$	**A1**	2.1
		(4)	
			(4 marks)
Notes:			
B1: Correct area formula applied, including the $\frac{1}{2}$ **M1:** Attempts the integration, cosh to sinh and vice versa, or in terms of exponentials. **M1:** Applies the limits to the integral and uses exponential definitions to achieve answer in suitable form. **A1:** Correct answer.			

Question	Scheme	Marks	AOs
2(a)	$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$	B1	1.1b
		(1)	
(b)	$\mathrm{e}^{\mathrm{e}^x - 1} = 1 + \left(\mathrm{e}^x - 1\right) + \frac{\left(\mathrm{e}^x - 1\right)^2}{2!} + \frac{\left(\mathrm{e}^x - 1\right)^3}{3!} + \ldots$ or $\mathrm{e}^{\mathrm{e}^x - 1} = \mathrm{e}^{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots - 1}$	M1	1.1b
	$= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \right) + \frac{1}{2}\left(x + \frac{x^2}{2!} + \ldots\right)^2 + \frac{1}{6}\left(x + \ldots\right)^3 + \ldots$	M1	3.1a
	$= 1 + x + \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{1}{6} + \frac{1}{2} \times 2 \times \frac{1}{2} + \frac{1}{6}\right)x^3 + \ldots$	M1	1.1b
	$= 1 + x + x^2 + \frac{5}{6}x^3 + \ldots$	A1 A1	1.1b 2.1
		(5)	
			(6 marks)

Notes:

(a)

B1: Correct series (ignore terms beyond x^3).

(b)

M1: Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! but must have 3! or 6 in the cube term.

M1: Attempts the exponential Maclaurin series twice and cancels the 1's in the "power" expansion. Allow if the 3! is incorrect for this mark, but a polynomial in *x* must have been achieved.

M1: Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power).

A1: Any two correct from coefficients of x, x^2 and x^3, need not be simplified.

A1: Fully correct answer with simplified terms..

NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark.

Question	Scheme	Marks	AOs
3(a)	$\mathbf{M}^2+11\mathbf{M}=\begin{pmatrix}a&0\\0&a\end{pmatrix}\Rightarrow\begin{pmatrix}34&5k-10\\6k-12&k^2+30\end{pmatrix}+\begin{pmatrix}-22&55\\66&11k\end{pmatrix}=\begin{pmatrix}a&0\\0&a\end{pmatrix}$	M1	1.1b
	$\Rightarrow a=12$	A1	2.2a
	$5k-10+55=0\Rightarrow 5k=-45\Rightarrow k=-9*$ or $6k-12+66=0\Rightarrow 6k=-54\Rightarrow k=-9*$ or $k^2+11k+30=12\Rightarrow k^2+11k+18=0\Rightarrow k=-2,-9*$, $k\neq-2$ as $4\times-2-10+55\neq 0$	A1*	2.1
		(3)	
(b)	$\begin{pmatrix}-2&5\\6&-9\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}X\\mX+c\end{pmatrix}\Rightarrow\begin{cases}-2x+5(mx+c)=X\\6x-9(mx+c)=mX+c\end{cases}$	M1	1.1b
	$\Rightarrow 6x-9mx-9c=-2mx+5m^2x+5mc+c$	M1	3.1a
	$\Rightarrow\left(5m^2+7m-6\right)x+\left(5m+10\right)c=0$	A1	1.1b
	$\Rightarrow 5m^2+7m-6=0\Rightarrow(m+2)(5m-3)\Rightarrow m=-2,\frac{3}{5}$	M1	1.1b
	$m=\frac{3}{5}\Rightarrow 5m+10\neq 0$ so need $c=0$ hence $y=\frac{3}{5}x$ is a fixed line	A1	2.2a
	$m=-2\Rightarrow 5m+10=0$ so *c* can be anything, so $y=-2x+c$ for any *c* is fixed.	A1	2.2a
		(6)	
(c)	($(0,c)\to(5c,-9c)$ so need *c* = 0,) $(1,m)\to(-2+5m,6-9m)$ so need $5m=3$ hence $y=\frac{3}{5}x$ contains fixed points.	B1	3.2a
		(1)	
			(10 marks)

Notes:

(a)

M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of *a* deduced from upper left entries.

A1*: Correct work to show $k=-9$. If off diagonals are used no further justification is needed (they are "given" the result is true). If the bottom right entry is used there must be a valid reason for rejecting -2 as a solution (ie checking the off diagonal).

(b)

M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates "*X*" to get a linear equation in "*x*".

A1: Correct equation.

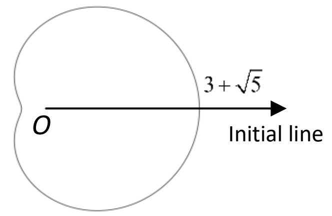
M1: Solves the equation in *m* by any valid means.

A1: Deduces $y=\frac{3}{5}x$ is a fixed line (where *c* = 0). If the value for *m* here is wrong, allow this A for $y=-2x$ if the general case for the final A is not scored.

A1: Deduces $y=-2x+c$ is a fixed line where *c* can be any value. Must include all the lines.

(c)

B1: Identifies $y=\frac{3}{5}x$ is a line of fixed points with reason. Allow if *c* = 0 is assumed. See scheme for one possible reason, others may be given.

Question	Scheme		Marks	AOs
4(a)		Recalls correct shape for the type of curve.	**B1**	1.2
		Correct position with labelling of pole, initial line and point.	**B1**	1.1b
			(2)	
(b)	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(3\sin\theta+\sqrt{5}\sin\theta\cos\theta\right)=A\cos\theta+B\cos 2\theta$ (oe)		**M1**	1.1b
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}\left(3\sin\theta+\sqrt{5}\sin\theta\cos\theta\right)=3\cos\theta+\sqrt{5}\cos 2\theta$ (oe)		**A1**	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 3\cos\theta+\sqrt{5}\left(2\cos^2\theta-1\right)=0\Rightarrow 2\sqrt{5}\cos^2\theta+3\cos\theta-\sqrt{5}=0$ $\Rightarrow\cos\theta=\frac{-3\pm\sqrt{9-4\left(2\sqrt{5}\right)\left(-\sqrt{5}\right)}}{4\sqrt{5}}=\ldots$		**M1**	3.1a
	$\cos\theta=\frac{-3\pm 7}{4\sqrt{5}}$, quadrant 1 needs $\cos\theta>0$ so $\cos\theta=\frac{1}{\sqrt{5}}$		**A1**	2.3
			(4)	
(c)	$r=4$		**B1**	1.1b
			(1)	
				(7 marks)

Notes:

(a)

B1: Recalls the correct cardioid shape for this type of polar curve.

B1: Correctly placed with the pole, initial line and point where curve crosses the initial line all indicated in some way.

(b)

M1: Uses $y=r\sin\theta$ with the curve and attempts to differentiate. Accept any correct form but may have slips in coefficients, so e.g. as shown or $A\cos\theta+B\cos^2\theta+C\sin^2\theta$ can score M1.

A1: Correct differentiation. Accept equivalents, e.g. $3\cos\theta+\sqrt{5}\cos^2\theta-\sqrt{5}\sin^2\theta$

M1: Sets their derivative equal to zero and attempts to find $\cos\theta$ (allow if $r\cos\theta$ was used)

A1: Selects the correct value for $\cos\theta$. If the other value is given it is A0 unless clearly rejected.

(c)

B1: Correct value for *r*

Question	Scheme	Marks	AOs
5(a)	$\alpha = \frac{z_1+z_2}{2} = \frac{35-25\text{i}-29+39\text{i}}{2} = \ldots$	M1	1.1b
	$= 3+7\text{i}\,*$	A1*	1.1b
		(2)	
(b)	$\beta(z_1-\alpha) = \left(\frac{1+\text{i}}{64}\right)(35-25\text{i}-(3+7\text{i})) = \left(\frac{1+\text{i}}{64}\right)(32-32\text{i}) =$ $= \frac{1}{64}\left(32-32\text{i}+32\text{i}-32\text{i}^2\right) = \frac{1}{64}(32-32\text{i}+32\text{i}+32)$	M1	1.1b
	$= \frac{1}{64}(64) = 1*$	A1*	1.1b
		(2)	
(c)	Roots are $1\text{e}^{\text{i}\frac{k\pi}{3}}, \quad k = 0,1,2,3,4,5$	B1	1.1b
		(1)	
(d)	$w = \beta(z-\alpha) = \text{e}^{\text{i}\frac{k\pi}{3}} \Rightarrow z = \frac{\text{e}^{\text{i}\frac{k\pi}{3}}}{\beta} + \alpha$	M1	3.1a
	$\Rightarrow z = \frac{64\left(\cos\frac{k\pi}{3} + \text{i}\sin\frac{k\pi}{3}\right)(1-\text{i})}{(1+\text{i})(1-\text{i})} + 3 + 7\text{i} = \ldots$	M1	1.1b
	Two of $19+16\sqrt{3}+\left(16\sqrt{3}-9\right)\text{i},\ 16\sqrt{3}-13+\left(23+16\sqrt{3}\right)\text{i}$, $-13-16\sqrt{3}+\left(23-16\sqrt{3}\right)\text{i},\ 19-16\sqrt{3}-\left(9+16\sqrt{3}\right)\text{i}$	A1	2.5
	All four of $19+16\sqrt{3}+\left(16\sqrt{3}-9\right)\text{i},\ 16\sqrt{3}-13+\left(23+16\sqrt{3}\right)\text{i}$, $-13-16\sqrt{3}+\left(23-16\sqrt{3}\right)\text{i},\ 19-16\sqrt{3}-\left(9+16\sqrt{3}\right)\text{i}$	A1	2.2a
		(4)	
			(11 marks)

Notes:

(a)

M1: Attempts the midpoint of z_1 and z_2

A1*: Correct point.

(b)

M1: Substitutes into the equation with z_1 and α and β, simplifies and expands and applies $i^2 = -1$

A1*: Correct answer.

(c)

B1: Correct roots, accept all 6 listed or given in general form as in the scheme. Need not show the 1.

(d)

M1: Realises the need to set the roots of unity equal to $\beta(z-\alpha)$ and solve for *z*. Must be attempted at least once with any of their roots.

M1: Finds the Cartesian form from their equation for at least one of the roots other than z_1 and z_2

A1: At least two correct other roots than z_1 and z_2 in Cartesian form.

A1: Deduces all four correct in Cartesian form.

Question	Scheme	Marks	AOs
6	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{2x}\sinh x + \mathrm{e}^{2x}\cosh x = \mathrm{e}^{2x}\left(a\sinh x + b\cosh x\right)$	**M1**	2.2a
	$= \mathrm{e}^{2x}\left(\frac{3+1}{2}\sinh x + \frac{3-1}{2}\cosh x\right)$ so the result is true for n = 1	**A1**	2.4
	(Assume the result is true for n = k, then) $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = 2\mathrm{e}^{2x}\left(\frac{3^k+1}{2}\sinh x + \frac{3^k-1}{2}\cosh x\right) + \mathrm{e}^{2x}\left(\frac{3^k+1}{2}\cosh x + \frac{3^k-1}{2}\sinh x\right)$	**M1**	2.1
	$=\mathrm{e}^{2x}\left(\left(3^k+1+\frac{3^k-1}{2}\right)\sinh x + \left(3^k-1+\frac{3^k+1}{2}\right)\cosh x\right)$ $= \mathrm{e}^{2x}\left(\frac{3\times3^k+1}{2}\sinh x + \frac{3\times3^k-1}{2}\cosh x\right)$	**dM1**	1.1b
	$= \mathrm{e}^{2x}\left(\frac{3^{k+1}+1}{2}\sinh x + \frac{3^{k+1}-1}{2}\cosh x\right)$	**A1**	2.1
	Hence the result is also <u>true for n = k +1</u>, so <u>if true for n = k then true for n = k + 1</u>, and as also <u>true for n = 1</u>, so the result is <u>true for all positive integers</u>.	**A1**	2.4
		(6)	
			(6 marks)

Notes:

M1: Attempts the first derivative and factors out the exponential

A1: Correct derivative and reaches appropriate form to deduce the result is true for n = 1

M1: (Makes the inductive assumption and) attempts the (k +1)-th derivative from the k-th derivative. Allow slips in coefficients.

dM1: Factors out the exponential and gathers the sinh x and cosh x terms. Accept either form shown or equivalent.

A1: Reaches the correct form from correct work. Must have the "k+1" showing. Depends on the previous two method marks.

A1: Makes appropriate concluding sentence covering the points indicated in scheme. Depends on all method marks having been scored. Must have reached at least the second line shown in the dM mark, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown).

Question	Scheme	Marks	AOs
7(a)	$V=\pi\int_{-1.545}^{1.257} x^2\,\mathrm{d}y$	**B1**	1.1a
	$\int x^2\,\mathrm{d}y=\frac{1}{16}\int 6-3y^2+y\cos\left(\frac{5}{2}y\right)\mathrm{d}y \to Ky-Ly^3+\ldots$	**M1**	3.1a
	$\int y\cos\left(\frac{5}{2}y\right)\mathrm{d}y=y.\frac{2}{5}\sin\left(\frac{5}{2}y\right)-\int 1.\frac{2}{5}\sin\left(\frac{5}{2}y\right)\mathrm{d}y$ $=\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)(+c)$	**M1**	1.1b
	$\int x^2\,\mathrm{d}y=\frac{1}{16}\left(6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right)(+c)$	**A1**	1.1b
	$\int_{-1.545}^{1.257} x^2\,\mathrm{d}y=\frac{1}{16}\left[6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right]_{-1.545}^{1.257}$ $=\frac{1}{16}(5.3954\ldots-(-6.1101\ldots))=\ldots$	**dM1**	3.4
	Volume = $\pi\times\frac{11.505\ldots}{16}=2.26$ cm³ (2.2591159…)	**A1**	3.2a
		(6)	
(b)	Max volume for 100 berries (as we know volume of the largest) is $100\times 2.26 \approx 226$ cm³	**B1ft**	1.1b
	e.g. but not all the berry will become juice (e.g. skin, flesh, seeds may not pulp) and not all will be as big as the largest, so the berries are not likely to produce 200 cm³ of juice.	**B1**	2.2b
		(1)	
			(8 marks)

Notes:

(a)

B1: Selects the correct volume of revolution formula to use, with correct limits in evidence.

M1: Makes x^2 the subject of the equation and attempts to integrate with the correct form for the constant and term in y^2

M1: Applies integration by parts fully on the $y\cos\left(\frac{5}{2}y\right)$ term in the correct direction. Allow slips in the coefficients, but the form must be correct.

A1: Correct integration of the x^2 equation.

dM1: Applies the limits to their integral. No need for the π for this mark.

A1: Correct volume including units. Accept awrt 2.26 cm^3. Allow 0.719π cm^3.

(b)

B1ft: Attempts to estimate the volume of juice produced by 100 berries - look for their (a) multiplied by 100.

B1: Draws a suitable conclusion with reason given. Accept as shown, but also allow a reason along the lines of 200 ml is less than 90% of the possible 226 ml, so probably will fill the cup with juice.

Question	Scheme	Marks	AOs
8(a)	One possibility is (that all three roots lie on) the real axis.	**B1**	2.2a
	The other possibility is that all three roots have the same real part so lie on a vertical line.	**B1**	3.1a
		(2)	
(b)	Other roots are $\frac{3}{2}$ and $\frac{3}{2}-\frac{3}{2}\mathrm{i}$	**B1**	3.2a
		(1)	
(c)(i)	Common root must be $\frac{3}{2}$	**B1**	2.2a
		(1)	
(ii)	So $\mathrm{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z+\alpha)$	**M1**	1.1b
	$-\frac{3}{2}\times 4\times\alpha=12\Rightarrow\alpha=-2$	**M1** **A1**	3.1a 1.1b
		(3)	
(d)	$\mathrm{f}(z)=8\left(z-\frac{3}{2}\right)\left(z-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)\left(z-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)=8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)$	**M1**	1.1b
	$\mathrm{f}(z)=\mathrm{g}(z)\Rightarrow 8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)=\left(z-\frac{3}{2}\right)(z+4)(z-2)$ $\Rightarrow 8z^2-24z+36=(z+4)(z-2)\quad\left(\text{or } z=\frac{3}{2}\right)$	**M1**	3.1a
	$\Rightarrow 7z^2-26z+44\Rightarrow z=\frac{26\pm\sqrt{26^2-4\times7\times44}}{14}=\ldots$	**M1**	1.1b
	So solutions are $\frac{3}{2}$, $\frac{13\pm\mathrm{i}\sqrt{139}}{7}$	**A1**	1.1b
		(4)	
			(11 marks)

Notes:

(a)

B1: Identifies the case that all three roots could be real so lie on the real axis. Accept as a diagram or equation given. Mention of real axis is sufficient.

B1: Identifies the case of vertical lines (where there is a complex conjugate pair and the third root must lie on the real axis with same real part as the pair).

(b)

B1: Interprets the conclusion from (a) in context by identifying the correct two roots.

(c)

B1: Deduces the real root is the one in common.

M1: Forms an expression for g(*z*) using the two known roots. Alternatively, if product of roots is used first, it is for forming a cubic expression for g(*z*) from their roots.

M1: Uses the given form for g(*z*) in the question to find the third root/value for α in their expression. May be scored before the previous M if product of roots is used to find the third root before forming the cubic.

A1: Correct expression for g(*z*), either factorised or expanded.

(d)

M1: Uses their roots of f(*z*) to form a cubic expression for f(*z*), and expands to at least a linear term times a quadratic with real coefficients (which may be seen later).

M1: Sets their expressions equal and factorises out or cancels the common term to achieve a quadratic expression in *z*. Allow if f(*z*) is not yet expanded.

M1: Expands, gathers terms and solves the resulting quadratic. Allow this mark if the $z = \frac{3}{2}$ solution is not given.

A1: All three correct solutions given.

Question	Scheme	Marks	AOs
9(a)	$\frac{d^2y}{dt^2} = 0.032\frac{dx}{dt} - 0.025\frac{dy}{dt}$ oe e.g. $\frac{dx}{dt} = \frac{1}{0.032}\left(\frac{d^2y}{dt^2} + 0.025\frac{dy}{dt}\right)$	B1	1.1b
	$\frac{d^2y}{dt^2} = 0.032(0.025y - 0.045x + 2) - 0.025\frac{dy}{dt}$ $= 0.0008y - \frac{0.00144}{0.032}\left(\frac{dy}{dt} + 0.025y\right) + 0.064 - 0.025\frac{dy}{dt}$ Or $\frac{1}{0.032}\left(\frac{d^2y}{dt^2} + 0.025\frac{dy}{dt}\right) = 0.025y - \frac{0.045}{0.032}\left(\frac{dy}{dt} + 0.025y\right) + 2$	M1	1.1b
	$40\,000\frac{d^2y}{dt^2} + 2800\frac{dy}{dt} + 13y = 2560$*	A1*	2.1
		(3)	
(b)	$40000m^2 + 2800m + 13 = 0 \Rightarrow m = ...$	M1	3.4
	$\text{CF}: y = Ae^{m_1t} + Be^{m_2t}$	M1	1.1b
	$\text{CF}: y = Ae^{\frac{-t}{200}} + Be^{\frac{-13t}{200}}$	A1	1.1b
	PI: Try $y = k \Rightarrow 13k = 2560 \Rightarrow k = \frac{2560}{13}$	M1	3.4
	GS: $y = Ae^{\frac{-t}{200}} + Be^{\frac{-13t}{200}} + \frac{2560}{13}$	A1ft	1.1b
		(5)	
(c)	$t = 0, y = 0 \Rightarrow 0 = A + B + \frac{2560}{13}$	M1	3.4
	$t = 0, y = 0, x = 0 \Rightarrow \frac{dy}{dt} = 0.032 \times 0 - 0.025 \times 0 = 0$	B1	3.4
	$\Rightarrow \frac{dy}{dt} = -\frac{A}{200}e^{\frac{-t}{200}} - \frac{13B}{200}e^{\frac{-13t}{200}} = 0 \Rightarrow -\frac{A}{200} - \frac{13B}{200} = 0 \Rightarrow A = -13B$	M1	1.1b
	$y = -\frac{640}{3}e^{\frac{-t}{200}} + \frac{640}{39}e^{\frac{-13t}{200}} + \frac{2560}{13}$	A1	1.1b
		(4)	
(d)	As $t \to \infty, e^{-kt} \to 0$ for $k > 0$ so $y \to ...,$	M1	1.1b

	$y \to \frac{2560}{13} \approx 196.92$ so the rate of administration is sufficient to reach the required level.	**A1**	3.2b
		(2)	
			(14 marks)

Notes:

(a)

B1: Differentiates the second equation with respect to t correctly. May have rearranged to make x the subject first. The dot notation for derivatives may be used.

M1: Uses the second equation to eliminate x to achieve an equation in y, $\frac{dy}{dt}, \frac{d^2y}{dt^2}$.

A1*: Achieves the printed answer with no errors.

(b)

M1: Uses the model to form and attempt to solve the auxiliary equation (Accept a correct equation followed by two values for m as an attempt to solve.)

M1: Forms the complementary function correct for their roots (so if repeated or complex roots found, award for appropriate form for CF). Must be in terms of t only (not x)

A1: Correct CF

M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI

A1ft: Combines their CF (which need not be correct) with the correct PI to give y in terms of t so look for y = their CF + $\frac{2560}{13}$, accepting awrt 197.

(c)

M1: Uses the initial conditions of the model to set up an equation in A and B from their general solution.

B1: Uses the initial conditions of the model to find the value of $\frac{dy}{dt}$ when $t = 0$

M1: Differentiates their general solution and substitutes $t = 0$ to form another equation in A and B and proceed at least as far as finding A in terms of B oe.

A1: Correct particular solution.

(d)

M1: Uses the limit of the exponential terms is zero to find the long term limit of the concentration

A1: Correct limit and concludes the rate of administration is sufficient to achieve the required level.

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