NAME:

Date to be handed in:

MARK (out of 100):

| Qu | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|
|    |   |   |   |   |   |   |   |   |   |    |    |    |    |

# Mathematics Advanced Subsidiary Paper 1: Pure Mathematics Practice Paper D: Time 2 hours You must have: Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

# **Questions to revise:**

# PAPER D

 $f(x) = x^3 - 3x - 2$ .

The figure below shows a sketch of part of the curve with equation y = f(x).



(a) On a separate set of axes, sketch the curve with equation y = f(2x) showing the location and coordinates of the images of points A, B, C and D.

(2)

(b) On a separate set of axes, sketch the curve with equation y = f(-x) showing the location and coordinates of the images of points *A*, *B*, *C* and *D*.

(2)

(Total 4 marks)

2. Find 
$$\int (5-3\sqrt{x})^2 dx$$
.

1.

(Total 5 marks)

3. Solve algebraically, showing each step of your working, the equation

 $(8^{x-1})^2 - 18(8^{x-1}) + 32 = 0.$ 

(Total 5 marks)

4 A buoy is a device which floats on the surface of the sea and moves up and down as waves pass.

For a certain buoy, its height, above its position in still water, y in metres, is modelled by a sine function of the form  $y = \frac{1}{2} \sin 180t^\circ$ , where t is the time in seconds.

(a) Sketch a graph showing the height of the buoy above its still water level for  $0 \le t \le 10$  showing the coordinates of points of intersection with the *t*-axis.

(3)

(b) Write down the number of times the buoy is 0.4 m above its still water position during the first 10 seconds.

(1)

(c) Give one reason why this model might not be realistic.

(1)

### (Total 5 marks)

$$f(x) = x^3 - 4x^2 - 35x + 20.$$

Find the set of values of x for which f(x) is increasing.

(Total 5 marks)

6. The speed,  $v \text{ ms}^{-1}$ , of a rollercoaster at time *t* s is given by  $v(t) = \frac{1}{20} (50 \sqrt{t} + 20t^2 - t^3)$ , where  $0 \le t \le 20$ .

The distance, s m, travelled by the rollercoaster in the first 20 s is given by  $s = \int_{0}^{20} v(t) dt$ .

Find the value of *s*, giving your answer to 3 significant figures.

### (Total 5 marks)

7.

$$f(x) = x^2 - (k+8)x + (8k+1).$$

(a) Find the discriminant of f(x) in terms of k giving your answer as a simplified quadratic.

(3)

(2)

(3)

- (b) If the equation f(x) = 0 has two equal roots, find the possible values of k.
- (c) Show that when k = 8, f(x) > 0 for all values of x.

### (Total 8 marks)

- 8. The equations of two circles are  $x^2 + 10x + y^2 12y = 3$  and  $x^2 6x + y^2 2qy = 9$ .
  - (a) Find the centre and radius of each circle, giving your answers in terms of q where necessary.
  - (b) Given that the distance between the centres of the circles is  $\sqrt{80}$ , find the two possible values of q.

(3)

(6)

### (Total 9 marks)

- 9. The graph of  $y = ab^x$  passes through the points (2, 400) and (5, 50).
  - (a) Find the values of the constants a and b.

(b) Given that  $ab^x < k$ , for some constant k > 0, show that  $x > \frac{\log\left(\frac{1600}{k}\right)}{\log 2}$ , where log means log to any valid base.

(4)

(5)

### (Total 9 marks)

- 10. (a) Calculate the value of  $-2 \tan(-120^\circ)$ .
  - (b) On the same set of axes sketch the graphs of  $y = 2 \sin (x 60^{\circ})$  and  $y = -2 \tan x$ , in the interval  $-180^{\circ} \le x \le 180^{\circ}$ , showing the coordinates of points of intersection with the coordinate axes in exact form.
    - (7)

(1)

(c) Explain how you can use the graph to identify solutions to the equations

$$y = 2\sin(x - 60^\circ) + 2\tan x = 0$$
 in the interval  $-180^\circ \le x \le 180^\circ$ . (1)

(d) Write down the number of solutions of the equation

$$y = 2\sin(x - 60^\circ) + 2\tan x = 0$$
 in the interval  $-180^\circ \le x \le 180^\circ$ .

(1)

11. A curve C has equation  $y = x^3 - x^2 - x + 2$ .

The point *P* has *x*-coordinate 2.

(a) Find 
$$\frac{dy}{dx}$$
 in terms of x.

(2)

(4)

(4)

(b) Find the equation of the tangent to the curve C at the point P.

The normal to C at P intersects the x-axis at A.

(c) Find the coordinates of A.

(Total 10 marks)

 $f(x) = x^3 + x^2 + px + q$ , where *p* and *q* are constants.

Given that f(5) = 0 and f(-3) = 8,

12.

- (a) find the values of p and q,
- (b) factorise f(x) completely.

(5)

(7)

(Total 12 marks)

# **13.** OACB is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ .

The points M, S, N and T divide OB, BC, CA and AO in the ratio 1 : 4 respectively.

The lines ST and MN intersect at the point D.



| ( <i>a</i> ) | Express $\overrightarrow{MN}$ in terms of <b>a</b> and <b>b</b> . | (2) |
|--------------|---|-----|
| ( <i>b</i> ) | Express $\overrightarrow{ST}$ in terms of <b>a</b> and <b>b</b> . | (2) |
| (c)          | Show that the lines <i>MN</i> and <i>ST</i> bisect one another.   | (9) |

(Total 13 marks)

## END OF PAPER (TOTAL: 100 MARKS)