Pearson
Edexcel

A Level Further Mathematics

Specification

Pearson Edexcel Level 3 Advanced GCE in Further Mathematics (9FM0)

First teaching from September 2017

First certification from 2019

Issue 3

Summary of Pearson Edexcel Level 3 Advanced GCE in Further Mathematics Specification Issue 3 changes

Summary of changes made between previous issue and this current issue	Page number/s
Content and assessment overview - information about choosing optional papers added in	4
The tables showing the assessment overview for the optional papers has been updated.	5
Paper 1 and Paper 2: Core Pure Mathematics, Section 2.1 – correction to the guidance for deducing roots for cubics.	9
Paper 4A: Further Pure Mathematics 2, Section 2.1 – $\mathrm{d}y$ in the integral changed to $\mathrm{d}x$	23
Paper 3B: Further Statistics 1, Section 4.1 – changed to bold text	27
Student malpractice – How to report candidate malpractice updated	56
Staff/centre malpractice – How to report staff/centre malpractice updated	56
Awarding and reporting – information added about taking more than two optional papers.	56
Appendix 9: Entry codes for optional routes – information about permitted entry routes and entry codes updated.	85

Earlier issues show previous changes.

If you need further information on these changes or what they mean, contact us via our website at: qualifications.pearson.com/en/support/contact-us.html.

Contents

1 Introduction

Why choose Edexcel A Level Further Mathematics?

We have listened to feedback from all parts of the mathematics subject community, including higher education. We have used this opportunity of curriculum change to redesign a qualification that reflects the demands of a wide variety of end users as well as retaining many of the features that have contributed to the increasing popularity of GCE Mathematics in recent years.

We will provide:

- **Simple, intuitive specifications** that enable co-teaching and parallel delivery. Increased pressure on teaching time means that it's important you can cover the content of different specifications together. Our specifications are designed to help you co-teach A and AS Level, as well as deliver Maths and Further Maths in parallel.
- **Clear, familiar, accessible exams with specified content in each paper**. Our new exam papers will deliver everything you'd expect from us as the leading awarding body for maths. They'll take the most straightforward and logical approach to meet the government's requirements. You and your students will know which topics are covered in each paper so there are no surprises. They'll use the same clear design that you've told us makes them so accessible, while also ensuring a range of challenge for all abilities.
- **A wide range of exam practice** to fully prepare students and help you track progress. With the new linear exams your students will want to feel fully prepared and know how they're progressing. We'll provide lots of exam practice to help you and your students understand and prepare for the assessments, including secure mock papers, practice papers and free topic tests with marking guidance.
- **Complete support and free materials** to help you understand and deliver the specification. Change is easier with the right support, so we'll be on-hand to listen and give advice on how to understand and implement the changes. Whether it's through our Launch, Getting Ready to Teach, and Collaborative Networks events or via the renowned Maths Emporium; we'll be available face to face, online or over the phone throughout the lifetime of the qualification. We'll also provide you with free materials like schemes of work, topic tests and progression maps.
- **The published resources you know and trust**, fully updated for 2017. Our new A Level Maths and Further Maths textbooks retain all the features you know and love about the current series, whilst being fully updated to match the new specifications. Each textbook comes packed with additional online content that supports independent learning, and they all tie in with the free qualification support, giving you the most coherent approach to teaching and learning.

Supporting you in planning and implementing this qualification

Planning

- Our **Getting Started** guide gives you an overview of the new A Level qualification to help you to get to grips with the changes to content and assessment as well as helping you understand what these changes mean for you and your students.
- We will give you a **course planner** and **scheme of work** that you can adapt to suit your department.
- **Our mapping documents** highlight the content changes between the legacy modular specification and the new linear specifications.

Teaching and learning

There will be lots of free teaching and learning support to help you deliver the new qualifications, including:

- topic guides covering new content areas
- teaching support for problem solving, modelling and the large data set
- student guide containing information about the course to inform your students and their parents.

Preparing for exams

We will also provide a range of resources to help you prepare your students for the assessments, including:

- specimen papers written by our senior examiner team
- practice papers made up from past exam questions that meet the new criteria
- secure mock papers
- marked exemplars of student work with examiner commentaries.

ResultsPlus and Exam Wizard

ResultsPlus provides the most detailed analysis available of your students' exam performance. It can help you identify the topics and skills where further learning would benefit your students.

Exam Wizard is a data bank of past exam questions (and sample paper and specimen paper questions) allowing you to create bespoke test papers.

Get help and support

Mathematics Emporium — Support whenever you need it

The renowned Mathematics Emporium helps you keep up to date with all areas of maths throughout the year, as well as offering a rich source of past questions, and of course access to our in-house Pearson Edexcel Maths team.

Sign up to get Emporium emails

Get updates on the latest news, support resources, training and alerts for entry deadlines and key dates direct to your inbox. Just email mathsemporium@pearson.com to sign up

Emporium website

Over 12 000 documents relating to past and present Pearson/Edexcel Mathematics qualifications available free. Visit www.edexcelmaths.com/ to register for an account.

Qualification at a glance

Content and assessment overview

This Pearson Edexcel Level 3 Advanced GCE in Further Mathematics builds on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics and the subject content for the Pearson Edexcel Level 3 Advanced Subsidiary and Advanced GCE Mathematics qualifications. Assessments will be designed to reward students for demonstrating the ability to provide responses that draw together different areas of their knowledge, skills and understanding from across the full course of study for the AS further mathematics qualification and also from across the AS Mathematics qualification. Problem solving, proof and mathematical modelling will be assessed in further mathematics in the context of the wider knowledge which students taking A level further mathematics will have studied.

The Pearson Edexcel Level 3 Advanced GCE in Further Mathematics consists of four externally-examined papers. Students must take Paper 1 and Paper 2, the two mandatory Core Pure papers, and two optional papers. Students are permitted to take more than the two optional papers if they want to extend their course of study. See page 56 for details of how their grade will be awarded.

Students must complete all assessments in May/June in any single year.

Paper 1: Core Pure Mathematics 1 (*Paper code: 9FM0/01) **Paper 2: Core Pure Mathematics 2 (*Paper code: 9FM0/02)**
Each paper is: ***1 hour and 30 minutes written examination*** ***25% of the qualification*** ***75 marks***
Content overview Proof, Complex numbers, Matrices, Further algebra and functions, Further calculus, Further vectors, Polar coordinates, Hyperbolic functions, Differential equations
Assessment overview • Paper 1 and Paper 2 may contain questions on any topics from the Pure Mathematics content. • Students must answer all questions. • Calculators can be used in the assessment.

Further Mathematics Optional Papers (*Paper codes: 9FM0/3A-3D, 9FM0/4A-4D)	
Each paper is: ***Written examination: 1 hour and 30 minutes*** ***25% of the qualification*** ***75 marks***	
Content overview Students take **two** options from the following eight:	
Option 1 Papers	**Option 2 Papers**
3A: Further Pure Mathematics 1	4A: Further Pure Mathematics 2
3B: Further Statistics 1	4B: Further Statistics 2
3C: Further Mechanics 1	4C: Further Mechanics 2
3D: Decision Mathematics 1	4D: Decision Mathematics 2
There are restrictions on which papers can be taken together. Students choose a pair of options, either: • any two Option 1 papers, or • a matching pair of Option 1 and Option 2 papers This makes a total of ten different option pairs.	
Assessment overview • Students must answer all questions. • Calculators can be used in the assessment.	

*See *Appendix 8: Codes* for a description of this code and all other codes relevant to this qualification.

**There will be restrictions on which papers can be taken together, see page 83, *Appendix 9*.

2 Subject content and assessment information

Qualification aims and objectives

The aims and objectives of this qualification are to enable students to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development

Overarching themes

The overarching themes should be applied along with associated mathematical thinking and understanding, across the whole of the detailed content in this specification.

These overarching themes are inherent throughout the content and students are required to develop skills in working scientifically over the course of this qualification. The skills show teachers which skills need to be included as part of the learning and assessment of the students.

Overarching theme 1: Mathematical argument, language and proof

A Level Mathematics students must use the mathematical notation set out in the booklet *Mathematical Formulae and Statistical Tables* and be able to recall the mathematical formulae and identities set out in *Appendix 1*.

	Knowledge/Skill
OT1.1	Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable
OT1.2	Understand and use mathematical language and syntax as set out in the glossary
OT1.3	Understand and use language and symbols associated with set theory, as set out in the glossary
OT1.4	Understand and use the definition of a function; domain and range of functions
OT1.5	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

Overarching theme 2: Mathematical problem solving

	Knowledge/Skill
OT2.1	Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved
OT2.2	Construct extended arguments to solve problems presented in an unstructured form, including problems in context
OT2.3	Interpret and communicate solutions in the context of the original problem
OT2.6	Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle
OT2.7	Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems

Overarching theme 3: Mathematical modelling

	Knowledge/Skill
OT3.1	Translate a situation in context into a mathematical model, making simplifying assumptions
OT3.2	Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)
OT3.3	Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)
OT3.4	Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate]
OT3.5	Understand and use modelling assumptions

Paper 1 and Paper 2: Core Pure Mathematics

To support the co-teaching of this qualification with the AS Mathematics qualification, common content has been highlighted in bold.

<table>
<tr><th rowspan="2">Topic</th><th colspan="3">What students need to learn:</th></tr>
<tr><th colspan="2">Content</th><th>Guidance</th></tr>
<tr><td>1
Proof</td><td>1.1</td><td>Construct proofs using mathematical induction.
Contexts include sums of series, divisibility and powers of matrices.</td><td>To include induction proofs for
(i) summation of series
e.g. show $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$
or
show $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$
(ii) divisibility e.g. show $3^{2n}+11$ is divisible by 4
(iii) matrix products e.g. show
$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix}$</td></tr>
<tr><td rowspan="2">2
Complex numbers</td><td>2.1</td><td>Solve any quadratic equation with real coefficients.
Solve cubic or quartic equations with real coefficients.</td><td>Given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics, for example:
(i) $\mathrm{f}(z) = 2z^3 + 5z^2 + 7z + 10$
Given that $z+2$ is a factor of $\mathrm{f}(z)$, use algebra to solve $\mathrm{f}(z) = 0$ completely.
(ii) $\mathrm{g}(x) = x^4 - x^3 + 6x^2 + 14x - 20$
Given $\mathrm{g}(1) = 0$ and $\mathrm{g}(-2) = 0$, use algebra to solve $\mathrm{g}(x) = 0$ completely.</td></tr>
<tr><td>2.2</td><td>Add, subtract, multiply and divide complex numbers in the form $x+\mathrm{i}y$ with x and y real.
Understand and use the terms ‘real part’ and ‘imaginary part’.</td><td>Students should know the meaning of the terms, ‘modulus’ and ‘argument’.</td></tr>
</table>

Topic	What students need to learn:		
	Content		Guidance
2 **Complex numbers** *continued*	2.3	**Understand and use the complex conjugate.** **Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.**	**Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then $z_1{}^*$ is also a root.**
	2.4	**Use and interpret Argand diagrams.**	**Students should be able to represent the sum or difference of two complex numbers on an Argand diagram.**
	2.5	**Convert between the Cartesian form and the modulus-argument form of a complex number.**	Knowledge of radians is assumed.
	2.6	**Multiply and divide complex numbers in modulus argument form.**	**Knowledge of the results** $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$, $\left\lvert \frac{z_1}{z_2} \right\rvert = \frac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ Knowledge of radians and compound angle formulae is assumed.
	2.7	**Construct and interpret simple loci in the argand diagram such as $\lvert z-a \rvert > r$ and $\arg(z-a) = \theta$.**	**To include loci such as $\lvert z-a \rvert = b$, $\lvert z-a \rvert = \lvert z-b \rvert$, $\arg(z-a) = \beta$, and regions such as $\lvert z-a \rvert \le \lvert z-b \rvert$, $\lvert z-a \rvert \le b$, $\alpha < \arg(z-a) < \beta$** Knowledge of radians is assumed.

Topic	What students need to learn:		
	Content		Guidance
2 **Complex numbers** *continued*	2.8	Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.	To include using the results, $z + \frac{1}{z} = 2\cos\theta$ and $z - \frac{1}{z} = 2\mathrm{i}\sin\theta$ to find $\cos p\theta$, $\sin q\theta$ and $\tan r\theta$ in terms of powers of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and powers of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of multiple angles. For sums of series, students should be able to show that, for example, $1 + z + z^2 + \ldots + z^{n-1} = 1 + \mathrm{i}\cot\left(\frac{\pi}{2n}\right)$ where $z = \cos\left(\frac{\pi}{n}\right) + \mathrm{i}\sin\left(\frac{\pi}{n}\right)$ and n is a positive integer.
	2.9	Know and use the definition $\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = r\mathrm{e}^{\mathrm{i}\theta}$	Students should be familiar with $\cos\theta = \frac{1}{2}(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta})$ and $\sin\theta = \frac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta})$
	2.10	Find the n distinct nth roots of $r\mathrm{e}^{\mathrm{i}\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon in the Argand diagram.	
	2.11	Use complex roots of unity to solve geometric problems.	
3 **Matrices**	3.1	**Add, subtract and multiply conformable matrices.** **Multiply a matrix by a scalar.**	
	3.2	**Understand and use zero and identity matrices.**	

Topic	What students need to learn:		
	Content		**Guidance**
3 **Matrices** *continued*	3.3	**Use matrices to represent linear transformations in 2-D.** **Successive transformations.** **Single transformations in 3-D.**	**For 2-D, identification and use of the matrix representation of single and combined transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation through any angle about $(0, 0)$, stretches parallel to the x-axis and y-axis, and enlargement about centre $(0, 0)$, with scale factor k, $(k \neq 0)$, where $k \in \mathbb{R}$.** **Knowledge that the transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$.** **3-D transformations confined to reflection in one of $x = 0, y = 0, z = 0$ or rotation about one of the coordinate axes.** **Knowledge of 3-D vectors is assumed.**
	3.4	**Find invariant points and lines for a linear transformation.**	**For a given transformation, students should be able to find the coordinates of invariant points and the equations of invariant lines.**
	3.5	**Calculate determinants of** **2 x 2 and 3 x 3 matrices and interpret as scale factors, including the effect on orientation.**	**Idea of the determinant as an area scale factor in transformations.**
	3.6	**Understand and use singular and non-singular matrices.** **Properties of inverse matrices.** **Calculate and use the inverse of non-singular 2 x 2 matrices and 3 x 3 matrices.**	**Understanding the process of finding the inverse of a matrix is required.** **Students should be able to use a calculator to calculate the inverse of a matrix.**

Topic	What students need to learn:		
	Content		Guidance
3 **Matrices** *continued*	3.7	**Solve three linear simultaneous equations in three variables by use of the inverse matrix.**	
	3.8	**Interpret geometrically the solution and failure of solution of three simultaneous linear equations.**	**Students should be aware of the different possible geometrical configurations of three planes, including cases where the planes,** **(i) meet in a point** **(ii) form a sheaf** **(iii) form a prism or are otherwise inconsistent**
4 **Further algebra and functions**	4.1	**Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.**	**For example, given a cubic polynomial equation with roots α, β and γ students should be able to evaluate expressions such as,** **(i)** $\alpha^2+\beta^2+\gamma^2$ **(ii)** $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ **(iii)** $(3+\alpha)(3+\beta)(3+\gamma)$ **(iv)** $\alpha^3+\beta^3+\gamma^3$
	4.2	**Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).**	
	4.3	**Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series.**	**For example, students should be able to sum series such as** $\sum_{r+1}^{n} r^2(r+2)$

Topic	What students need to learn:		
	Content		**Guidance**
4 **Further algebra and functions** *continued*	4.4	Understand and use the method of differences for summation of series including use of partial fractions.	Students should be able to sum series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ by using partial fractions such as $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$
	4.5	Find the Maclaurin series of a function including the general term.	
	4.6	Recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$, and be aware of the range of values of x for which they are valid (proof not required).	To include the derivation of the series expansions of compound functions.
5 **Further calculus**	5.1	**Derive formulae for and calculate volumes of revolution.**	**Both** $\pi \int y^2 \, dx$ **and** $\pi \int x^2 \, dy$ **are required.** Students should be able to find a volume of revolution given either Cartesian equations or parametric equations.
	5.2	Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.	For example, $\int_0^{\infty} e^{-x} \, dx, \int_0^2 \frac{1}{\sqrt{x}} \, dx$
	5.3	Understand and evaluate the mean value of a function.	Students should be familiar with the mean value of a function f(x) as, $\frac{1}{b-a} \int_a^b f(x) \, dx$
	5.4	Integrate using partial fractions.	Extend to quadratic factors $ax^2 + c$ in the denominator

Topic	What students need to learn:		
	Content		Guidance
5 **Further calculus** *continued*	5.5	Differentiate inverse trigonometric functions.	For example, students should be able to differentiate expressions such as, $\arcsin x + x\sqrt{(1-x^2)}$ and $\frac{1}{2}\arctan x^2$
	5.6	Integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 - x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions.	
6 **Further vectors**	6.1	**Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.**	**The forms, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and** $\dfrac{x-a_1}{b_1} = \dfrac{x-a_2}{b_2} = \dfrac{x-a_3}{b_3}$ **Find the point of intersection of two straight lines given in vector form.** **Students should be familiar with the concept of skew lines and parallel lines.**
	6.2	**Understand and use the vector and Cartesian forms of the equation of a plane.**	**The forms** **$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and $\mathbf{a}x + \mathbf{b}y + \mathbf{c}z = d$**
	6.3	**Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.**	$\mathbf{a}.\mathbf{b} = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$ **The form $\mathbf{r}.\mathbf{n} = k$ for a plane.**
	6.4	**Check whether vectors are perpendicular by using the scalar product.**	**Knowledge of the property that $\mathbf{a}.\mathbf{b} = 0$ if the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.**

Topic	What students need to learn: Content		Guidance
6 **Further vectors** *continued*	6.5	**Find the intersection of a line and a plane.** **Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.**	**The perpendicular distance from** (α, β, γ) **to** $n_1x + n_2y + n_3z + d = 0$ **is** $\dfrac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d\rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$
7 **Polar coordinates**	7.1	Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.	
	7.2	Sketch curves with r given as a function of θ, including use of trigonometric functions.	The sketching of curves such as $r = p \sec(\alpha - \theta)$, $r = a$, $r = 2a \cos\theta$, $r = k\theta$, $r = a(1 \pm \cos\theta)$, $r = a(3 + 2\cos\theta)$, $r = a \cos 2\theta$ and $r^2 = a^2 \cos 2\theta$ may be set.
	7.3	Find the area enclosed by a polar curve.	Use of the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2\,\mathrm{d}\theta$ for area. The ability to find tangents parallel to, or at right angles to, the initial line is expected.
8 **Hyperbolic functions**	8.1	Understand the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs.	For example, $\cosh x = \frac{1}{2}\left(\mathrm{e}^x + \mathrm{e}^{-x}\right)$
	8.2	Differentiate and integrate hyperbolic functions.	For example, differentiate $\tanh 3x$, $x \sinh^2 x$, $\dfrac{\cosh 2x}{\sqrt{(x+1)}}$

Topic	What students need to learn:		
	Content		Guidance
8 **Hyperbolic functions** *continued*	8.3	Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.	$\text{arsinh}x = \ln\left[x+\sqrt{x^2+1}\right]$ $\text{arcosh}x = \ln\left[x+\sqrt{x^2-1}\right],\ x \geqslant 1$ $\text{artanh}x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right],\ -1 < x < 1$
	8.4	Derive and use the logarithmic forms of the inverse hyperbolic functions.	
	8.5	Integrate functions of the form $\left(x^2+a^2\right)^{-\frac{1}{2}}$ and $\left(x^2-a^2\right)^{-\frac{1}{2}}$ and be able to choose substitutions to integrate associated functions.	
9 **Differential equations**	9.1	Find and use an integrating factor to solve differential equations of form $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$ and recognise when it is appropriate to do so.	The integrating factor $\mathrm{e}^{\int P(x)\,\mathrm{d}x}$ may be quoted without proof.
	9.2	Find both general and particular solutions to differential equations.	Students will be expected to sketch members of the family of solution curves.
	9.3	Use differential equations in modelling in kinematics and in other contexts.	
	9.4	Solve differential equations of form $y'' + ay' + by = 0$ where a and b are constants by using the auxiliary equation.	

Topic	What students need to learn:		
	Content		Guidance
9 **Differential equations** ***continued***	9.5	Solve differential equations of form $y''+ay'+by=\mathrm{f}(x)$ where a and b are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $\mathrm{f}(x)$ is a polynomial, exponential or trigonometric function).	$\mathrm{f}(x)$ will have one of the forms $k\mathrm{e}^{px}$, $A + Bx$, $p + qx + cx^2$ or $m \cos \omega x + n \sin \omega x$
	9.6	Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation.	
	9.7	Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion.	
	9.8	Model damped oscillations using second order differential equations and interpret their solutions.	Damped harmonic motion, with resistance varying as the derivative of the displacement, is expected. Problems may be set on forced vibration.

Topic	What students need to learn:		
	Content		Guidance
9 **Differential equations** *continued*	9.9	Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.	Restricted to coupled first order linear equations of the form, $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by + \mathrm{f}(t)$ $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy + \mathrm{g}(t)$

Assessment information

- First assessment: May/June 2019.
- The assessment is 1 hour 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators can be used in the assessment.
- The booklet '*Mathematical Formulae and Statistical* Tables' will be provided for use in the assessment.

Synoptic assessment

Synoptic assessment requires students to work across different parts of a qualification and to show their accumulated knowledge and understanding of a topic or subject area.

Synoptic assessment enables students to show their ability to combine their skills, knowledge and understanding with breadth and depth of the subject.

This paper assesses synopticity.

Sample assessment materials

A sample paper and mark scheme for this paper can be found in the *Pearson Edexcel Level 3 Advanced GCE in Further Mathematics Sample Assessment Materials (SAMs)* document.

Paper 3 and Paper 4: Further Mathematics Options

Paper 3A: Further Pure Mathematics 1

Topics	What students need to learn:		
	Content		Guidance
1 **Further Trigonometry**	1.1	**The *t*-formulae**	**The derivation and use of** $\sin\theta \equiv \dfrac{2t}{1+t^2}$, $\cos\theta \equiv \dfrac{1-t}{1+t^2}$, $\tan\theta \equiv \dfrac{2t}{1-t^2}$, **where** $t = \tan\dfrac{\theta}{2}$
	1.2	**Applications of *t*-formulae to trigonometric identities**	**E.g. show that** $\dfrac{1+\operatorname{cosec}\theta}{\cot\theta} \equiv \dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}$
	1.3	**Applications of *t*-formulae to solve trigonometric equations**	**E.g. the solution of equations of the form** $a\cos x + b\sin x = c$
2 **Further calculus**	2.1	Derivation and use of Taylor series.	The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x-\pi)$ up to and including the term in $(x-\pi)^3$.
	2.2	Use of series expansions to find limits.	E.g. $\lim\limits_{x\to 0}\dfrac{x-\arctan x}{x^3}$, $\lim\limits_{x\to 0}\dfrac{e^{2x^2}-1}{x^2}$
	2.3	Leibnitz's theorem.	Leibnitz's theorem for differentiating products.
	2.4	L'Hospital's Rule.	The use of derivatives to evaluate limits of indeterminate forms. Repeated applications and/or substitutions may be required. E.g. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{x-\sin x}$, $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x$

Topics	What students need to learn:		
	Content		Guidance
2 **Further calculus** *continued*	2.5	The Weierstrass substitution for integration.	The use of tangent half angle substitutions to find definite and indefinite integrals. E.g. $\int \operatorname{cosec} x \, \mathrm{d}x$, $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{1+\sin x-\cos x} \mathrm{d}x$ using $t = \tan \frac{x}{2}$.
3 **Further differential equations**	3.1	Use of Taylor series method for series solution of differential equations.	For example, derivation of the series solution in powers of x, as far as the term in x^4, of the differential equation $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$, where $y = 1, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 0$
	3.2	Differential equations reducible by means of a given substitution.	Differential equations reducible to the types as specified in section 9 of the A level Further Pure Mathematics content for papers 1 and 2.
4 **Coordinate systems**	4.1	**Cartesian and parametric equations for the parabola and rectangular hyperbola**, ellipse and hyperbola.	Students should be familiar with the equations: $y^2 = 4ax$; $x = at^2$, $y = 2at$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $x = a$ x$\cos t$, $y = b \sin t$. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $x = a \sec t$, $y = b \tan t$, $x = a \cosh t, y = b \sinh t$
	4.2	**The focus-directrix properties of the parabola**, ellipse and hyperbola, including the eccentricity.	For example, students should know that, for the ellipse $b^2 = a^2(1-e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\frac{a}{e}$ and $x = -\frac{a}{e}$
	4.3	**Tangents and normals to these curves.**	**The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.** Students are expected to be able to use algebraic differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for these curves.
	4.4	**Loci problems.**	

Topics	What students need to learn:		
	Content		**Guidance**
5 **Further vectors**	5.1	**The vector product $\mathbf{a} \times \mathbf{b}$ of two vectors.**	To include the interpretation of $\lvert \mathbf{a} \times \mathbf{b} \rvert$ as an area.
	5.2	**The scalar triple product $\mathbf{a}.\mathbf{b} \times \mathbf{c}$**	**Students should be able to use the scalar triple product to find the volume of a tetrahedron and a parallelepiped.**
	5.3	Applications of vectors to three dimensional geometry involving points, lines and planes.	To include the equation of a line in the form $(\mathbf{r} - \mathbf{b}) \times \mathbf{b} = \mathbf{0}$. Direction ratios and direction cosines of a line.
6 **Further numerical methods**	6.1	**Numerical solution of first order and second order differential equations.**	**The approximations** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{(y_1 - y_0)}{h}$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{(y_{-1} - y_1)}{2h}$ $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \frac{(y_1 - 2y_0 + y_{-1})}{h^2}$
	6.2	Simpson's rule.	
7 **Inequalities**	7.1	**The manipulation and solution of algebraic inequalities and inequations**, including those involving the modulus sign.	**The solution of inequalities such as** $\frac{1}{x-a} > \frac{x}{x-b}$, $\lvert x^2 - 1 \rvert > 2(x+1)$

Paper 4A: Further Pure Mathematics 2

Topics	What students need to learn:		
		Content	Guidance
1 **Groups**	1.1	**The Axioms of a group.**	**The terms 'binary operation, closure, associativity, identity and inverse'.**
	1.2	**Examples of groups.** **Cayley tables.** **Cyclic groups.**	**For example, symmetries of geometrical figures, non-singular matrices, integers modulo *n* with operation addition, and/or multiplication permutation groups.**
	1.3	**The order of a group and the order of an element.** **Subgroups.**	
	1.4	**Lagrange's theorem.**	
	1.5	Isomorphism.	Isomorphisms will be restricted to groups that have a maximum order of 8.
2 **Further calculus**	2.1	Further Integration — Reduction formulae.	Students should be able to derive formulae such as: $nI_n = (n-1)I_{n-2}, n \geq 2$, for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$, $I_{n+2} = \frac{2\sin(n+1)x}{n+1} + I_n$ for $I_n = \left\{\int \frac{\sin nx}{\sin x} dx\right\}$ [CP1]
	2.2	The calculation of arc length and the area of a surface of revolution.	The equation of the curve may be given in Cartesian, parametric or polar form.
3 **Further matrix algebra**	3.1	**Eigenvalues and eigenvectors of 2 × 2** and 3 × 3 **matrices.**	**Understand the term *characteristic equation* for a 2 × 2 matrix.** **Repeated eigenvalues and complex eigenvalues.** **Normalised vectors may be required.**

Topics	What students need to learn:		
		Content	Guidance
3 **Further matrix algebra** *continued*	3.2	**Reduction of matrices to diagonal form.**	**Students should be able to find a matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{AP}$ is diagonal** **Symmetric matrices and orthogonal diagonalisation.**
	3.3	**The use of the Cayley-Hamilton theorem.**	**Students should understand and be able to use the fact that, every 2×2 or 3×3 matrix satisfies its own characteristic equation.**
4 **Further complex numbers**	4.1	**Further loci and regions in the Argand diagram.**	**To include loci such as** $\lvert z - a \rvert = k \lvert z - b \rvert$, $\arg \frac{z-a}{z-b} = \beta$ **and** **regions such as** $\alpha \leq \arg(z - z_1) \leq \beta$ **and** $p \leq \mathrm{Re}(z) \leq q$
	4.2	Elementary transformations from the z-plane to the w-plane.	Transformations such as $w = z^2$ and $w = \frac{az+b}{cz+d}$, where $a, b, c, d \in \mathbb{C}$ may be set.
5 **Number theory**	5.1	**An understanding of the division theorem and its application to the Euclidean Algorithm and congruences.**	**Students should be able to apply the algorithm to find the highest common factor of two numbers.**
	5.2	**Bezout's identity.**	**Students should be able to use back substitution to identity the Bezout's identity for two numbers.**
	5.3	**Modular arithmetic. Understanding what is meant by two integers a and b to be congruent modulo an integer n. Properties of congruences.**	**The notation $a \equiv b \pmod{n}$ is expected.** **Knowledge of the following properties:** $a \equiv a \pmod{n}$ **if** $a \equiv b \pmod{n}$ **then** $b \equiv a \pmod{n}$ **if** $a \equiv b \pmod{n}$ **and** $b \equiv c \pmod{n}$ **then** $a \equiv c \pmod{n}$ **Addition and subtraction laws for congruences.** **Multiplication and power laws.**
	5.4	Fermat's Little Theorem.	For example, students should be able to find the least positive residue of 4^{20} modulo 7 Proof is not required.

Topics	What students need to learn:		
	Content		**Guidance**
5 **Number theory** *continued*	5.5	**Divisibility Tests.**	**For divisibility by $2, 3, 4, 5, 6, 9, 10$ and 11.**
	5.6	Solution of congruence equations.	Conditions under which solutions exist should be known. Use of Bezout's identity to find multiplicative inverses.
	5.7	Combinatorics: counting problems, permutations and combinations.	The multiplicative principle of counting. Set notation is expected; the number of subsets of a set should be known. Addition and subtraction principles. Students should, for example, be able to determine the number of positive integers less than 1000 containing the digit 3 (at least once). Understanding both ${}^{n}\mathrm{P}_{r}$ and ${}^{n}\mathrm{C}_{r}$ and when to use them. For example, to determine how many ways there are to select a team of 11 players from a squad of 21 (a) regardless of playing positions and (b) if positions matter
6 **Further sequences and series**	6.1	**First** and second **order recurrence relations.**	**Equations of the form** $u_{n+1} + \mathrm{f}(n)u_n = \mathrm{g}(n)$ and $u_{n+2} + \mathrm{f}(n)u_{n+1} + \mathrm{g}(n)u_n = \mathrm{h}(n)$
	6.2	**The solution of recurrence relations to obtain closed forms.**	**Students should be able to solve relations such as:** $u_{n+1} - 5u_n = 8,\ u_1 = 1,$ $2u_{n+2} + 7u_{n+1} - 15u_n = 6,\ u_1 = 10,\ u_2 = -17$ **The terms, particular solution, complementary function and auxiliary equation should be known.** **Use of recurrence relations to model applications, e.g. population growth.**
	6.3	**Proof by induction of closed forms.**	**For example** **if $u_{n+1} - 3u_n = 4$ with $u_1 = 1$ prove by mathematical induction that $u_n = 3^n - 2$**

Paper 3B: Further Statistics 1*

Topic	What students need to learn:		
		Content	Guidance
1 **Discrete probability distributions**	1.1	**Calculation of the mean and variance of discrete probability distributions.** **Extension of expected value function to include $\mathrm{E}(\mathrm{g}(X))$**	**Use of $\mathrm{E}(X) = \mu = \sum x\mathrm{P}(X = x)$** **and** $\mathrm{Var}(X) = \sigma^2 = \sum x^2\mathrm{P}(X = x) - \mu^2$ **The formulae used to define $\mathrm{g}(x)$ will be consistent with the level required in A level Mathematics and A level Further Mathematics.** **Questions may require candidates to use these calculations to assess the suitability of models.**
2 **Poisson & binomial distributions**	2.1	**The Poisson distribution** **The additive property of Poisson distributions**	**Students will be expected to use this distribution to model a real-world situation and to comment critically on the appropriateness.** **Students will be expected to use their calculators to calculate probabilities including cumulative probabilities.** **Students will be expected to use the additive property of the Poisson distribution. E.g. if X = the number of events per minute and $X \sim \mathrm{Po}(\lambda)$, then the number of events per 5 minutes ~ $\mathrm{Po}(5\lambda)$.** **If X and Y are independent random variables with $X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(\mu)$, then $X + Y \sim \mathrm{Po}(\lambda + \mu)$** **No proofs are required.**
	2.2	**The mean and variance of the binomial distribution and the Poisson distribution.**	**Knowledge and use of :** **If $X \sim \mathrm{B}(n, p)$, then $\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1 - p)$** **If $Y \sim \mathrm{Po}(\lambda)$, then $\mathrm{E}(Y) = \lambda$ and $\mathrm{Var}(Y) = \lambda$** **Derivations are not required.**
	2.3	**The use of the Poisson distribution as an approximation to the binomial distribution.**	**When n is large and p is small the distribution $\mathrm{B}(n, p)$ can be approximated by $\mathrm{Po}(np)$.** **Derivations are not required.**

Topic	What students need to learn:		
	Content		Guidance
3 **Geometric and negative binomial distributions**	3.1	Geometric and negative binomial distributions.	Models leading to the distributions $\mathrm{p}(x) = p(1-p)^{x-1}, x = 1, 2 \ldots$ and $\mathrm{p}(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ $x = r, r+1, r+2, \ldots$
	3.2	Mean and variance of a geometric distribution with parameter *p.*	Use of $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$
	3.3	Mean and variance of negative binomial distribution with $\mathrm{P}(X = x)$ $= \binom{x-1}{r-1} p^r (1-p)^{(x-r)}$	Use of $\mu = \frac{r}{p}$ and $\sigma^2 = \frac{r(1-p)}{p^2}$
4 **Hypothesis Testing**	4.1	**Extend ideas of hypothesis tests to test for the mean of a Poisson distribution**	**Hypotheses should be stated in terms of a population parameter μ or λ**
	4.2	Extend hypothesis testing to test for the parameter p of a geometric distribution.	Hypotheses should be stated in terms of p
5 **Central Limit Theorem**	5.1	Applications of the Central Limit Theorem to other distributions.	For a population with mean μ and variance σ^2, for large n $\overline{X} \approx \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ Applications may involve any of the distributions in A level Mathematics or A level Further Statistics 1 No proofs required.

Topic	What students need to learn:		
	Content		Guidance
6 **Chi Squared Tests**	6.1	**Goodness of fit tests and Contingency Tables** **The null and alternative hypotheses.** **The use of** $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ **as an approximate** χ^2 **statistic.** **Degrees of freedom.**	**Applications to include the discrete uniform, binomial, Poisson** and geometric **distributions.** **Lengthy calculations will not be required.** **Students will be expected to determine the degrees of freedom when one or more parameters are estimated from the data. Cells should be combined when** $E_i < 5$ **Students will be expected to obtain** p-**values from their calculator or use tables to find critical values.**
7 **Probability generating functions**	7.1	Definitions, derivations and applications. Use of the probability generating function for the negative binomial, geometric, binomial and Poisson distributions.	
	7.2	Use to find the mean and variance.	Proofs of standard results may be required.
	7.3	Probability generating function of the sum of independent random variables.	$\mathrm{G}_{X+Y}(t) = \mathrm{G}_X(t) \times \mathrm{G}_Y(t)$ Derivation is not required.
8 **Quality of tests**	8.1	Type I and Type II errors. Size and Power of Test. The power function.	Calculation of the probability of a Type I or Type II error. Use of Type I and Type II errors and power function to indicate effectiveness of statistical tests. Questions will use any of the distributions in A level Mathematics or A level Further Statistics 1

*This paper is also the **Paper 4 option 4B** paper and will have the title '*Paper 4, Option 4B: Further Statistics 1*'. *Appendix 9*, '*Entry codes for optional routes*' shows how each optional route incorporates the optional papers.

Paper 4B: Further Statistics 2

Topics	What students need to learn:		
	Content		Guidance
1 **Linear Regression**	**1.1**	**Least squares linear regression. The concept of residuals and minimising the sum of squares of residuals.**	**Students should have an understanding of the process involved in linear regression.** **They should be able to calculate the regression coefficients for the equation y on x using standard formulae.**
	1.2	**Residuals.** **The residual sum of squares (RSS)**	**An intuitive use of residuals to check the reasonableness of linear fit and to find possible outliers.** **Use in refinement of mathematical models.** **The formula RSS =** $S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$ **Derivations are not required**
2 **Continuous probability distributions**	**2.1**	**The concept of a continuous random variable.** **The probability density function and the cumulative distribution function for a continuous random variable.**	**Students will be expected to link with their knowledge of histograms and frequency polygons.** **Use of the probability density function f(x), where** $\mathrm{P}(a < X \leqslant b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$ **Use of the cumulative distribution function** $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \int_{-\infty}^{x_0} \mathrm{f}(x)\,\mathrm{d}x$. The formulae used in defining $\mathrm{f}(x)$ and the calculus required will be consistent with the level expected in A level Mathematics and A level Further Mathematics.

Topics	What students need to learn:		
	Content		Guidance
2 **Continuous probability distributions** *continued*	**2.2**	**Relationship between probability density and cumulative distribution functions.**	$\mathrm{f}(x) = \dfrac{\mathrm{dF}(x)}{\mathrm{d}x}$.
	2.3	**Mean and variance of continuous random variables.** **Extension of expected value function to include $\mathrm{E}(\mathrm{g}(X))$** **Mode, median and percentiles of continuous random variables.** **Idea of skewness**	The formulae used to define $\mathrm{g}(x)$ will be consistent with the level required in A level Mathematics and A level Further Mathematics **Questions may require candidates to use these calculations to assess the suitability of models.** **Candidates will be expected to describe the skewness as positive, negative or zero and give a suitable justification.**
	2.4	**The continuous uniform (rectangular) distribution.**	**Including the derivation of the mean, variance and cumulative distribution function.**
3 **Correlation**	**3.1**	**Use of formulae to calculate the product moment correlation coefficient.** **Knowledge of the conditions for the use of the product moment correlation coefficient.** **A knowledge of the effects of coding will be expected.**	**Students will be expected to be able to use the formula to calculate the value of a coe fficient given summary statistics.**
	3.2	**Spearman's rank correlation coefficient, its use, interpretation.**	**Use of** $r_s = 1 - \dfrac{6\sum d^2}{n(n^2-1)}$ **Numerical questions involving ties may be set. An understanding of how to deal with ties will be expected.** **Students will be expected to calculate the resulting correlation coefficient on their calculator or using the formula.**

Topics	What students need to learn:		
	Content		Guidance
3 **Correlation** ***continued***	3.3	**Testing the hypothesis that a correlation is zero using either Spearman's rank correlation or the product moment correlation coefficient.**	**Hypotheses should be in terms of ρ or ρ_s and test a null hypothesis that ρ or $\rho_s = 0$.** **Use of tables for critical values of Spearman's and product moment correlation coefficients.** **Students will be expected to know that the critical values for the product moment correlation coefficient require that the data comes from a population having a bivariate normal distribution. Formal verification of this condition is not required.**
4 **Combinations of random variables**	4.1	Distribution of linear combinations of independent Normal random variables.	If $X \sim \mathrm{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^2)$ independently, then $aX \pm bY \sim \mathrm{N}(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$ No proofs required.
5 **Estimation, confidence intervals and tests using a normal distribution**	5.1	Concepts of standard error, estimator, bias. Quality of estimators	The sample mean, $\bar{x}$, and the sample variance, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, as unbiased estimates of the corresponding population parameters. Candidates will be expected to compare estimators and look for features such as unbiasedness and small variance.
	5.2	Concept of a confidence interval and its interpretation.	Link with hypothesis tests.
	5.3	Confidence limits for a Normal mean, with variance known.	Candidates will be expected to know how to apply the Normal distribution and use the standard error and obtain confidence intervals for the mean, rather than be concerned with any theoretical derivations.
	5.4	Hypothesis test for the difference between the means of two Normal distributions with variances known.	Use of $\dfrac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0, 1)$

Topics	What students need to learn:		
	Content		Guidance
5 **Estimation, confidence intervals and tests using a normal distribution** *continued*	5.5	Use of large sample results to extend to the case in which the population variances are unknown.	Use of Central Limit Theorem and use of $\dfrac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{S_x^2}{n_x} + \dfrac{S_y^2}{n_y}}} \approx \sim \text{N}(0,1)$
6 **Other Hypothesis Tests and confidence intervals**	6.1	Hypothesis test and confidence interval for the variance of a Normal distribution.	Use of $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ Candidates may use χ^2 –tables or their calculators to find critical values or p-values.
	6.2	Hypothesis test that two independent random samples are from Normal populations with equal variances.	Use of $\dfrac{S_1^{\,2}}{S_2^{\,2}} \sim F_{n_1-1,\, n_2-1}$ under H_0. Candidates may use tables of the *F*-distribution or their calculators to find critical values or p-values.
7 **Confidence intervals and tests using the *t* – distribution**	7.1	Hypothesis test and confidence interval for the mean of a Normal distribution with unknown variance.	Use of $\dfrac{\overline{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$ Candidates may use *t*-tables or their calculators to calculate critical or p-values.
	7.2	Paired *t*-test.	
	7.3	Hypothesis test and confidence interval for the difference between two means from independent Normal distributions when the variances are equal but unknown. Use of the pooled estimate of variance.	Use of *t*-distribution. $\dfrac{\overline{X} - \overline{Y} - (\mu_x - \mu_y)}{S\sqrt{\dfrac{1}{n_x} + \dfrac{1}{n_y}}} \sim t_{n_x + n_y - 2}$, under H_0. $s^2 = \dfrac{(n_1-1)s_1^{\,2} + (n_2-1)s_2^{\,2}}{n_1 + n_2 - 2}$

Paper 3C: Further Mechanics 1*

Topics	What students need to learn:		
	Content		Guidance
1 **Momentum and impulse**	1.1	**Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two spheres colliding directly.**	**The spheres may be modelled as particles.**
	1.2	Momentum as a vector. The impulse-momentum principle in vector form.	
2 **Work, energy and power**	2.1	**Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.**	**Problems involving motion under a variable resistance and/or up and down an inclined plane may be set.**
3 **Elastic strings and springs and elastic energy**	3.1	Elastic strings and springs. Hooke's law.	
	3.2	Energy stored in an elastic string or spring.	Problems using the work-energy principle involving kinetic energy, potential energy and elastic energy may be set.
4 **Elastic collisions in one dimension**	4.1	**Direct impact of elastic spheres. Newton's law of restitution. Loss of kinetic energy due to impact.**	**Students will be expected to know and use the inequalities $0 \le e \le 1$ (where e is the coefficient of restitution).** The spheres may be modelled as particles.
	4.2	**Successive direct impacts of spheres and/or a sphere with a smooth plane surface.**	**The spheres may be modelled as particles.**

*This paper is also the **Paper 4 option 4C** paper and will have the title '*Paper 4, Option 4C: Further Mechanics 1*'. *Appendix 9*, '*Entry codes for optional routes*' shows how each optional route incorporates the optional papers.

Topics	What students need to learn:		
	Content		**Guidance**
5 **Elastic collisions in two dimensions**	5.1	Oblique impact of smooth elastic spheres and a smooth sphere with a fixed surface. Loss of kinetic energy due to impact.	Problems will only involve spheres with the same radius. Problems may be set in vector form. The spheres may be modelled as particles.
	5.2	Successive oblique impacts of a sphere with smooth plane surfaces.	The sphere may be modelled as particle.

Paper 4C: Further Mechanics 2

Topics	What students need to learn:		
		Content	**Guidance**
1 **Motion in a circle**	1.1	**Angular speed. $v = r\omega$.** **Uniform motion of a particle moving in a horizontal circle.** **Radial acceleration in circular motion.** **The forms $r\omega^2$ and $\frac{v^2}{r}$ are required.**	**Problems involving the 'conical pendulum', an elastic string, motion on a banked surface, as well as other contexts, may be set.**
	1.2	Motion of a particle in a vertical circle. Radial and tangential acceleration in circular motion. Kinetic and potential energy and the conservation of energy principle applied to motion in a vertical circle.	Questions may be set which involve complete or incomplete circles.
2 **Centres of mass of plane figures**	2.1	**Moment of a force.** **Centre of mass of a discrete mass distribution in one and two dimensions.**	
	2.2	**Centre of mass of uniform plane figures, and of composite plane figures.** **Centre of mass of frameworks.** **Equilibrium of a plane lamina or framework under the action of coplanar forces.**	**The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required.** **Questions may involve non-uniform composite plane figures/frameworks.** **Figures may include the shapes referred to in the formulae book. Unless otherwise stated in the question, results given in the formulae book may be quoted without proof.**

Topics	What students need to learn:		
	Content		Guidance
3 **Further centres of mass**	3.1	Centre of mass of uniform and non-uniform rigid bodies and composite bodies.	Problems which involve the use of integration and/or symmetry to determine the centre of mass of a uniform or non-uniform body may be set. The level of calculus will be consistent with that required in A level Mathematics and Further Mathematics. Unless otherwise stated in the question, results given in the formulae book may be quoted without proof.
	3.2	Equilibrium of rigid bodies under the action of coplanar forces.	To include (i) suspension of a body from a fixed point, (ii) a rigid body placed on a horizontal or inclined plane.
	3.3	Toppling and sliding of a rigid body on a rough plane.	
4 **Further dynamics**	4.1	Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.	The solution of the resulting equations will be consistent with the level of calculus required in A level Mathematics and Further Mathematics. Problems may involve the law of gravitation, i.e. the inverse square law.
	4.2	Simple harmonic motion. Oscillations of a particle attached to the end of elastic string(s) or spring(s). Kinetic, potential and elastic energy in the context of SHM.	Proof that a particle moves with simple harmonic motion in a given situation may be required (i.e. showing that $\ddot{x} = -\omega^2 x$). Students will be expected to be familiar with standard formulae, which may be quoted without proof. e.g. $x = a\sin\omega t,\ x = a\cos\omega t,$ $v^2 = \omega^2\left(a^2 - x^2\right),\ T = \dfrac{2\pi}{\omega}$

Topics	What students need to learn:		
	Content		Guidance
5 **Further kinematics**	5.1	**Kinematics of a particle moving in a straight line when the acceleration is a function of the** displacement (x), or **time (t) or velocity (v).**	**The setting up and solution of equations where** $\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{f}(t)$ **or** $\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{f}(v)$**,** $v\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{f}(v)$ or $v\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{f}(x)$**,** $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(x)$ **or** $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(t)$ **will be consistent with the level of calculus required in A level Mathematics** and Further Mathematics.

Paper 3D: Decision Mathematics 1*

Topics	What students need to learn:		
	Content		Guidance
1 **Algorithms and graph theory**	1.1	**The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text.**	**The meaning of the order of an algorithm is expected. Students will be expected to determine the order of a given algorithm and the order of standard network problems.**
	1.2	**Bin packing, bubble sort and quick sort.**	**When using the quick sort algorithm, the pivot should be chosen as the middle item of the list.**
	1.3	**Use of the order of the nodes to determine whether a graph is Eulerian, semi-Eulerian or neither.**	**Students will be expected to be familiar with the following types of graphs: complete (including K notation), planar and isomorphic.**
	1.4	The planarity algorithm for planar graphs.	Students will be expected to be familiar with the term 'Hamiltonian cycle'.
2 **Algorithms on graphs**	2.1	**The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's algorithm.**	**Matrix representation for Prim's algorithm is expected. Drawing a network from a given matrix and writing down the matrix associated with a network may be required.**
	2.2	**Dijkstra's** and Floyd's **algorithm for finding the shortest path.**	When applying Floyd's algorithm, unless directed otherwise, students will be expected to complete the first iteration on the first row of the corresponding distance and route problems, the second iteration on the second row and so on until the algorithm is complete.
3 **Algorithms on graphs II**	3.1	**Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex (The Route Inspection Algorithm).**	**Also known as the 'Chinese postman' problem.** **Students will be expected to use inspection to consider all possible pairings of odd nodes. The network will contain at most four odd nodes.** If the network has more than four odd nodes then additional information will be provided that will restrict the number of pairings that will need to be considered.

*This paper is also the **Paper 4 option 4D** paper and will have the title '*Paper 4, Option 4D: Decision Mathematics 1*'. *Appendix 9*, '*Entry codes for optional routes*' shows how each optional route incorporates the optional papers.

Topics	What students need to learn:		
	Content		Guidance
3 **Algorithms on graphs II** *continued*	3.2	The practical and classical Travelling Salesman problems. The classical problem for complete graphs satisfying the triangle inequality.	The use of short cuts to improve upper bound is included.
		Determination of upper and lower bounds using minimum spanning tree methods.	The conversion of a network into a complete network of shortest 'distances' is included.
		The nearest neighbour algorithm.	
4 **Critical path analysis**	4.1	**Modelling of a project by an activity network, from a precedence table.**	**Activity on arc will be used. The use of dummies is included.**
	4.2	**Completion of the precedence table for a given activity network.**	**In a precedence network, precedence tables will only show immediate predecessors.**
	4.3	**Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities. Identification of critical activities and critical path(s).**	**Calculating the lower bound for the number of workers required to complete the project in the shortest possible time is required.**
	4.4	**Calculation of the total float of an activity. Construction of Gantt (cascade) charts.**	**Each activity will require only one worker.**
	4.5	Construct resource histograms (including resource levelling) based on the number of workers required to complete each activity.	The number of workers required to complete each activity of a project will be given and the number or workers required may not necessarily be one.

<table>
<tr><th rowspan="2">Topics</th><th colspan="3">What students need to learn:</th></tr>
<tr><th colspan="2">Content</th><th>Guidance</th></tr>
<tr><td>4
Critical path analysis
continued</td><td>4.6</td><td>Scheduling the activities using the least number of workers required to complete the project.</td><td></td></tr>
<tr><td rowspan="4">5
Linear programming</td><td>5.1</td><td>Formulation of problems as linear programs including the meaning and use of slack, surplus and artificial variables.</td><td>For example,
$3x+2y \leqslant 20 \Rightarrow 3x+2y+s_1=20$
$2x+5y \leqslant 35 \Rightarrow 2x+5y+s_2=35$
$x+y \geqslant 5 \Rightarrow x+y-s_3+t_1=5$
where s_1, s_2 are slack variables,
s_3 is a surplus variable and t_1 is an artificial variable.</td></tr>
<tr><td>5.2</td><td>Graphical solution of two variable problems using objective line and vertex methods including cases where integer solutions are required.</td><td></td></tr>
<tr><td>5.3</td><td>The Simplex algorithm and tableau for maximising and minimising problems with $\leq$ constraints.</td><td>Problems will be restricted to those with a maximum of four variables (excluding slack variables) and four constraints, in addition to non-negativity conditions.</td></tr>
<tr><td>5.4</td><td>The two-stage Simplex and big-M methods for maximising and minimising problems which may include both $\leq$ and $\geq$ constraints.</td><td>Problems will be restricted to those with a maximum of four variables (excluding slack, surplus and artificial variables) and four constraints, in addition to any non-negativity conditions.</td></tr>
</table>

Glossary for Decision Mathematics 1

Algorithms and graph theory

The **efficiency** of an algorithm is a measure of the 'run-time' of the algorithm and in most cases is proportional to the number of operations that must be carried out.

The **size** of a problem is a measure of its complexity and so in the case of algorithms on graphs it is likely to be the number of vertices on the graph.

The **order** of an algorithm is a measure of its efficiency as a function of the size of the problem.

In a list containing N items the 'middle' item has position $[\frac{1}{2}(N+1)]$ if N is odd $[\frac{1}{2}(N+2)]$ if N is even, so that if $N = 9$, the middle item is the 5th and if $N = 6$ it is the 4th.

A **graph** G consists of points (**vertices** or **nodes**) which are connected by lines (**edges** or **arcs**).

A **subgraph** of G is a graph, each of whose vertices belongs to G and each of whose edges belongs to G.

If a graph has a number associated with each edge (usually called its **weight**) then the graph is called a **weighted graph** or **network**.

The **degree** or **valency** of a vertex is the number of edges incident to it. A vertex is **odd** (**even**) if it has **odd** (**even**) degree.

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more then once.

A **cycle** (**circuit**) is a closed path, i.e. the end vertex of the last edge is the start vertex of the first edge.

Two vertices are **connected** if there is a path between them. A graph is **connected** if all its vertices are connected.

If the edges of a graph have a direction associated with them they are known as **directed edges** and the graph is known as a **digraph**.

A **tree** is a connected graph with no cycles.

A **spanning tree** of a graph G is a subgraph which includes all the vertices of G and is also a tree.

An **Eulerian graph** is a graph with every vertex of even degree. An **Eulerian cycle** is a cycle that includes every edge of a graph exactly once.

A **semi-Eulerian graph** is a graph with exactly two vertices of odd degree.

A **Hamiltonian cycle** is a cycle that passes through every vertex of a graph once and only once, and returns to its start vertex.

A graph that can be drawn in a plane in such a way that no two edges meet each other, except at a vertex to which they are both incident, is called a **planar graph**.

Two graphs are **isomorphic** if they have the same number of vertices and the degrees of corresponding vertices are the same.

Algorithms on graphs II

The **travelling salesman problem** is 'find a route of minimum length which visits every vertex in an undirected network'. In the '**classical**' problem, each vertex is visited once only. In the '**practical**' problem, a vertex may be revisited.

For three vertices A, B and C, the **triangular inequality** is 'length $AB \leqslant$ length AC + length CB', where AB is a longest length.

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

Critical path analysis

The **total float** $F(i,j)$ of activity (i,j) is defined to be $F(i,j) = l_j - e_i -$ duration (i,j), where e_i is the earliest time for event i and l_j is the latest time for event j.

Linear programming

Constraints of a linear programming problem will be re-written as

$$3x + 2y \leqslant 20 \Rightarrow 3x + 2y + s_1 = 20$$

$$2x + 5y \leqslant 35 \Rightarrow 2x + 5y + s_2 = 35$$

$$x + y \geqslant 5 \Rightarrow x + y - s_3 + t_1 = 5$$

where s_1, s_2 are slack variables, s_3 is a surplus variable and t_1 is an artificial variable.

The **simplex tableau** for the linear programming problem:

Maximise $P = 14x + 12y + 13z$,

Subject to $4x + 5y + 3z \leq 16$,

$5x + 4y + 6z \leq 24$,

will be written as

Basic variable	x	y	z	s_1	s_2	Value
s_1	4	5	3	1	0	16
s_2	5	4	6	0	1	24
P	–14	–12	–13	0	0	0

where S_1 and S_2 are slack variables.

Paper 4D: Decision Mathematics 2

Topics	What students need to learn:		
	Content		Guidance
1 **Transportation problems**	1.1	The north-west corner method for finding an initial basic feasible solution.	Problems will be restricted to a maximum of four sources and four destinations.
	1.2	Use of the stepping-stone method for obtaining an improved solution. Improvement indices.	The ideas of dummy locations and degeneracy are required. Students should identify a specific entering cell and a specific exiting cell.
	1.3	Formulation of the transportation problem as a linear programming problem.	
2 **Allocation (assignment) problems**	2.1	**Cost matrix reduction.**	**Students should reduce rows first.**
		Use of the Hungarian algorithm to find a least cost allocation.	**Ideas of a dummy location is required.** **The adaption of the algorithm to manage incomplete data is required.**
		Modification of the Hungarian algorithm to deal with a maximum profit allocation.	**Students should subtract all the values (in the original matrix) from the largest value (in the original matrix).**
	2.2	Formulation of the Hungarian algorithm as a linear programming problem.	
3 **Flows in networks**	3.1	**Cuts and their capacity.**	**Only networks with directed arcs will be considered.**
	3.2	**Use of the labelling procedure to augment a flow to determine the maximum flow in a network.**	**The arrow in the same direction as the arc will be used to identify the amount by which the flow along that arc can be increased. The arrow in the opposite direction will be used to identify the amount by which the flow in the arc could be reduced.**
	3.3	**Use of the max–flow min–cut theorem to prove that a flow is a maximum flow.**	
	3.4	Multiple sources and sinks. Vertices with restricted capacity.	Problems may include vertices with restricted capacity.

Topics	What students need to learn:		
	Content		Guidance
3 **Flows in networks** *continued*	3.5	Determine the optimal flow rate in a network, subject to given constraints.	Problems may include both upper and lower capacities.
4 **Dynamic programming**	4.1	Principles of dynamic programming. Bellman's principle of optimality.	Students should be aware that any part of the shortest/longest path from source to sink is itself a shortest/longest path, that is, any part of an optimal path is itself optimal.
		Stage variables and State variables. Use of tabulation to solve maximum, minimum, minimax or maximin problems.	Both network and table formats are required.
5 **Game theory**	5.1	**Two person zero-sum games and the pay-off matrix.**	**A pay-off matrix will always be written from the row player's point of view unless directed otherwise.**
	5.2	**Identification of play safe strategies and stable solutions (saddle points).**	**Students should be aware that in a zero-sum game there will be a stable solution if and only if the row maximin = the column minimax** **The proof of the stable solution theorem is not required.**
	5.3	Reduction of pay-off matrices using dominance arguments.	
	5.4	**Optimal mixed strategies for a game with no stable solution by use of graphical methods for $2 \times n$ or $n \times 2$ games where $n = 1, 2, 3$ or 4**	
	5.5	Optimal mixed strategies for a game with no stable solution by converting games to linear programming problems that can be solved by the Simplex algorithm.	

Topics	What students need to learn:		
	Content		Guidance
6 **Recurrence relations**	6.1	**Use of recurrence relations to model appropriate problems.**	
	6.2	**Solution of first** and second **order linear homogeneous and non-homogeneous recurrence relations.**	**Students should be able to solve relations such as,** $\boldsymbol{u_{n+1} - 5u_n = 8,\ u_1 = 1}$ $2u_{n+2} + 7u_{n+1} - 15u_n = 6,\ u_1 = 10,\ u_2 = -17$ **The terms, particular solution, complementary function and auxiliary equation should be known.** **Use of recurrence relations to model applications e.g. population growth**
7 **Decision analysis**	7.1	Use, construct and interpret decision trees.	Students should be familiar with the terms *decision nodes*, *chance nodes* and *pay-offs*.
	7.2	Use of expected monetary values (EMVs) and utility to compare alternative courses of action.	

Glossary for Decision Mathematics 2

Transportation problems

In the **north-west corner method,** the upper left-hand cell is considered first and as many units as possible sent by this route.

The **stepping stone method** is an iterative procedure for moving from an initial feasible solution to an optimal solution.

Degeneracy occurs in a transportation problem, with m rows and n columns, when the number of occupied cells is less than $(m + n - 1)$.

In the transportation problem:

The **shadow costs** R_i , for the ith row, and K_j , for the jth column, are obtained by solving $R_i + K_j = C_{ij}$ for **occupied cells**, taking $R_1 = 0$ arbitrarily.

The **improvement index** I_{ij} for an **unoccupied cell** is defined by $I_{ij} = C_{ij} - R_i - K_j$.

Flows in networks

A **cut**, in a network with source S and sink T, is a set of arcs (edges) whose removal separates the network into two parts X and Y, where X contains at least S and Y contains at least T. The **capacity of a cut** is the sum of the capacities of those arcs in the cut which are directed from X to Y.

If a problem contains both **upper** and **lower capacities** then the capacity of the cut is the sum of the upper capacities for arcs that cross the cut in the direction from S to T minus the sum of the lower capacities for arcs that cross the cut in the direction from T to S.

If a vertex has a **restricted capacity** then this can be replaced by two unrestricted vertices connected by an edge of the relevant capacity.

If a network has several sources S_1 , S_2 , . . ., then these can be connected to a single **supersource** S. The capacity of the edge joining S to S_1 is the sum of the capacities of the edges leaving S_1.

If a network has several sinks T_1 , T_2 , . . ., then these can be connected to a **supersink** T. The capacity of the edge joining T_1 to T is the sum of the capacities of the edges entering T_1.

Dynamic programming

Bellman's principle for dynamic programming is 'Any part of an optimal path is optimal.'

The **minimax route** is the one in which the maximum length of the arcs used is as small as possible.

The **maximin route** is the one in which the minimum length of the arcs used is as large as possible.

Game theory

A **two-person game** is one in which there are exactly two players.

A **zero-sum** game is one in which the sum of the losses for one player is equal to the sum of the gains for the other player.

Decision analysis

At a **decision node**, the user must choose the most favourable outcome.

At a **chance node**, the Expected Monetary Value is calculated.

The **end nodes** show the profit for each outcome.

Assessment information

- First assessments: May/June 2019.
- The assessments are 1 hour 30 minutes.
- The assessments are out of 75 marks.
- Students must answer all questions.
- Calculators can be used in the assessment.
- The booklet '*Mathematical Formulae and Statistical* Tables' will be provided for use in the assessments.

Synoptic assessment

Synoptic assessment requires students to work across different parts of a qualification and to show their accumulated knowledge and understanding of a topic or subject area.

Synoptic assessment enables students to show their ability to combine their skills, knowledge and understanding with breadth and depth of the subject.

These papers assesses synopticity.

Sample assessment materials

A sample paper and mark scheme for these papers can be found in the *Pearson Edexcel Level 3 Advanced GCE in Further Mathematics Sample Assessment Materials (SAMs)* document.

Assessment Objectives

Students must:		% in GCE A Level
AO1	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions	48-52%
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inferences; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts'* *(AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*	At least 15%
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; • translate situations in context into mathematical models; • Use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*	At least 15%
Total		**100%**

Further guidance on the interpretation of these assessment objectives is given in *Appendix 4*.

Breakdown of Assessment Objectives

There are ten different routes through the Advanced GCE in Further Mathematics qualification.

Route A

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Pure Mathematics 1	11.67-13.00	6.00-7.33	5.33-6.67	23–27%
Paper 4 Further Pure Mathematics 2	11.67-13.00	8.33-9.67	3.00-4.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route B

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Pure Mathematics 1	11.67-13.00	6.00-7.33	5.33-6.67	23–27%
Paper 4 Further Statistics 1	11.67-13.00	5.00-6.33	6.33-7.67	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route C

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Pure Mathematics 1	11.67-13.00	6.00-7.33	5.33-6.67	23–27%
Paper 4 Further Mechanics 1	11.67-13.00	3.00-4.33	8.67-10.00	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route D

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Pure Mathematics 1	11.67-13.00	6.00-7.33	5.33-6.67	23–27%
Paper 4 Decision Mathematics 1	11.67-13.00	7.67-9.00	3.00-4.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route E

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	**AO2 %**	**AO3 %**	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Statistics 1	11.67-13.00	5.00-6.33	6.33-7.67	23–27%
Paper 4 Further Mechanics 1	11.67-13.00	3.00-4.33	8.67 -10.00	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route F

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	**AO2 %**	**AO3 %**	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Statistics 1	11.67-13.00	5.00-6.33	6.33-7.67	23–27%
Paper 4 Decision Mathematics 1	11.67-13.00	7.67-9.00	3.00-4.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route G

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Statistics 1	11.67-13.00	5.00-6.33	6.33-7.67	23–27%
Paper 4 Further Statistics 2	11.67-13.00	7.00-8.33	3.67-5.00	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route H

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Mechanics 1	11.67-13.00	3.00-4.33	8.67-10.00	23–27%
Paper 4 Decision Mathematics 1	11.67-13.00	7.67-9.00	3.00-4.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route J

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Further Mechanics 1	11.67-13.00	3.00-4.33	8.67-10.00	23–27%
Paper 4 Further Mechanics 2	11.67-13.00	5.33-6.67	6.00-7.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

Route K

Paper	Assessment Objectives			Total for all Assessment Objectives
	AO1 %	AO2 %	AO3 %	
Paper 1 Core Pure Mathematics 1	11.67-13.00	4.67-6.00	6.67-8.00	23–27%
Paper 2 Core Pure Mathematics 2	11.67-13.00	5.67-7.00	5.33-6.67	23–27%
Paper 3 Decision Mathematics 1	11.67-13.00	7.67-9.00	3.00-4.33	23–27%
Paper 4 Decision Mathematics 2	11.67-13.00	6.33-7.67	5.00-6.33	23–27%
Total for GCE A Level	**48–52%**	**At least 15%**	**At least 15%**	**100%**

NB Totals have been rounded either up or down.

3 Administration and general information

Entries

Details of how to enter students for the examinations for this qualification can be found in our *UK Information Manual*. A copy is made available to all examinations officers and is available on our website: qualifications.pearson.com

Discount code and performance tables

Centres should be aware that students who enter for more than one GCE qualification with the same discount code will have only one of the grades they achieve counted for the purpose of the school and college performance tables. This will be the grade for the larger qualification (i.e. the A Level grade rather than the AS grade). If the qualifications are the same size, then the better grade will be counted (please see *Appendix 8*: *Codes*).

Please note that there are two codes for AS GCE qualifications; one for Key Stage 4 (KS4) performance tables and one for 16–19 performance tables. If a KS4 student achieves both a GCSE and an AS with the same discount code, the AS result will be counted over the GCSE result.

Students should be advised that if they take two GCE qualifications with the same discount code, colleges, universities and employers they wish to progress to are likely to take the view that this achievement is equivalent to only one GCE. The same view may be taken if students take two GCE qualifications that have different discount codes but have significant overlap of content. Students or their advisers who have any doubts about their subject combinations should check with the institution they wish to progress to before embarking on their programmes.

Access arrangements, reasonable adjustments, special consideration and malpractice

Equality and fairness are central to our work. Our equality policy requires all students to have equal opportunity to access our qualifications and assessments, and our qualifications to be awarded in a way that is fair to every student.

We are committed to making sure that:

- students with a protected characteristic (as defined by the Equality Act 2010) are not, when they are undertaking one of our qualifications, disadvantaged in comparison to students who do not share that characteristic
- all students achieve the recognition they deserve for undertaking a qualification and that this achievement can be compared fairly to the achievement of their peers.

Language of assessment

Assessment of this qualification will be available in English. All student work must be in English.

Access arrangements

Access arrangements are agreed before an assessment. They allow students with special educational needs, disabilities or temporary injuries to:

- access the assessment
- show what they know and can do without changing the demands of the assessment.

The intention behind an access arrangement is to meet the particular needs of an individual student with a disability, without affecting the integrity of the assessment. Access arrangements are the principal way in which awarding bodies comply with the duty under the Equality Act 2010 to make 'reasonable adjustments'.

Access arrangements should always be processed at the start of the course. Students will then know what is available and have the access arrangement(s) in place for assessment.

Reasonable adjustments

The Equality Act 2010 requires an awarding organisation to make reasonable adjustments where a person with a disability would be at a substantial disadvantage in undertaking an assessment. The awarding organisation is required to take reasonable steps to overcome that disadvantage.

A reasonable adjustment for a particular person may be unique to that individual and therefore might not be in the list of available access arrangements.

Whether an adjustment will be considered reasonable will depend on a number of factors, including:

- the needs of the student with the disability
- the effectiveness of the adjustment
- the cost of the adjustment; and
- the likely impact of the adjustment on the student with the disability and other students.

An adjustment will not be approved if it involves unreasonable costs to the awarding organisation, or affects timeframes or the security or integrity of the assessment. This is because the adjustment is not 'reasonable'.

Special consideration

Special consideration is a post-examination adjustment to a student's mark or grade to reflect temporary injury, illness or other indisposition at the time of the examination/ assessment, which has had, or is reasonably likely to have had, a material effect on a student's ability to take an assessment or demonstrate their level of attainment in an assessment.

Further information

Please see our website for further information about how to apply for access arrangements and special consideration.

For further information about access arrangements, reasonable adjustments and special consideration, please refer to the JCQ website: www.jcq.org.uk.

Malpractice

Student malpractice

Student malpractice refers to any act by a student that compromises or seeks to compromise the process of assessment or which undermines the integrity of the qualifications or the validity of results/certificates.

Student malpractice in examinations **must** be reported to Pearson using a *JCQ Form M1* (available at www.jcq.org.uk/exams-office/malpractice).

The form should be emailed to candidatemalpractice@pearson.com. Please provide as much information and supporting documentation as possible. Note that the final decision regarding appropriate sanctions lies with Pearson.

Failure to report malpractice constitutes staff or centre malpractice.

Staff/centre malpractice

Staff and centre malpractice includes both deliberate malpractice and maladministration of our qualifications. As with student malpractice, staff and centre malpractice is any act that compromises or seeks to compromise the process of assessment or which undermines the integrity of the qualifications or the validity of results/certificates.

All cases of suspected staff malpractice and maladministration **must** be reported immediately, before any investigation is undertaken by the centre, to Pearson on a *JCQ Form M2(a)* (available at www.jcq.org.uk/exams-office/malpractice).

The form, supporting documentation and as much information as possible should be emailed to pqsmalpractice@pearson.com. Note that the final decision regarding appropriate sanctions lies with Pearson.

Failure to report malpractice itself constitutes malpractice.

More detailed guidance on malpractice can be found in the latest version of the document *General and Vocational Qualifications Suspected Malpractice in Examinations and Assessments Policies and Procedures,* available at www.jcq.org.uk/exams-office/malpractice.

Awarding and reporting

This qualification will be graded, awarded and certificated to comply with the requirements of Ofqual's General Conditions of Recognition.

This A Level qualification will be graded and certificated on a six-grade scale from A* to E using the total combined marks (out of 300) for the two compulsory papers and the two optional papers chosen. The different routes within the qualification may have different grade thresholds.

Where students take more than two optional papers, the combination of papers that result in the best grade will be used. The combination of papers that is graded may not be the combination that gives the highest number of marks. The total mark will be the total from the combination that produces the best grade.

Students whose level of achievement is below the minimum judged by Pearson to be of sufficient standard to be recorded on a certificate will receive an unclassified U result.

The first certification opportunity for this qualification will be 2019.

Student recruitment and progression

Pearson follows the JCQ policy concerning recruitment to our qualifications in that:

- they must be available to anyone who is capable of reaching the required standard
- they must be free from barriers that restrict access and progression
- equal opportunities exist for all students.

Prior learning and other requirements

There are no prior learning or other requirements for this qualification.

Students who would benefit most from studying this qualification are likely to have a Level 3 GCE in Mathematics qualification.

Progression

Students can progress from this qualification to:

- a range of different, relevant academic or vocational higher education qualifications
- employment in a relevant sector
- further training.

Appendices

Appendix 1: Formulae

Formulae which students are expected to know for A Level Further Mathematics are given below and will not appear in the booklet '*Mathematical Formulae and Statistical Tables*' which will be provided for use with the paper.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) as equation $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n - 1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC:

Sine rule: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Mensuration

Circumference and area of circle, radius r and diameter d:

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' theorem:

In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse, $c^2 = a^2 + b^2$

Area of a trapezium = $\frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis.

Exponential Form: $e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix $\mathbf{AB}$ is the transformation represented by matrix $\mathbf{B}$ followed by the transformation represented by matrix $\mathbf{A}$.

For matrices $\mathbf{A}, \mathbf{B}$:

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Algebra

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$ with roots α and β:

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$$\sum \alpha = -\frac{b}{a} \qquad \sum \alpha\beta = \frac{c}{a} \qquad \alpha\beta\gamma = -\frac{d}{a}$$

Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$\sinh x \equiv \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

Calculus and Differential Equations

<u>Differentiation</u>

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
$\cosh kx$	$\frac{1}{k}\sinh kx+c$
$\sinh kx$	$\frac{1}{k}\cosh kx+c$
e^{kx}	$\frac{1}{k}\mathrm{e}^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$\mathrm{f}'(x)+\mathrm{g}'(x)$	$\mathrm{f}(x)+\mathrm{g}(x)+c$
$\mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)$	$\mathrm{f}(\mathrm{g}(x))+c$

Area under a curve $=\int_a^b y\ \mathrm{d}x\ \ (y\geqslant 0)$

Volumes of revolution about the x and y axes:

$$V_x=\pi\int_a^b y^2\mathrm{d}x \qquad V_y=\pi\int_c^d x^2\mathrm{d}y$$

Simple Harmonic Motion:

$$\ddot{x}=-\omega^2 x$$

Vectors

$$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the acute angle between the vectors $\mathbf{a}$ and $\mathbf{b}$

The equation of the line through the point with position vector $\mathbf{a}$ parallel to vector $\mathbf{b}$ is:

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

The equation of the plane containing the point with position vector $\mathbf{a}$ and perpendicular to vector $\mathbf{n}$ is:

$(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

Statistics

The mean of a set of data: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{\sum \mathrm{f}x}{\sum \mathrm{f}}$

The standard Normal variable: $Z = \dfrac{X - \mu}{\sigma}$ where $X \sim \mathrm{N}(\mu, \sigma^2)$

Mechanics

Forces and Equilibrium

Weight = mass $\times g$

Friction: $\mathrm{F} \le \mu R$

Newton's second law in the form: $\mathrm{F} = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt} \qquad a = \frac{dv}{dx} = \frac{d^2 r}{dt^2}$$

$$r = \int v \, \mathrm{d}t \qquad v = \int a \, \mathrm{d}t$$

Momentum $= mv$

Impulse $= mv - mu$

Kinetic energy $= \frac{1}{2} mv^2$

Potential energy $= mgh$

The tension in an elastic string $= \dfrac{\lambda x}{l}$

The energy stored in an elastic string $= \dfrac{\lambda x^2}{2l}$

For Simple harmonic motion:

$$\ddot{x} = -\omega^2 x,$$

$$x = a \cos \omega t \ \text{ or } \ x = a \sin \omega t,$$

$$v^2 = \omega^2(a^2 - x^2),$$

$$T = \frac{2\pi}{\omega}$$

Appendix 2: Notation

The tables below set out the notation used in A Level Further Mathematics examinations. Students will be expected to understand this notation without need for further explanation.

1	Set Notation	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
1.22	$(a, b]$	the interval $\{\{x \in \mathbb{R} : a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1.24	$\mathbb{C}$	the set of complex numbers

2	Miscellaneous Symbols	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

3	Operations	
3.1	$a+b$	a plus b
3.2	$a-b$	a minus b
3.3	$a\times b,\ ab,\ a\cdot b$	a multiplied by b
3.4	$a\div b,\ \dfrac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1+a_2+\ldots+a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1\times a_2\times\ldots\times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a\rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n\times(n-1)\times\ldots\times 2\times 1$, $n\in\mathbb{N}$; $0! = 1$
3.10	$\binom{n}{r},\ {}^{n}C_r,\ {}_{n}C_r$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n, r\in\mathbb{Z}_0^+, r\leqslant n$ or $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n\in\mathbb{Q}, r\in\mathbb{Z}_0^+$

4	Functions	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x\mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x)=\mathrm{g}(\mathrm{f}(x))$
4.5	$\lim\limits_{x\to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x,\ \delta x$	an increment of x
4.7	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x

4	Functions *continued*	
4.9	$f'(x),\ f''(x),\ \ldots,\ f^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $f(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y\ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y\ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

5	Exponential and Logarithmic Functions	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x,\quad \log_{\mathrm{e}} x$	natural logarithm of x

6	Trigonometric Functions	
6.1	sin, cos, tan, cosec, sec, cot	the trigonometric functions
6.2	$\left.\begin{array}{l}\sin^{-1},\ \cos^{-1},\ \tan^{-1}\\ \arcsin,\ \arccos,\ \arctan\end{array}\right\}$	the inverse trigonometric functions
6.3	°	degrees
6.4	rad	radians
6.5	$\left.\begin{array}{l}\mathrm{cosec}^{-1},\ \sec^{-1},\ \cot^{-1}\\ \mathrm{arccosec},\ \mathrm{arcsec},\ \mathrm{arccot}\end{array}\right\}$	the inverse trigonometric functions
6.6	$\left.\begin{array}{l}\sinh,\ \cosh,\ \tanh,\\ \mathrm{cosech},\ \mathrm{sech},\ \coth\end{array}\right\}$	the hyperbolic functions
6.7	$\left.\begin{array}{l}\sinh^{-1},\ \cosh^{-1},\ \tanh^{-1},\\ \mathrm{cosech}^{-1},\ \mathrm{sech}^{-1},\ \coth^{-1}\end{array}\right\}$ $\left.\begin{array}{l}\mathrm{arsinh},\ \mathrm{arcosh},\ \mathrm{artanh},\\ \mathrm{arcosech},\ \mathrm{arcsech},\ \mathrm{arcoth}\end{array}\right\}$	the inverse hyperbolic functions

7	Complex Numbers	
7.1	i, j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	the real part of z, $\mathrm{Re}(z) = x$
7.6	$\mathrm{Im}(z)$	the imaginary part of z, $\mathrm{Im}(z) = y$
7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta$, $-\pi < \theta \leq \pi$
7.9	z^*	the complex conjugate of z, $x - \mathrm{i}y$

8	Matrices	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	$\det \mathbf{M}$ or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	$\mathbf{Mr}$	Image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$

9	Vectors	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment **AB**
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert \mathbf{a} \rvert$, a	the magnitude of $\mathbf{a}$
9.6	$\lvert \overrightarrow{AB} \rvert$, AB	the magnitude of $\overrightarrow{AB}$

9	Vectors *continued*	
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i}+b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector
9.12	$\mathbf{a.b}$	the scalar product of **a** and **b**
9.13	$\mathbf{a}\times\mathbf{b}$	the vector product of **a** and **b**
9.14	$\mathbf{a.b}\times\mathbf{c}$	the scalar triple product of **a**, **b** and **c**

10	Differential Equations	
10.1	ω	angular speed

11	Probability and Statistics	
11.1	A, B, C, etc.	events
11.2	$\mathrm{A}\cup\mathrm{B}$	union of the events A and B
11.3	$\mathrm{A}\cap\mathrm{B}$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A\mid B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X=x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X

11	Probability and Statistics *continued*	
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim \mathrm{N}(0,1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0, 1)$
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\overline{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population
11.32	$\mathrm{Po}(\lambda)$	Poisson distribution with parameter λ
11.33	$\mathrm{Geo}(p)$	geometric distribution with parameter p
11.34	$\mathrm{G}_X(t)$	probability generating function of the random variable X
11.35	χ_v^2	chi squared distribution with v degrees of freedom
11.36	t_n	t distribution with n degrees of freedom
11.37	F_{v_1,v_2}	F distribution with v_1 and v_2 degrees of freedom

12	Mechanics	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, $\mathrm{m\,s^{-1}}$	metres per second (velocity)
12.5	$\mathrm{m/s^2}$, $\mathrm{m\,s^{-2}}$	metres per second per second (acceleration)
12.6	F	Force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

Appendix 3: Use of calculators

Students may use a calculator in all A Level Further Mathematics examinations. Students are responsible for making sure that their calculators meet the guidelines set out in this appendix.

The use of technology permeates the study of A level Further mathematics. Calculators used **must** include the following features:

- an iterative function
- the ability to perform calculations with matrices up to at least order 3×3
- the ability to compute summary statistics and access probabilities from standard statistical distributions

In addition, students **must** be told these regulations before sitting an examination:

<table>
<tr>
<td>Calculators must be:
• of a size suitable for use on the desk;
• either battery or solar powered;
• free of lids, cases and covers which have printed instructions or formulas.</td>
<td rowspan="2">Calculators must not:
• be designed or adapted to offer any of these facilities:
o language translators;
o symbolic algebra manipulation;
o symbolic differentiation or integration;
o communication with other machines or the internet;
• be borrowed from another student during an examination for any reason;*
• have retrievable information stored in them – this includes:
o databanks;
o dictionaries;
o mathematical formulas;
o text.</td>
</tr>
<tr>
<td>The student is responsible for the following:
• the calculator’s power supply;
• the calculator’s working condition;
• clearing anything stored in the calculator.</td>
</tr>
</table>

Advice: *An invigilator may give a student a replacement calculator

Appendix 4: Assessment objectives

The following tables outline in detail the strands and elements of each assessment objective for A Level Further Mathematics, as provided by Ofqual in the document *GCE Subject Level Guidance for Further Mathematics.*

- A 'strand' is a discrete bullet point that is formally part of an assessment objective
- An 'element' is an ability that the assessment objective does not formally separate, but that could be discretely targeted or credited.

Assessment Objectives 2 and 3 contain further detail which can be found on page 48 (the italicised text).

AO1: Use and apply standard techniques. ***50% (A Level) 60% (AS)***

Learners should be able to:

- **select and correctly carry out routine procedures**
- **accurately recall facts, terminology and definitions**

Strands	Elements
1. select and correctly carry out routine procedures	1a – select routine procedures
	1b – correctly carry out routine procedures
2. accurately recall facts, terminology and definitions	This strand is a single element

AO2: Reason, interpret and communicate mathematically ***At least 15% (A Level) At least 10% (AS)***

Learners should be able to:

- **construct rigorous mathematical arguments (including proofs)**
- **make deductions and inferences**
- **assess the validity of mathematical arguments**
- **explain their reasoning**
- **use mathematical language and notation correctly**

Strands	Elements
1. construct rigorous mathematical arguments (including proofs)	This strand is a single element
2. make deductions and inferences	2a – make deductions
	2b – make inferences
3. assess the validity of mathematical arguments	This strand is a single element
4. explain their reasoning	This strand is a single element
5. use mathematical language and notation correctly	This strand is a single element

AO3: Solve problems within mathematics and in other contexts Learners should be able to: ▪ translate problems in mathematical and non-mathematical contexts into mathematical processes ▪ interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations ▪ translate situations in context into mathematical models ▪ use mathematical models ▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them	*At least 15% (A Level)* *At least 10% (AS)*
Strands	**Elements**
1. translate problems in mathematical and non-mathematical contexts into mathematical processes	1a – translate problems in mathematical contexts into mathematical processes
	1b – translate problems in non-mathematical contexts into mathematical processes
2. interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations	2a – interpret solutions to problems in their original context
	2b – where appropriate, evaluation the accuracy and limitations of solutions to problems
3. translate situations in context into mathematical models	This strand is a single element
4. use mathematical models	This strand is a single element
5. evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them	5a – evaluate the outcomes of modelling in context
	5b – recognise the limitations of models
	5c – where appropriate, explain how to refine models

Assessment objectives coverage

There will be full coverage of all elements of the assessment objectives, with the exceptions of AO3.2b and AO3.5c, in each set of A Level Further Mathematics assessments offered by Pearson. Elements AO3.2b and AO3.5c will be covered in each route through the qualification within three years.

Appendix 5: The context for the development of this qualification

All our qualifications are designed to meet our World Class Qualification Principles[1] and our ambition to put the student at the heart of everything we do.

We have developed and designed this qualification by:

- reviewing other curricula and qualifications to ensure that it is comparable with those taken in high-performing jurisdictions overseas
- consulting with key stakeholders on content and assessment, including learned bodies, subject associations, higher-education academics, teachers and employers to ensure this qualification is suitable for a UK context
- reviewing the legacy qualification and building on its positive attributes.

This qualification has also been developed to meet criteria stipulated by Ofqual in their documents *GCE Qualification Level Conditions and Requirements* and *GCE Subject Level Conditions and Requirements for* Further Mathematics published in April 2016.

[1] Pearson's World Class Qualification Principles ensure that our qualifications are:

- **demanding**, through internationally benchmarked standards, encouraging deep learning and measuring higher-order skills
- **rigorous**, through setting and maintaining standards over time, developing reliable and valid assessment tasks and processes, and generating confidence in end users of the knowledge, skills and competencies of certified students
- **inclusive**, through conceptualising learning as continuous, recognising that students develop at different rates and have different learning needs, and focusing on progression
- **empowering**, through promoting the development of transferable skills, see *Appendix 6*.

From Pearson's Expert Panel for World Class Qualifications

May 2014

" The reform of the qualifications system in England is a profoundly important change to the education system. Teachers need to know that the new qualifications will assist them in helping their learners make progress in their lives.

When these changes were first proposed we were approached by Pearson to join an 'Expert Panel' that would advise them on the development of the new qualifications.

We were chosen, either because of our expertise in the UK education system, or because of our experience in reforming qualifications in other systems around the world as diverse as Singapore, Hong Kong, Australia and a number of countries across Europe.

We have guided Pearson through what we judge to be a rigorous qualification development process that has included:

- extensive international comparability of subject content against the highest-performing jurisdictions in the world
- benchmarking assessments against UK and overseas providers to ensure that they are at the right level of demand
- establishing External Subject Advisory Groups, drawing on independent subject-specific expertise to challenge and validate our qualifications
- subjecting the final qualifications to scrutiny against the DfE content and Ofqual accreditation criteria in advance of submission.

Importantly, we have worked to ensure that the content and learning is future oriented. The design has been guided by what is called an 'Efficacy Framework', meaning learner outcomes have been at the heart of this development throughout.

We understand that ultimately it is excellent teaching that is the key factor to a learner's success in education. As a result of our work as a panel we are confident that we have supported the development of qualifications that are outstanding for their coherence, thoroughness and attention to detail and can be regarded as representing world-class best practice. "

Sir Michael Barber (Chair)

Chief Education Advisor, Pearson plc

Bahram Bekhradnia

President, Higher Education Policy Institute

Dame Sally Coates

Principal, Burlington Danes Academy

Professor Robin Coningham

Pro-Vice Chancellor, University of Durham

Dr Peter Hill

Former Chief Executive ACARA

Professor Lee Sing Kong

Director, National Institute of Education, Singapore

Professor Jonathan Osborne

Stanford University

Professor Dr Ursula Renold

Federal Institute of Technology, Switzerland

Professor Bob Schwartz

Harvard Graduate School of Education

All titles correct as at May 2014

Appendix 6: Transferable skills

The need for transferable skills

In recent years, higher education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.

The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as 'the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully and consistently perform an activity or task and can be built upon and extended through learning.' [1]

To support the design of our qualifications, the Pearson Research Team selected and evaluated seven global 21st-century skills frameworks. Following on from this process, we identified the National Research Council's (NRC) framework as the most evidence-based and robust skills framework. We adapted the framework slightly to include the Program for International Student Assessment (PISA) ICT Literacy and Collaborative Problem Solving (CPS) Skills.

The adapted National Research Council's framework of skills involves: [2]

Cognitive skills

- **Non-routine problem solving** – expert thinking, metacognition, creativity.
- **Systems thinking** – decision making and reasoning.
- **Critical thinking** – definitions of critical thinking are broad and usually involve general cognitive skills such as analysing, synthesising and reasoning skills.
- **ICT literacy** – access, manage, integrate, evaluate, construct and communicate. [3]

Interpersonal skills

- **Communication** – active listening, oral communication, written communication, assertive communication and non-verbal communication.
- **Relationship-building skills** – teamwork, trust, intercultural sensitivity, service orientation, self-presentation, social influence, conflict resolution and negotiation.
- **Collaborative problem solving** – establishing and maintaining shared understanding, taking appropriate action, establishing and maintaining team organisation.

Intrapersonal skills

- **Adaptability** – ability and willingness to cope with the uncertain, handling work stress, adapting to different personalities, communication styles and cultures, and physical adaptability to various indoor and outdoor work environments.
- **Self-management and self-development** – ability to work remotely in virtual teams, work autonomously, be self-motivating and self-monitoring, willing and able to acquire new information and skills related to work.

Transferable skills enable young people to face the demands of further and higher education, as well as the demands of the workplace, and are important in the teaching and learning of this qualification. We will provide teaching and learning materials, developed with stakeholders, to support our qualifications.

---

[1] OECD – *Better Skills, Better Jobs, Better Lives* (OECD Publishing, 2012)

[2] Koenig J A, National Research Council – *Assessing 21st Century Skills: Summary of a Workshop* (National Academies Press, 2011)

[3] PISA – *The PISA Framework for Assessment of ICT Literacy* (2011)

Appendix 7: Level 3 Extended Project qualification

What is the Extended Project?

The Extended Project is a standalone qualification that can be taken alongside GCEs. It supports the development of independent learning skills and helps to prepare students for their next step – whether that be higher education or employment. The qualification:

- is recognised by higher education for the skills it develops
- is worth half of an Advanced GCE qualification at grades A*–E
- carries UCAS points for university entry.

The Extended Project encourages students to develop skills in the following areas: research, critical thinking, extended writing and project management. Students identify and agree a topic area of their choice for in-depth study (which may or may not be related to a GCE subject they are already studying), guided by their teacher.

Students can choose from one of four approaches to produce:

- a dissertation (for example an investigation based on predominately secondary research)
- an investigation/field study (for example a practical experiment)
- a performance (for example in music, drama or sport)
- an artefact (for example creating a sculpture in response to a client brief or solving an engineering problem).

The qualification is coursework based and students are assessed on the skills of managing, planning and evaluating their project. Students will research their topic, develop skills to review and evaluate the information, and then present the final outcome of their project.

The Extended Project has 120 guided learning hours (GLH) consisting of a 40-GLH taught element that includes teaching the technical skills (for example research skills) and an 80-GLH guided element that includes mentoring students through the project work. The qualification is 100% internally assessed and externally moderated.

How to link the Extended Project with further mathematics

The Extended Project creates the opportunity to develop transferable skills for progression to higher education and to the workplace, through the exploration of either an area of personal interest or a topic of interest from within the mathematics qualification content.

Through the Extended Project, students can develop skills that support their study of mathematics, including:

- conducting, organising and using research
- independent reading in the subject area
- planning, project management and time management
- defining a hypothesis to be tested in investigations or developing a design brief
- collecting, handling and interpreting data and evidence
- evaluating arguments and processes, including arguments in favour of alternative interpretations of data and evaluation of experimental methodology
- critical thinking.

In the context of the Extended Project, critical thinking refers to the ability to identify and develop arguments for a point of view or hypothesis and to consider and respond to alternative arguments.

Types of Extended Project related to further mathematics

Students may produce a dissertation on any topic that can be researched and argued. In mathematics this might involve working on a substantial statistical project or a project which requires the use of mathematical modelling.

Projects can give students the opportunity to develop mathematical skills which can't be adequately assessed in exam questions.

- ***Statistics*** – students can have the opportunity to plan a statistical enquiry project, use different methods of sampling and data collection, use statistical software packages to process and investigate large quantities of data and review results to decide if more data is needed.
- ***Mathematical modelling*** – students can have the opportunity to choose modelling assumptions, compare with experimental data to assess the appropriateness of their assumptions and refine their modelling assumptions until they get the required accuracy of results.

Using the Extended Project to support breadth and depth

In the Extended Project, students are assessed on the quality of the work they produce and the skills they develop and demonstrate through completing this work. Students should demonstrate that they have extended themselves in some significant way beyond what they have been studying in further mathematics. Students can demonstrate extension in one or more dimensions:

- ***deepening understanding*** – where a student explores a topic in greater depth than in the specification content. This could be an in-depth exploration of one of the topics in the specification
- ***broadening skills*** – where a student learns a new skill. This might involve learning the skills in statistics or mathematical modelling mentioned above or learning a new mathematical process and its practical uses.
- ***widening perspectives*** – where the student's project spans different subjects. Projects in a variety of subjects need to be supported by data and statistical analysis. Students studying mathematics with design and technology can do design projects involving the need to model a situation mathematically in planning their design.

A wide range of information to support the delivery and assessment of the Extended Project, including the specification, teacher guidance for all aspects, an editable scheme of work and exemplars for all four approaches, can be found on our website.

Appendix 8: Codes

Type of code	Use of code	Code
Discount codes	Every qualification eligible for performance tables is assigned a discount code indicating the subject area to which it belongs. Discount codes are published by the DfE.	Please see the GOV.UK website*
Regulated Qualifications Framework (RQF) codes	Each qualification title is allocated an Ofqual Regulated Qualifications Framework (RQF) code. The RQF code is known as a Qualification Number (QN). This is the code that features in the DfE Section 96 and on the LARA as being eligible for 16–18 and 19+ funding, and is to be used for all qualification funding purposes. The QN will appear on students' final certification documentation.	The QN for this qualification is: 603/1499/0
Subject codes	The subject code is used by centres to enter students for a qualification. Centres will need to use the entry codes only when claiming students' qualifications.	9FM0
Paper codes	These codes are provided for reference purposes. Students do not need to be entered for individual papers.	Paper 1: 9FM0/01 Paper 2: 9FM0/02 Optional Papers: 9FM0/3A-3D 9FM0/4A-4D

*www.gov.uk/government/publications/2018-performance-tables-discount-code

Appendix 9: Entry codes for optional routes

All students must take the two Core Pure Papers 9FM0 01 and 9FM0 02 and **at least** two of the optional papers from 9FM0 3A-3D and 9FM0/4A-4D to be award the A level in Further Mathematics.

Mandatory Core Pure Papers		Optional Papers
9FM0 01: Core Pure Mathematics 1	9FM0 02: Core Pure Mathematics 2	9FM0 3A: Further Pure Mathematics 1
		9FM0 3B: Further Statistics 1
		9FM0 3C: Further Mechanics 1
		9FM0 3D: Decision Mathematics 1
		9FM0 4A: Further Pure Mathematics 2
		9FM0 4B: Further Statistics 2
		9FM0 4C: Further Mechanics 2
		9FM0 4D: Decision Mathematics 1

Students must be entered for 9FM0 along with the entry code for the optional route that they wish to take. The entry codes for each optional route are provided in the *UK Information Manual*.

For students taking more than the two optional papers (see page 4 'Content and assessment overview'), entry codes are provided for each combination of options they may wish to take.

TM23/06/21 9781446956489_GCE2017_AL_FURMATHS_ISSUE 3.DOC.1-92/0

Edexcel, BTEC and LCCI qualifications

Edexcel, BTEC and LCCI qualifications are awarded by Pearson, the UK's largest awarding body offering academic and vocational qualifications that are globally recognised and benchmarked. For further information, please visit our qualifications website at qualifications.pearson.com. Alternatively, you can get in touch with us using the details on our contact us page at qualifications.pearson.com/contactus

About Pearson

Pearson is the world's leading learning company, with 22,500 employees in more than 70 countries working to help people of all ages to make measurable progress in their lives through learning. We put the learner at the centre of everything we do, because wherever learning flourishes, so do people. Find out more about how we can help you and your learners at qualifications.pearson.com

This specification is Issue 3. We will inform centres of any changes to this issue. The latest issue can be found on the Pearson website: qualifications.pearson.com

References to third party material made in this specification are made in good faith. Pearson does not endorse, approve or accept responsibility for the content of materials, which may be subject to change, or any opinions expressed therein. (Material may include textbooks, journals, magazines and other publications and websites.)

All information in this specification is correct at time of publication.

Original origami artwork: Mark Bolitho
Origami photography: Pearson Education Ltd/Naki Kouyioumtzis

ISBN 978 1 446 95648 9

For information about Pearson Qualifications, including Pearson Edexcel, BTEC and LCCI qualifications visit qualifications.pearson.com

Edexcel and BTEC are registered trademarks of Pearson Education Limited

Pearson Education Limited. Registered in England and Wales No. 872828
Registered Office: 80 Strand, London WC2R 0RL

VAT Reg No GB 278 537121

Pearson
Edexcel

SBN 978-1-4469-5648-9