Pearson Edexcel

Examiners' Report

Principal Examiner Feedback

Summer 2023

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 02 Core Pure Mathematics 2

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Question 1

On the whole this question was really well done with only a few candidates with an insecure knowledge of the relevant formula. Most candidates scored full marks.

Errors were with the arithmetic, usually in adding exponentials incorrectly, multiplying through by 2 and missing doing this to the constant or by being unable to square the expression. Errors in the latter involved squaring twice (usually each term the second time) or by square rooting each term.

The vast majority of candidates did the integration in sinh$x$ and cosh$x$ and converted to exponentials after integrating. Some candidates lost the final mark. A common error seen was

$$2\left(\frac{e^{\pi}+e^{-\pi}}{2}+\frac{e^{\pi}-e^{-\pi}}{2}-1\right)=2e^{\pi}-1$$

Some candidates that did not gain full marks, incorrectly quoted the definitions of sinhx and coshx with the inclusion of i leading to an answer not of the correct form. Whereas some candidates made a slight error in distributing the factor of 2 to both limits.

The majority of candidates were gaining full marks. Of those that didn't, some failed to start with the correct formula (missing the half in front, or the squaring of r), but the vast majority of lost marks were through poor accuracy in substituting the limits.

The multiples of $\frac{1}{2}$ and 4 caused some confusion with candidates not being sure whether to cancel them or multiply them out.

Question 2

(a) The large majority of candidates scored this mark. Some chose to derive this using differentiation techniques but most quoted the correct form as given in the formulae booklet. The only error seen was candidates writing the x^3 term over 3.

(b) Many candidates stopped after one application of Maclaurin's series, giving an answer in terms of e^x rather than a polynomial as requested. Of those that progressed further, the answers were mainly fully correct. Candidates could have simplified their working by rejecting higher powers earlier. With those methods involving expanding a lot of brackets, some inaccuracies occurred.

There were also several candidates which multiplied their expression by 6 to remove the fractions, despite the expression not being equal to anything.

The first approach expressed $e^x - 1$ as a power series using the answer to (a). Most following this route then used the expansion from (a) with this power series for $e^x - 1$ substituted for the variable, expanded the brackets, and collected like terms. Another route from this starting point was to split the exponential into the product of three exponentials and use the series expansions to multiply out brackets.

A second commonly used approach was to use $e^x - 1$ directly in the series expansion from (a) and to expand brackets and rearrange to get a sum of exponentials. Many candidates who were unable to arrive at the final polynomial form typically achieved a function containing e^x, e^{2x} and e^{3x} and believed this was the final solution, not identifying a polynomial in x was required. Of those who then used the answer to (a) to express the exponentials as power series and collect like terms, many could produce the correct series.

The candidates that manipulated the given function to $e^{-1}e^{e^x}$ were unable to make meaningful progress after applying the Maclaurin expansion once to e^{e^x}.

Those that had $e^{x+\frac{x^2}{2}+\frac{x^3}{6}}$ were generally more successful in getting a cubic polynomial, but again the accuracy in the expansion of brackets was not particularly good overall.

Question 3

(a) The majority of students answered part (a) successfully, with most able to evaluate $\mathbf{M}^2$ and use this to find values of a and k using the off diagonals. Some slips were seen leading to incorrect values and a few candidates used the bottom right entry only to find the given value of k, which was not enough to justify this. A few candidates were careless when evaluating $\mathbf{M}^2$, particularly in the top left, leading to an incorrect value for a.

(b) There appeared to be confusion between a line of invariant points and an invariant line. A surprising number of students found the line of invariant points

$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix}$ and quite a few produced an incomplete solution by using $\begin{pmatrix} x \\ mx \end{pmatrix} \to \begin{pmatrix} X \\ mX \end{pmatrix}$ instead of the required $\begin{pmatrix} x \\ mx \end{pmatrix} \to \begin{pmatrix} X \\ mX + c \end{pmatrix}$. Special cases for the first 2 cases were incorporated into the mark scheme and candidates were able to score some marks.

However, a majority did take the correct approach to find right values for m. Having done so quite a few thought that $c = 0$ for both lines and so produced

$y = \frac{3}{5}x$ and $y = -2x$ rather than $y = -2x + c$.

(c) Many candidates correctly thought that $y = \frac{3}{5}x$ was the line of invariant points, but had difficulty justifying their decision.

<u>Question 4</u>

(a) Candidates were successful in recalling the correct shape for the curve. The overall dimple shape of the cardioid and its orientation was almost universally known. A common error was omitting all requested labels, often misidentifying the pole by drawing a line at $\theta = \frac{\pi}{2}$ and labelling as pole. Very few candidates scored full marks because they failed to label the diagram as requested with the pole, the initial line and the intercept point.

Parts (b) and (c) were completed to a high standard with only a few candidates using $r\cos\theta$ instead of $r\sin\theta$.

(b) The majority of candidates were able to differentiate $r\sin\theta$ correctly and then used correct trigonometric identities to form a quadratic for $\cos\theta$.

Most candidates reasoned that as θ was acute, the result for $\cos\theta$ would be positive and that the second solution was not valid, leading to successful rejection of the second solution. Some candidates did fail to reject the other solution of their quadratic.

(c) Virtually all candidates correctly scored this mark, even those who did not attempt (b), with only a small handful of candidates attempting $\cos\left(\frac{1}{\sqrt{5}}\right)$ leading to an incorrect answer. A few candidates arrived at a solution of 3 due to a simple calculation error.

<u>Question 5</u>

Parts (a) and (b) were generally completed with full marks. However, a significant number of candidates failed to write the roots correctly in (c)(i) and (c)(ii) was very poorly attempted with many candidates having no idea how to begin, nor spotted the connection to the previous parts of the question.

(a) A large majority of candidates managed to show the given results; recognising that the centre is the midpoint of the two complex numbers. Those who tried doing this with distances were unsuccessful. An occasional wrong response was to show 3 + 7i was equidistant from the 2 points rather than their midpoint.

(b) Nearly all candidates knew what they were doing here. The majority of candidates successfully attempted part (b) with most arriving to the desired result with no errors; some omitted necessary working in their proofs achieving the solution but unable to achieve the final A1 mark in part (b). Many skipped stages in showing the result true, including expanding the brackets and using $\mathrm{i}^2 = -1$.

(c) This was poorly answered. Many tried complex methods and many missed the i off in the power. Some thought there were π radians in a full turn, giving answers over 6 instead of 3. Many candidates failed to spot the relationship between the previous parts in part (ii); some tried alternative methods but few of these were successful.

(c)(i) required a listing of the sixth roots of unity. This part was answered correctly by many students. However, often some roots were omitted, or the spacing of the arguments of the roots was $\frac{\pi}{6}$ instead of $\frac{\pi}{3}$. A few candidates tried to write as function $\mathrm{e}^{n\frac{\pi}{3}}$ but many did not define *n*.

(c)(ii) The final part of the question proved challenging with only the most able students being able to attempt this question. Very few were able to gain full marks. They did not realise that they need to set the roots of unity equal to $\beta(z-\alpha)$ and solve for *z*. Those who did make headway in this part sometimes did not gain full credit either because they resorted to decimals or because they did not give their answer in the requested Cartesian form.

If a candidate attempted the problem, they were able to achieve the required roots in the correct form; some omitted i within some solutions, missing the B1 mark. For the remainder of the problem candidates made attempts to rotate their roots understanding the need to equate their roots to $\beta(z-\alpha)$ or some similar form.

Using transformation, if candidates successfully transformed the centre to the origin, they often were able to arrive at the desired solutions. However, many candidates failed to move the centre to the origin, unable to note the requirement of being centred about the origin before applying their methods.

If candidates were unable to identify the required method, the initial M1 mark was earned by some respondents in the alternative mark scheme for this problem, determining the modulus and argument.

A lot of candidates struggled to start (c)(ii), or if they did attempt it, they failed to gain marks as they did not deal with the transformation by (3+7i) in their method. There was a lot of processing required here unless candidates used their calculators effectively.

Question 6

This question was performed relatively well by candidates overall, with most understanding the structure required for a proof by induction. Some candidates however did not sufficiently justify the

n =1 case, either by not showing the substitution into the hypothesis or by simply claiming that the result from their substitution was $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Candidates appreciated the need to differentiate the $n = k$ case in order to make progress. It was pleasing to see so many fully complete differentiation attempts with adequate evidence of rearrangement to prove the truth of the $n = k$ +1 case.

A few candidates incorrectly thought that $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}^k y}{\mathrm{d}x^k} \times \frac{\mathrm{d}y}{\mathrm{d}x}$.

Drawing the strand of the inductive proof together into a convincing statement was well done; some students chose to give their inductive statement as a narrative – with the "true for n = 1" statement appearing earlier in the proof – and this was given full credit.

Those whose final statement included a phrase like "true for $n = k$, true for $n = k + 1$" and thus omitting the implication that truth for the first implies truth for the second did lose the final mark.

Question 7

(a) Most candidates knew the correct formula for the volume of revolution and were not put off by rotation about the y-axis. The majority integrated the correct function of y. Generally, there was no problem integrating $6 - 3y^2$ and most were able to integrate $y\cos\left(\frac{5y}{2}\right)$ correctly using integration by parts. Common mistakes were missing the y out of the sine term and having the incorrect sign in front of the $\frac{4}{25}\cos\left(\frac{5y}{2}\right)$. Some candidates either did not recognise the need to integrate by parts, or attempted it without success.

Substitution of limits did create some problems. Candidates need to realise how easy it is to slip up when using a calculator to do the entire calculation and should be encouraged to show their working in order to demonstrate that they are performing the expected evaluation by showing the substitution of limits.

A significant cohort failed to mention units in their final answer, which was required and would frequently be penalised.

Some candidates had clearly used their calculator to get the ‘correct’ answer with the incorrect integration.

(b) The majority of candidates correctly multiplied their answer to (a) by 100 but do need an explanation relating this to 200 and noting that this is the largest possible or that not all of the grapes will turn to juice – a simple yes or no without further explanation is not enough.

Question 8

This was a really good question for testing understanding. If a candidate scored on part (a) they mostly went on to achieve (b), (c)(i) and (c)(ii). Of those who attempted part (d), about half didn’t account for the fact that the coefficient of z^3 was 8 so scored a maximum of 1 out of 4; the most successful solutions were from those who used sum and product of root results to obtain both cubics and then solved the resulting equation.

In (a), many candidates only realised to answer with the line $x = k$ where k is a real number and failed to use the real axis for both marks. Those candidates who were successful with part (a) had the most success with this question. If candidates were unable to successfully identify the vertical line, the remainder of the question proved challenging to restart. A few candidates drew a diagram to show a vertical line which was acceptable.

In (b), most candidates understood that the complex conjugate of the given root was required but unsuccessful candidates were unable to correctly identify the real root. This part did link with part (a) and candidates are reminded to look back at previous parts of the questions.

In (c)(i), candidates who scored marks in (b) also scored B1 for the common root.

In (c)(ii), many candidates chose the appropriate method but a significant number set the product of roots = 12 instead of –12.

Part (d) was very poorly answered with many candidates not understanding how to proceed with the question. Those that did were generally successful although there was a tendency to slip into decimals which lost them the final mark.

Quite a few candidates forgot that f(z) needed a coefficient of 8 on the cube term, losing most of the marks. Omitting the factor of 8 often led candidates to a linear equation where equating $\mathrm{f}(z)$ and $\mathrm{g}(z)$ preventing method marks being achieved later in this part.

A few candidates made errors in identifying the coefficients of g(*z*) but most were successful in achieving a cubic form for *g*(*z*).

A notable number of candidates who were able to successfully arrive at the required roots provided their roots in a decimal form rather than the exact form which was required.

Many candidates expanded, rather than removing the $\left(z-\frac{3}{2}\right)$ and this often led to a decimal solution for the roots (losing a mark) because they only solved the solution in their calculator.

Question 9

(a) There were many different methods used by the candidates. The algebra being complicated by unpleasant decimals/fractions which led to many careless errors. Most candidates found a correct expression involving $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, and after some manipulation substituted for *x*. There were quite a few cases, following slips, where the given differential equation managed to appear, which did not seem to follow the working. In these cases, candidates should go back at the end and try to correct their working.

(b) This part of the question was mostly successful and often the only part candidates attempted. Many of the candidates successfully found the correct CF and proceeded to find a correct GS, although there were some errors seen in selecting a correct PI.

(c) Most candidates substituted $t = 0$ and $y = 0$ to find one simple equation; finding the second proved to be a little more problematic. This was a good discriminator with many choosing the efficient method of using $\frac{dy}{dx} = 0$ and gaining two equations in A and B. Some took the more arduous root of finding an equation for x in terms of t, to varying degrees of success. Once candidates had simultaneous equations, they generally went on to solve them, with overall success dependent upon the accuracy of their equations.

(d) For most candidates who correctly found y as $t \rightarrow$ infinity, it did not matter what errors were made in (c) since the result depended only on the constant term. They then drew the correct conclusion. Attempting to find y for a random value of t was generally not successful.

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