Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 01 Core Pure Mathematics 1

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9FM01 2023 Examiner's Report

General

This paper provided plenty of opportunity for candidates to demonstrate what they had learnt. All questions provided access at all levels and the earlier questions were often expected and were a good source of marks for many. Later question offered greater challenge, particularly questions 7 and 8 with question 7 perhaps being the most demanding question on the paper.

It is important for candidates to recognise the importance of instances where the warnings in bold are given at the start of a question. This was particularly true for question 3 where a significant number of candidates simply resorted to using a calculator to answer part (b).

There were instances throughout the paper where marks were lost unnecessarily. Examples are;

- question 1 where the final equation was lacking the "= 0" or was not written in terms of w
- question 2 where the final mark was lost due to incorrect simplification
- question 3 where insufficient work was seen for the base case and the conclusion was lacking the correct structure
- question 5 where candidates often failed to give a vector **equation** for a line

Question 1

This question was very well answered with a majority of candidates achieving full marks, particularly by those candidates who chose to use a substitution method.

Candidates who applied $x = w - 2$ and substituted would often score the majority of the marks available. Errors that were seen with this method included algebraic slips in expanding brackets and also a failure to write the final equation with the "= 0". Some candidates mistakenly used $x = w + 2$ or even $x = x + 2$.

Candidates who used the sum, pair, product method were less successful and it was not uncommon for them to get sufficiently lost in the algebra to get no further marks but there were still many fully correct answers. Some candidates failed to find three correct equations for the B1 mark and this led to mistakes in their final answer. Those candidates who used this method could often then get the first method mark for finding the new sum, pair sum and product. There was mixed success in identifying the positive and negative nature of their p, q and r. Other final errors were in failing to write an equation including "= 0" as with the main scheme.

Another successful but less common method was also seen which involved candidates using the substitution $w = x + 2$ into the target cubic and comparing coefficients to derive the values of p, q and r.

A few candidates attempted a very long-winded approach, where they attempted to expand the new equation in the form $\{(x - (\alpha + 2)\}\{x - (\beta + 2)\}\{x - (\gamma + 2)\}$ then used the sum, pair sum and product of roots in their new equation. These attempts rarely scored more than the first two marks.

Question 2

This question proved accessible to most candidates, with many scoring full marks.

In part (a), almost all candidates completed the square successfully with candidates in general being able to write the answer with no working. Any errors were very rare.

In part (b), most candidates left their answer in terms of arcosh, usually working directly from the integral having used part (a) to rearrange the denominator. Many wrote the answer down using a standard result with no working but some could not obtain the required form and wrote down e.g. $\text{arcosh}\left(\frac{x}{3}+2\right)$ or $\text{arcosh}\left(\frac{x+2}{9}\right)$ instead of the correct $\text{arcosh}\left(\frac{x+2}{3}\right)$. Relatively few candidates gave their solution just as a logarithm, though a few gave both forms. Those who used the logarithmic form made more slips. Rather than use a standard result, some candidates chose to use a substitution

e.g. $u = x + 2$ or $x + 2 = \sec\theta$. A few responses were seen where arcsin or arsinh was mistakenly given.

In part (c), most candidates knew how to find the mean value of a function. Those who failed to do that could often obtain B0M1A0 with the correct use of limits with their integral. There were some incorrect substitutions and collection and/or simplification of answers with some failing to divide both parts of their final fraction by 3 where appropriate.

Question 3

Part (a) was mainly answered correctly with most candidates writing the modulus as $4\sqrt{2}$ and the argument as $\frac{3\pi}{4}$. However, some gave an argument in an incorrect quadrant such as $-\frac{\pi}{4}$ or $\frac{7\pi}{4}$.

In part (b), correct answers only with no working scored no marks because the candidates were required to show all stages of their working and not rely on calculator technology. A few candidates did not heed these warnings but generally sufficient working was shown. In part (i), most candidates found $\frac{z_1}{z_2}$ by dividing the modulii and subtracting the arguments. Those that tried to divide the two complex numbers in Cartesian form, often failed to show sufficient working and resorted to using a calculator or could not negotiate the processing required to achieve an answer in the required exact form.

In part (ii) most candidates attempted to apply de Moivre’s Theorem correctly. Candidates knew to raise the power of the modulus and multiply the argument by 4. There were some failed attempts to write their answer in the required form. Candidates who were most successful in part (b) often resorted to writing the complex numbers in exponential form.

In part (c), around one third of candidates scored full marks. Axes were sometimes poorly drawn and in some cases z_1 and z_2 appeared to be equidistant from O or z_2 was wrongly positioned within the quadrant. Of those who did not give the required forms in part (b), some also positioned $\frac{z_1}{z_2}$ in an incorrect quadrant. In part (ii) quite a number of candidates did not draw a line or have any shading to indicate the region required. A few realised that a perpendicular bisector was required but failed to do any shading. Others drew a line across z_1z_2 but it was not a perpendicular bisector. For those who did identify the perpendicular bisector, very few specified which side they believed to be the required area. Only a small number of responses were seen where the wrong side was clearly stated. Incorrect areas included areas between arcs from z_1 and z_2 and wedge-shaped areas between the lines from the origin to the points in their diagram.

Question 4

This question was accessible to nearly all candidates and full marks was commonly seen. There was a large number of extremely well executed proofs with candidates obviously well-rehearsed in such questions.

All candidates who attempted this question started by trying to show that the result was true for $n = 1$. However, a significant number did not explicitly show the substitution of $n = 1$ in the right-hand matrix (by writing, for example, -2×1) and thereby lost the first mark. Most candidates then set up the assumption statement and many went on to prove the result was true for $n = k + 1$ using their assumption. Most successfully achieved the result, although a small number did not factorise $-2 -2k$ to show equivalence. A minority of candidates set up a target result and then showed that they had achieved this with a "meet in the middle" approach.

The greatest difficulty for some came with the final statement where a significant number of candidates struggled to put together all the required elements in a clear and cohesive form. Some candidates omitted to state that it was true for all n, or stated incorrectly that it was true for all real numbers. Some candidates who did not give an acceptable conclusion at the end were still able to gain this final accuracy mark, as the mark scheme allowed it to be gained if all the key points were seen as a narrative in their solution. A significant number of candidates lost the final mark for conclusions that did not convey the "if…then" nature of the proof and stated, for example, "true for $n = k$ and true for $n = k + 1$ and true for $n = 1$ therefore true for all n."

Question 5

This question yielded a varied range of mark profiles and also saw a wide variety of methods employed in the different parts of the question.

In part (a), the vast majority of candidates achieved full marks by substituting the parametric form of the line into the plane equation to find a value for their parameter. They then substituted this value into the equation of their line to find the coordinates of the point of intersection. A few attempted to rearrange the Cartesian equation of the line to express y and z in terms of x and then substituted these expressions into the equation of the plane to find the value of x (then finding y and z).

In part (b), the majority of candidates executed their solutions competently to achieve the given result.

Most set up the equation of a new line using the normal to the plane and a point on the given line (usually $(-5, -4, 3)$) and attempted to find the value of their parameter where this line met the plane. They then proceeded to find the image point and the equation of the line as required. Those who had used an initial point other than (−5, −4, 3) generally realised that their resulting direction vector was a multiple of the required direction vector. When writing the equation of the reflected line, a significant number did not write "**r** =" (despite it being a given result), either omitting it entirely or writing e.g.

$l_2 =$. This omission lost them the final accuracy mark.

A relatively small number of candidates identified a point on l_1 and found the perpendicular distance from this point to the plane. Many did not progress any further, but some went on to gain full marks by considering the modulus of the normal to the plane which in turn enabled them to identify the image point and ultimately the equation of the line as required. A few candidates found the angle between the line and the plane, but this very rarely resulted in any creditworthy work.

In part (c), relatively few students used their working from part (b) to establish a second point on the line of intersection of the two planes, which enabled the equation of the line of intersection to be found very efficiently. Instead, there was a wide variety of (quite time consuming) attempts, many using a vector cross product to find the vector equation of the second plane and solve this simultaneously with the first. Many such attempts, however, stopped short of providing a vector equation of a line and so did not score any marks. Often seen amongst the successful attempts was the relatively efficient method of using the vector product twice; first using the direction vectors of the two reflected lines to find the normal to plane 2, then once again using this normal vector with the normal to plane 1 to find the direction vector of the line of intersection. The vector product is not on the specification, but its use was entirely acceptable.

Many candidates were unsure of what to do in part (d) and did not attempt a solution. A common method was finding (often using earlier work) the vector equation of the second plane, then using consistency arguments by considering a linear combination of the equations of the three planes. Others candidates who had also established the three equations of the planes used a determinant method to find the value of *a*. However, these candidates were often unsure how to proceed to find the value of *b*. Again, the easier methods were seen quite rarely. Those with two points on the line of intersection were able to find the values of *a* and *b* very efficiently by substituting these position vectors into the equation of the third plane. A few candidates recognised that the normal to the third plane was perpendicular to the direction vector of their line of intersection found in part (c) and they used the scalar product to find the value of *a*. These candidates usually went on to use a point on the line of intersection to find *b*.

Question 6

In part (a), a number of candidates failed to score any marks here, with the majority failing to set up the required model in terms of "*k*". There were some descriptive attempts and some candidates attempted to write an equation for *V* rather than for $\frac{\mathrm{d}V}{\mathrm{d}t}$. The link between the word "rate" and $\frac{\mathrm{d}V}{\mathrm{d}t}$ seemed to be poorly understood. Of those that set up the correct equation the majority scored full marks. A small number used $\frac{\mathrm{d}V}{\mathrm{d}t} = +3.$ Others attempted this part by starting with the answer given and substituting in the given values to show that they worked. These verification methods gained no marks. Some candidates just ignored this part, even if they scored marks in other parts.

Part (b) saw more success although some candidates did not attempt this part. The majority of attempts were a success, but a minority failed to use the chain rule to obtain $e^{0.4t}$ in the numerator. In general, the coefficient was correct. Incorrect squaring of $e^{0.4t}$ to give $e^{0.8t^2}$ was seen occasionally.

In part (c), a pleasing majority rearranged the differential equation and recognised the technique required and established the correct integrating factor. Most went on to obtain the first method mark, but some lost this, either as they stopped before integrating the LHS or failed to multiply the RHS by the integrating factor. Of those who did, most scored the second method mark, almost always for integrating the first term, though a noticeable minority had an incorrect coefficient, usually because they multiplied by the 0.4. Success with the second term was more mixed, even when part (b) had been answered correctly - the coefficient being a problem for many. Some candidates who failed to differentiate $\arctan e^{0.4t}$, did integrate successfully, occasionally using a substitution. Some candidates lost the final two marks as they did not include the constant of integration or did not deal with it correctly when dividing by $e^{0.4t}$. A few did not give an exact value for c, losing the last mark. Some did not use the correct initial conditions when trying to find a value for c, and also lost the last two marks.

In part (d), although the correct value for V when t = 10 was rarely seen, most candidate who reached an answer for part (c) attempted this and made an appropriate comment. A minority of responses clearly substituted the wrong value for t, usually 8. For a small number of candidates, whose model gave negative value, rather than checking their working, the conclusion appeared to be that it wasn't a good model. Some forgot to make a comment evaluating the model.

Question 7

The "Explain why" in part (a) led many candidates to very wordy and unclear answers. Those that seemed to see it more as a "show that" question often scored the method mark and some had a convincing argument to demonstrate the equivalence of both sides of the equation. There was generally an idea about odd terms being negative and evens positive, but this was not always described well enough for the mark with 'alternating' used quite a lot. Those that kept it simple by listing, then grouping and showing RHS = LHS fared best. Very few managed to also justify why the upper limit of the summation needed to be change to n.

Part (b) stretched both algebraic skills and problem solving with the requirement to spot that $(-1)^{2r}$ must be even, proving beyond a substantial number of candidates. Quite a few chose to ignore it in their solution, whilst others took it out as a factor from two of the terms resulting from multiplying out the quadratic. Some attempted to use the result from part (a) on $(-1)^{2r}$ and ended up with $\Sigma(2r - (2r - 1))$ leading to $\Sigma 1 = n$. Such errors meant that they could often only score a maximum of two marks in this part. It was also not uncommon for candidates to only multiply some of their terms in the quadratic by r. However, mistakes multiplying through by r didn't always preclude candidates from achieving some of the marks, depending on where their errors were. For those who progressed further, applying the result from part (a) was sometimes missed for the $4r(-1)^r$ term with work leading to cubic expansions being a common error here. Missing the change from upper limit of $2n$ to n also tripped up a few candidates, so the mark for applying the correct formulae for sum of integers and sum of cubes was often not awarded. These sources of errors and misconceptions led to some very low scoring responses to this part. However, those candidates that were not put off by the $(-1)^r$ were usually successful in applying $(-1)^{2r} = 1$ and using the result from part (a) to achieve the first 4 marks. There were errors seen, both when candidates kept their terms factorised as much as possible as well as when they multiplied everything out (to leave *n* multiplied by a cubic expression in *n*). Some clear, precise and correct solutions were seen, along with some with many lines of working.

Part (c) was very rarely answered completely correctly. The majority of candidates successfully split the sum to a difference of upper limit 50 and upper limit 13, thus achieving the first method mark. However, very few spotted the need for the upper limits to be even in order to apply the result from part (b). Of the very few that did, it was equally common to use 12 or 14 and make the appropriate adjustment, leading invariably to full marks for the question. Many candidates used $n = 25$ for their upper limit of 50 and $n = 6.5$ for their upper limit of 13, but many also tried using their upper limits instead. It was clear that several candidates had used their calculator to find the answer and so having missed out on the 2nd method mark, they had a subtraction sum that did not equal the correct answer they gave.

Question 8

This question was not attempted by all students, possibly due to the time constraints of the paper. The different parts generally proved accessible, even if the preceding parts had not been attempted. It seemed that many candidates were running out of time and were attempting the parts that they could quickly gain some credit for. A wide variety of marks was seen.

Many candidates either did not answer part (a) or gave an incorrect reason (commonly that the age ranges for each category were uneven, claims that the categories overlapped, or it was difficult to determine the exact age of the mammal). Good correct answers tended to be that some mammals would become breeders before 3 months, or would become infertile after a length of time.

For those who attempted part (b), this part was often done well and full marks awarded. However, a significant number of candidates failed to set up the initial matrix and apply it twice to form an equation for B_0. Others only applied the matrix once and were only able to score the first mark. Some wrote equations in N_0, J_0 and B_0 before finally realising that N_0 and J_0 were 0 which enabled them to proceed and solve for *b* and then *a*. Some candidates extracted the information required and formed a set of equations without writing down the matrix itself and this was an acceptable approach. Those who correctly obtained the value of 25 for B_0 usually proceeded to correctly show the value of *a* to be 0.8.

Most candidates who attempted part (c) made a good attempt at finding the inverse matrix, although some made small arithmetical slips or sign errors, either in calculating the value of the determinant, or with one of the elements in their inverse. No doubt time pressure contributed to their accuracy in some cases.

Using the inverse matrix and forming and solving an equation for the total number of mammals was the most common approach seen in part (d). This proved quite time consuming, but a significant number of candidates presented fully correct solutions. Working forwards using the original model, then solving the resulting equations was also commonly seen. Some of those with an incorrect inverse were able to score full marks in this question because they used the latter method. For the matrix approach, some multiplied the inverse by the values but never summed the resulting values so did not gain any marks.

Very few candidates attempted part (e) and while some realised that it was necessary to create two new categories, they then failed to use this to form a 4×4 matrix system. Those candidates who did form a

4×4 matrix were generally completely correct in their working. Several candidates tried to write about what they would do, but offered no actual matrix so gained no marks.

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